Application of selected methods of multi-criteria decision-making in the evaluation of company performance

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Abstract

Selected decomposition methods of multi-criteria decision making are applied in the paper. The aim of the article is to identify and verify key evaluation indicators for evaluating the performance of companies using multi-criteria decomposition methods. Selected indicators include indicators of profitability, solvency, liquidity and activity. A multi-criteria method of AHP decomposition (analytical hierarchy process) based on Saaty's approach of pair comparison and WSM (weighted sum method) are described. The described methods are then used not only to determine the preferences of indicators for evaluating the company's performance. Our findings reveal the resulting preferences of individual indicators of the company's performance evaluation, key evaluation indicators and the evaluation of the company's performance itself.

Key words

Decomposition methods, multi-criteria decision making, preference, indicators, evaluation of company performance

JEL Classification: C02, C4, G2, G11

1. Introduction

The existence and functioning of a company, including its management, is linked to decision-making. This decision is usually influenced by a number of factors, criteria. Multi-criteria decision-making is one of options how to choose the optimal variant of a certain sets of variants (Fotr, Dědina, Hrůzová, 2010), (Brožová, Houška, Šubrt, 2014), (Ramík 1999), (Saaty, 1980), (Raju, Kumar, 2014). Only very rarely it is possible to find the optimal variant which meets all specified criteria best. The solution of decision-making problem is more often a compromise variant, which meets just the most important criteria and does not meet all the specified criteria best. More and more frequently it is necessary to deal with problems when the solution variants should be assessed using a larger number of evaluation criteria (Saaty, 2006), (Ishizaka, Nemery, 2013), (Doumpos, Zopounidis, 2014). Such decision-making problems then have the character of multi-criteria decision-making.

The aim of the article is to determine and verify the key performance assessment indicators of companies by applying the decomposition multi-criteria methods. Among the assessed indicators there are the indicators of profitability, solvency, liquidity and activity. The decomposition multi-criteria AHP method (analytic hierarchy process) based on the Saaty pair comparison approach are described, including the computation procedure, and WSM

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(weighted sum method). The described methods are then applied to determine the preferences of the indicators and the evaluation of company performance.

Indicators influencing the company's performance are considered selected indicators of profitability, indebtedness, liquidity and activity. He deals with them in her work (Dluhošová, 2010), (Richtarová, Ptáčková, 2019).

2. Methodology

The aim of the application of the multi-criteria decision making evaluation of variants is primary finding the best (optimal) variant and layout of these variants from the best to the worst. The best option is usually a variant of the compromise. The compromise solution is the least distant one from the ideal variant, or the furthest away from the variants of basal, while the ideal option is the one that has all the criteria with the best possible value. On the contrary, variant with the worst values of the criteria is the basal variant. Ideal and basal variants are usually hypothetical. If the ideal variant really existed, it would be at the same time, a variant of the optimal solution. However, this situation usually does not occur and therefore any selected solution is the solution to the compromise. Compromise variant must be undominated in all tasks, which means that there is no dominating variant among decision-making variants (Ramík, 1999).

Alternatives are specified by using variants and the measurement of satisfaction depends on each variant. Determination of the criteria is difficult process, which requires certain knowledge of the area. The criteria used to selection of the most appropriate variants can be classified according to several aspect. Firstly it's possible to divide criteria as maximizing (income, profit) or minimizing (cost, loss) according to the level of desirable values. According to the type Secondly it is possible to divide criteria into qualitative and quantitative. These are expressed in the units of measurement.

For calculations and comparison it is usually desirable for specified criteria values yij to be normalized the unit interval, i.e. $x_{ij} \in [0;1]$. Generally, it is possible to obtain these values of the criteria from the sub-functions of the utility (value) as $x_{ij} = u(y_{ij})$. Utility of the criteria, which acquire the worst values is equal to 0 or close to 0, and the utility of the criteria with the best value is equal to 1.

2.1 Multi-attribute methods AHP and ANP

Saaty method AHP and ANP will be used in the application part of the study, therefore the following description will be focused on these methods.

The Saaty's method of weights determination of the criteria can be divided into two steps. The first step consists of a pairwise comparison when finding the preferential relations of criteria pairs. It is presented as so-called Saaty's matrix S. This matrix is symmetric with elements sij. It is possible to determine also the size of this preference expressed by a certain number of points from the selected point scale in addition to the direction of the preference of pair of criteria. Scale of relative importance (descriptors) was recommended by Saaty and it is shown in Table 1. Other values can be used to express sub-preferences. The strength of preferences is expressed in the interval $s_{i,j} \in [0;9]$. The result of this step is to obtain the right upper triangular part of the matrix size preferences (Saaty's matrix). The diagonal element have to be $s_{i,j} = 1$ and for the inverse elements (in the lower left triangular part of matrix) is true the following:

$$s_{i,j} = \frac{1}{s_{j,i}}. \tag{1}$$

The elements $s_{i,j}$ Saaty matrix are estimated shares of weights of criteria v_i and v_j, so:

$$s_{i,j} \cong \frac{v_i}{v_j}. \tag{2}$$

The scales can be obtained in the following manner:

$$\min F = \sum_{i}^{n}\sum_{j}^{n}\left(s_{i,j} - \frac{v_i}{v_j}\right)^2, \tag{3}$$

with the condition $\sum_{i}^{n} v_i = 1.$

Because of difficulty it is possible to obtain the weights using an algorithm based on the geometric average.

$$\min F = \sum_{i=1}^{n}\sum_{j>i}^{n}\left[\ln s_{i,j} - (\ln v_i - \ln v_j)\right]^2, \tag{4}$$

with the condition $\sum_{i}^{n} v_i = 1.$

The final solution is based on the geometric mean of rows (Saaty, 2010):

$$w_i = \frac{v_i}{\sum_{i}^{N} v_i} = \frac{\left[\prod_{j}^{N} s_{i,j}\right]^{\frac{1}{N}}}{\sum_{i}^{N}\left[\prod_{j}^{N} s_{i,j}\right]^{\frac{1}{N}}}, \tag{5}$$

Table 1: Recommended point of scale with the descriptors by Saaty

The number of points	**Descriptor**
1	Element A and B are equally important
3	Element A is moderately more important than element B
5	Element A is strongly more important than element B
7	Element A is very strongly more important than element B
9	Element A is extremely more important than element B

Source: Saaty (2006), authors´own processing

The sign of relevant evaluation is the consistency of Saaty's matrix, in other words when the elements satisfy the condition of transitivity the most. It should be emphasized that in many methods this aspect is not accounted. Consistency can be measured using the coefficient of consistency CR (Consistency Ratio). The coefficient for consistent evaluation should be $CR \leq 0{,}1$ (Saaty, 2012). Consistency ratio is calculated as following $CR = \frac{CI}{RI}$, where $CI = \frac{\lambda_{\max} - N}{N-1}$, (Saaty, 2010), (Zmeškal, 2009). The characteristic number of the matrix $\lambda_{\max}$ can be determined by various procedures. One option is $\lambda_{\max} = \frac{1}{N}\sum_{i}^{N}(S \cdot w)_i / w_i$, while w is a vector and $(S \cdot w)_i$ is the *i*-th element of the vector. Furtherly RI (Random Index) is derived from an empirical examination and reaches the following values depending on the number of criteria, see in Table 2.

Table 2: The value RI according to the number of criteria

N	1	2	3	4	5	6	7	8	9	10
RI	0.00	0.00	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49

Source: Saaty (2009), authors´ own processing

Weights or values of criteria are in the case of decomposition tasks set by gradual decomposition from the goal, global groups of criteria, sub-groups, to the the initial sub-criteria and variants. For AHP method these linkages may be linear and for ANP method in the shape of a pyramid or nonlinear with feedbacks. Evaluation of preferences (weights) of the criteria is carried out using the Saaty's method of pairwise comparison.

Local weights (preferences) of the subgroups or indicators with regard to the specified purpose are determined by using Saaty's method of pairwise comparison. The next step is calculation of the global weights including the initial sub-weights. The sum of all sub-weights is equal to one.

In AHP method can be used analytical procedure and also method of supermatrix. In the ANP, it is possible to calculate global weights by using only the method of supermatrix (Saaty, 2010).

For analytical method AHP the indicator subgroup weights are obtained as follows, $w_{i,j} = v_i \cdot v_{i,j}$ where $w_{i,j}$ is global weight of *j*-th indicator and *i*-th group, v_i is local weight of *i*-th group and $v_{i,j}$ is local weight of *j*-th indicator and *i*-th group. By this way we can gradually get all the global weights of primary indicators (Saaty, 2012), (Zmeškal, Dluhošová, Tichý, 2013).

2.2 Weighted sum method

The weighted sum method (WSM) requires cardinal information, a criterion matrix *Y* and a vector of criteria weights. It constructs an overall rating for each variant, so it can be used both to find one of the most advantageous variants and to arrange the variants from best to worst. The weighted sum method is a special case of the utility function method. It is based on the principle of maximizing utility. If the variant a_1 reaches a certain value y_{ij} according to the criterion *j*, it thus brings the user a benefit which can be expressed by means of a linear utility function. The total utility of a variant is expressed by the weighted sum of the values of the partial utility functions

$$u(a_i) = \sum_{i=1}^{m} v_j\, u_j(y_{ij}), \qquad (6)$$

where u_j are the partial functions of the utility of the individual criteria and v_j are the weights of the criteria. The procedure of the weighted sum method is given by the following steps.

We convert minimization criteria to maximization criteria, for example, according to the relation (7) and we thus receive an evaluation for each variant by how much it is better than the worst variant according to the relevant criterion. For simplicity, we will always denote the transformed criterion matrix Y. This adjustment is not necessary, it serves to simplify the next step.

$$y_{ij} = \max_{i=1,\dots,m}\left(y_{ij}\right) - y_{ij}. \qquad (7)$$

Then we determine the ideal variant H with evaluation (h_1, ..., h_m) and the basal variant D with evaluation (d_1, ..., d_n). Next, we create a standardized criterion matrix *R*, the elements of which we obtain using formula (8). The matrix R already represents a matrix of values of the utility function from the *i-th* variant according to the *j-th* criterion, because the elements of this matrix are linearly transformed criterion values such that $r_{ij} \in \langle 0; 1 \rangle$. Then the basal variant corresponds to a value of zero and the ideal variant to a value of one.

$$r_{ij} = \frac{y_{ij} - d_j}{h_j - d_j} \tag{8}$$

For individual variants, we calculate the aggregate utility function according to formula (9). We then sort the variants in descending order according to the values of $u(a_i)$ and consider the required number of variants with the highest values as a solution to the problem.

$$u(a_i) = \sum_{j=1}^{n} v_j r_j. \tag{9}$$

3. Data

The database for verification and performance assessment of company consists of indicators in the period from 2014 to 2018. Selected standard indicators of performance assessment of company: indicators of profitability, solvency, liquidity and activity, see Table 3.

Table 3: Simple Usage Matrix - Input values of business performance

Indicators	2014	2015	2016	2017	2018
Return on assets (ROA)	15,15	11,03	15,29	11,34	11,92
Return on equity (ROE)	28,41	14,83	23,44	15,62	17,01
Return on capital employed (ROCE)	26,86	15,80	22,04	15,42	16,82
Return on sales (ROS)	9,38	5,78	7,37	6,07	6,97
Number of days of assets	222,88	188,84	173,58	192,82	210,39
Number of days of inventory	8,78	8,82	7,87	8,54	10,36
Number of days of receivable	64,85	68,37	48,82	46,85	65,38
Number of days of payable	91,18	62,74	62,07	59,01	67,24
Debt ratio	51,77	43,64	47,45	41,45	43,74
Equity ratio	47,98	56,27	52,51	58,52	56,21
Degree of coverage of long-term assets	246,95	302,62	290,79	308,09	359,33
Interest coverage	23,51	56,18	116,83	179,41	119,53
Interest load	4,25	1,78	0,86	0,56	0,84
Current liquidity	2,22	3,72	3,81	4,64	4,43
Quick liquidity	2,1	3,48	3,57	4,36	4,15
Cash liquidity	1,21	1,64	2,08	2,8	2,35

4. Results

The first step in meeting the set goal is to determine the weights of the indicators. The weights of groups of indicators and subsequently the weights of individual selected indicators of company performance evaluation are determined. Based on the values of preferences, key indicators are selected. These will be half of the selected performance evaluation indicators of the company with higher preferences.

Table 4: Comparison of individual groups of indicators with respect to the goal (AHP)

	R	S	L	A		geomean	weights w	S.w	$(S.w)_i/w_i$
R	1	2	3	4		2,2134	**0,4668**	1,9562	4,1905
S	1/2	1	2	3		1,1892	**0,2508**	1,0573	4,2154
L	1/3	1/2	1	2		0,9306	**0,1963**	0,8134	4,1442
A	1/4	1/3	1/2	1		0,4082	**0,0861**	0,3518	4,0862
						4,7414	**1,0000**	l_{max}=	4,1591
						RI=	0,9000	CI=	0,0530
						N=	4,0000	CR=CI/RI	**0,0589**

Table 5: Comparison of indicators with regard to profitability (AHP)

	R_1	R_2	R_3	R_4		geomean	weights w	S.w	$(S.w)_i/w_i$
R_1	1	1/2	2	3		1,1892	**0,2508**	1,0573	4,2154
R_2	2	1	3	4		2,2134	**0,4668**	1,9562	4,1905
R_3	1/3	1/2	1	2		0,9306	**0,1963**	0,8134	4,1442
R_4	1/4	1/3	1/2	1		0,4082	**0,0861**	0,3518	4,0862
						4,7414	**1,0000**	l_{max}=	4,1591
						RI=	0,9000	CI=	0,0530
						N=	4,0000	CR=CI/RI	**0,0589**

Table 6: Comparison of indicators with regard to solvency (AHP)

	S_1	S_2	S_3	S_4	S_5		geomean	weights w	S.w	$(S.w)_i/w_i$
S_1	1	1	2	3	3		1,9757	**0,3256**	1,7586	5,4012
S_2	1	1	2	3	3		1,9757	**0,3256**	1,7586	5,4012
S_3	1/2	1/2	1	2	2		1,0000	**0,1648**	0,8361	5,0735
S_4	1/3	1/3	1/2	1	1		0,5587	**0,0920**	0,4608	5,0092
S_5	1/3	1/3	1/2	1	1		0,5587	**0,0920**	0,4608	5,0092
							6,0688	**1,0000**	l_{max}=	5,1789
							RI=	1,1200	CI=	0,0447
							N=	5,0000	CR=CI/RI	**0,0399**

Table 7: Comparison of indicators with regard to liquidity (AHP)

	L_1	L_2	L_3		geomean	weights w	S.w	$(S.w)_i/w_i$
L_1	1	2	3		1,8171	**0,5396**	1,6238	3,0092
L_2	1/2	1	2		1,0000	**0,297**	0,8936	3,0092
L_3	1/3	1/2	1		0,5503	**0,1634**	0,4918	3,0092
					3,3674	**1,0000**	l_{max}=	3,0092
					RI=	0,5800	CI=	0,0046
					N=	3,0000	CR=CI/RI	**0,0079**

Table 8: Comparison of indicators with regard to activity (AHP)

	A_1	A_2	A_3	A_4		geomean	weights w	S.w	$(S.w)_i/w_i$
A_1	1	1/2	1/3	1/3		0,4831	**0,1085**	0,4196	3,8673
A_2	2	1	1/2	1/2		0,8409	**0,1887**	0,7652	4,0551
A_3	3	2	1	1		1,5651	**0,3514**	1,4598	4,1542
A_4	3	2	1	1		1,5651	**0,3514**	1,4598	4,1542
						4,4542	**1,0000**	l_{max}=	4,0577

RI=	0,9000	CI=	0,0192
N=	4,0000	CR=CI/RI	**0,0213**

Table 9: The initial supermatrix (AHP) = The weighted supermatrix (AHP)

	Goal	R	S	L	A	R_1	R_2	R_3	R_4	S_1	S_2	S_3	S_4	S_5	L_1	L_2	L_3	A_1	A_2	A_3	A_4
Goal	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
R	0,4668	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S	0,2508	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
L	0,1963	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0,0861	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
R_1	0	0,2508	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
R_2	0	0,4668	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
R_3	0	0,1963	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
R_4	0	0,0861	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
S_1	0	0	0,3256	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
S_2	0	0	0,3256	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
S_3	0	0	0,1648	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
S_4	0	0	0,0920	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
S_5	0	0	0,0920	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
L_1	0	0	0	0,5396	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
L_2	0	0	0	0,2970	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
L_3	0	0	0	0,1634	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
A_1	0	0	0	0	0,1085	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
A_2	0	0	0	0	0,1887	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
A_3	0	0	0	0	0,3514	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
A_4	0	0	0	0	0,3514	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
$\sum$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Table 10: The limit supermatrix (AHP)

	Goal	R	S	L	A	R_1	R_2	R_3	R_4	S_1	S_2	S_3	S_4	S_5	L_1	L_2	L_3	A_1	A_2	A_3	A_4
Goal	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
R	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
L	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
R_1	0,1171	0,2508	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
R_2	0,2179	0,4668	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
R_3	0,0916	0,1963	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
R_4	0,0402	0,0861	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
S_1	0,0817	0	0,3256	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
S_2	0,0817	0	0,3256	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
S_3	0,0413	0	0,1648	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
S_4	0,0231	0	0,0920	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
S_5	0,0231	0	0,0920	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
L_1	0,1059	0	0	0,5396	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
L_2	0,0583	0	0	0,2970	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
L_3	0,0321	0	0	0,1634	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
A_1	0,0093	0	0	0	0,1085	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
A_2	0,0162	0	0	0	0,1887	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
A_3	0,0303	0	0	0	0,3514	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
A_4	0,0303	0	0	0	0,3514	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
$\sum$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Table 11: Weights of indicators

Elements	Local weights	Group´s weights	Global weights - analytic	Global weights - metrix
Goal			AHP	AHP
R	46,68 %	100,00 %		
S	25,08 %			
L	19,63 %			
A	8,61 %			
R_1	25,08 %	46,48 %	11,71 %	11,71 %
R_2	46,68 %		21,79 %	21,79 %
R_3	19,63 %		9,16 %	9,16 %
R_4	8,61 %		4,02 %	4,02 %
S_1	32,56 %	25,08 %	8,17 %	8,17 %
S_2	32,56 %		8,17 %	8.17 %
S_3	16,48 %		4,13 %	4,13 %
S_4	9,20 %		2,31 %	2,31 %
S_5	9,20 %		2,31 %	2,31 %
L_1	53,96 %	19,63 %	10,59 %	10,59 %
L_2	29,70 %		5,83 %	5,83 %
L_3	16,34 %		3,21 %	3,21 %
A_1	10,85 %	8,61 %	0,93 %	0,93 %
A_2	18,87 %		1,62 %	1,62 %
A_3	35,14 %		3,03 %	3,03 %
A_4	35,14 %		3,03 %	3,03 %
∑			100,00 %	100,00 %

Given the values of the identified preferences, calculated weights, indicators of evaluation of the company's performance, the key indicators are the indicators listed in Table 12 in the first eight positions.

Table 12: Order of indicators by weight

Ranking	Elements	Indicators	Global weights
1.	R_2	ROE	21,79 %
2.	R_1	ROA	11,71 %
3.	L_1	Current liquidity	10,59 %
4.	R_3	ROCE	9,16 %
5.	S_1	Debt ratio	8,17 %
6.	S_2	Equity ratio	8,17 %
7.	L_2	Quick liquidity	5,83 %
8.	S_3	Degree of coverage of long-term assets	4,13 %
9.	R_4	ROS	4,02 %
10.	L_3	Cash liquidity	3,21 %
11.	A_3	Number of days of receivable	3,03 %
12.	A_4	Number of days of payable	3,03 %
13.	S_4	Interest coverage	2,31 %
14.	S_5	Interest load	2,31 %
15.	A_2	Number of days of inventory	1,62 %
16.	A_1	Number of days of assets	0,93 %

The weighted sum method will now be used. Using its application, the values of the usefulness of individual variants will be determined, ie the years in which the company achieves certain performance values. The aim is to find out in which year the company's financial performance, measured by key indicators, is the best.

Table 13: Criteria matrix - input values of company performance

Indicators	2014	2015	2016	2017	2018
R_2	28,41	14,83	23,44	15,62	17,01
R_1	15,15	11,03	15,29	11,34	11,92
L_1	2,22	3,72	3,81	4,64	4,43
R_3	26,86	15,80	22,04	15,42	16,82
S_1	47,98	56,27	52,51	58,52	56,21
S_2	51,77	43,64	47,45	41,45	43,74
L_2	2,1	3,48	3,57	4,36	4,15
S_3	246,95	302,62	290,79	308,09	359,33

Table 14: Modified criteria matrix for the condition of maximizing criteria

Indicators	2014	2015	2016	2017	2018	H	D
R_2	28,41	14,83	23,44	15,62	17,01	28,41	14,83
R_1	15,15	11,03	15,29	11,34	11,92	15,29	11,03
L_1	2,42	0,92	0,83	0	0,21	2,42	0
R_3	26,86	15,80	22,04	15,42	16,82	26,86	15,42
S_1	47,98	56,27	52,51	58,52	56,21	58,52	47,98
S_2	0	8,13	4,32	10,32	8,03	10,32	0
L_2	2,26	0,88	0,79	0	0,21	2,26	0
S_3	246,95	302,62	290,79	308,09	359,33	359,33	246,95

Table 15: Standardized criterion matrix

Indicators	weights w	2014	2015	2016	2017	2018
R_2	0,2179	1	0	0,6340	0,0582	0,1605
R_1	0,1171	0,9671	0	1	0,0728	0,2089
L_1	0,1059	1	0,3802	0,3430	0	0,0868
R_3	0,0916	1	0,0332	0,5787	0	0,1224
S_1	0,0817	0	0,7865	0,4298	1	0,7808
S_2	0,0817	0	0,7878	0,4186	1	0,7781
L_2	0,0583	1	0,3894	0,3496	0	0,0929
S_3	0,0413	0	0,4954	0,3901	0,5440	1

Table 16: The order of variants according to the weighted sum method

	2014	2015	2016	2017	2018
Utility	0,5869	0,2151	0,4504	0,2071	0,2539
Ranking	1	4	2	5	3

5. Conclusions

The aim of the article was to identify and verify key indicators for evaluating the company's performance using multi-criteria decomposition methods. Among the evaluated indicators were indicators of profitability, solvency, liquidity and activity. A multicriteria method of AHP decomposition (analytical hierarchy process) based on Saaty's pairwise comparison approach was described, including the calculation procedure and WSM (weighted sum method). The described methods were then used to determine the preferences of indicators and evaluate the company's performance.

The key indicators were ROE, ROA, Current liquidity, ROCE, Debt ratio, Equity ratio, Quick liquidity and Degree of coverage of long-term assets.

The values of these indicators, as achieved by the company in the years 2014 - 2018, were subsequently assessed using the WSM method and the period with the best performance of the company according to the monitored indicators was determined. This revealed the year 2014.

Acknowledgments

This paper was supported by the SGS Project VSB – TU Ostrava SP2020/124 „Finanční modelování a rozhodování podniků a finančních institucí za rizika, flexibility a interakce".

References

[1] Borovcová, M. (2019). Determination and Verification of the Key Assessment Indicators for the Insurance Market by Applying the Decomposition Multi-attribute Methods and Regression Analysis. In: *Financial Assets and Investing*, Vol 10, No 1 (2019). Brno: Masaryk University.

[2] Brožová, H., Houška, M. a Šubrt T. (2014). *Modely pro vícekriteriální rozhodování*. Praha: ČZU v Praze, Provozně ekonomická fakulta.

[3] Dluhošová, D. (2010). Finanční řízení a rozhodování podniku. Praha: Ekopress.

[4] Doumpos, M. and Zopounidis, C. (2014). *Multicriteria Analysis in Finance*. Chania: Technical University of Crete, Springer.

[5] Fotr, J., Dědina, J. a Hrůzová, H. (2010). *Manažerské rozhodování*. 2. aktualiz. vyd. Praha: Ekopress.

[6] Ishizaka, A. and Nemery P. (2013). *Multi-Criteria Decision Analysis*. Chichester, West Sussex: Wiley.

[7] Raju, K.S. and Kumar, D.N. (2014). *Multicriterion Analysis in Engineering and Management.* New Delhi: PHI.

[8] Ramík, J. (1999). *Vícekriteriální rozhodování – analytický hierarchický process (AHP).* 1st ed. Karviná: Slezská univerzita v Opavě.

[9] Richtarová, D. and Ptáčková, B. (2019). Analysis of Economic Value Added development of Manufacturing Industry: the case in the Czech Republic. In*: 12th Internation Scientific Conference Financial Management of Firms and Financial Institutions.* Ostrava: VSB-TU Ostrava.

[10] Saaty, T. L. (1980). *The analytic hierarchy process: planning, setting priorities, resource allocation.* New York, McGraw-Hill International Book Co.

[11] Saaty, T. L. (2012). *Decision making form leaders: the analytic hierarchy process for decisions in a complex world*. 3rd rev. ed. Pittsburgh: RWS Publications.

[12] Saaty, T. L. (2006). *Fundamentals of decision making and priority theory with the analytic hierarchy process*. 1st ed. Pittsburgh: RWS Publications.

[13] Saaty, T. L. (2010). *Mathematical principles of decision making: generalization of the analytic network process to neural firing and synthesis*. 1st ed. Pittsburgh: RWS Publications.

[14] Zmeškal, Z., Dluhošová, D. a Tichý T. (2013). *Finanční modely: koncepty, metody, aplikace.* 3. přeprac. a rozšíř. vyd. Praha: Ekopress.

[15] Zmeškal, Z. (2009). Vícekriteriální hodnocení variant a analýza citlivosti při výběru produktů finančních institucí. Financial management of firms and financial institutions. 1. vyd. Ostrava: VŠB – Technická univerzita Ostrava.

Impact of global conventional monetary policy on the stock price indices worldwide[1]

Jiří Branžovský, Josef Novotný[2]

Abstract

The research aim of this paper examines the influence of the conventional monetary policy of top five central banks globally to the equity indices across the globe. It was the US monetary policy-maker Federal Reserve System, further followed by European Central Bank, Bank of England, Bank of Japan and Swiss National Bank that were taken into consideration. This research were about to analyse the relationship between interest rates changes, economic cycle dummy variable and their impact on 19 stock indices between 3 Jan 2000 and 30 Jun 2020 with about 5 300 daily observations. The author´s thoughts were whether it is only the US Fed impacting the equity markets globally, or other systemic central banks too. As a method, vector error correction model (VECM) was selected to analyse the long-term relationship among variables. This long-run relationship was concluded only for the BSE, SSE and Nikkei indices. Another stochastic model vector autoregressive (VAR) was used in the short-term for all the observed countries. For US, British and Swiss MP had been identified mostly positive relationship between interest rates and equity returns, whilst as for EU and Japanese MP this dependence was oppositely negative.

Key words

Vector autoregressive model, vector error correction model, cointegration, conventional monetary policy, interest rates, stock indices.

JEL Classification: C01, C32, E44, G10

1 Introduction

The research in this paper focuses on the impact of the interest rate changes of top five systemic central banks the Federal Reserve System (further as "Fed"), European Central Bank (ECB), Bank of England (BoE), Bank of Japan (BoJ) and Swiss National Bank (SNB) on 19 stock price indices globally. International Monetary Fund actually considers six global currencies to be the reserve currency, or systemic important ones, with People´s Bank of China included, but China had not been observed due to its real lack of transparency.

Stock markets are considered to be not only a driver of the real economy, but also a predictor of the future. The current approach and understanding is that monetary variables influence the real economy through several channels, e.g. liquidity channel, wealth channel or asset price channel. Proposed transmission channel of interest rates does affect equity market

[1] This paper has been elaborated in the framework of the grant programme „Support for Science and Research in the Moravia-Silesia Region 2018" (RRC/10/2018), financed from the budget of the Moravian-Silesian Region. This article has been also supported by SGS grant from VSB – Technical University of Ostrava no. SP2020/116 (project 'Economic policy challenges in developed countries').

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returns as follows: lower interest rates and expansionary forward guidance lead to greater corporate profits because of the cheaper credit links and loosen credit activities of the banks that result after all to greater investment capacity and households credit consumption (interest rate & credit channels) and vice versa. Additionally, lower rates represent alternatively cheaper discount rates effectively increasing present values of future cash flows (asset prices channel).

The research was run daily since 3 January 2000 up to 30 June 2020.

The contribution of this paper is at estimation of the long-term and short-term impact of interest rate changes on the national stock price indices globally. Such as evidence compatible with monetary theories would be beneficial not only for academic and banking researchers, but investors too.

2 Literature review

In the second half of 20th century there had been many papers written with the core focus aimed at monetary variables, especially the conventional monetary policy with interest rates in front while analyzing impact of their changes on the money markets on the stock markets.

Haitsma, Unalmis and Haan (2016) found an significant impact of ECB monetary policy on the stock prices in the European union and United kingdom. Very much the comparable results were obtained by Fausch and Sigonius (2017) specifically focusing on the German stock market. Balafas, Florackis and Kostakis (2018) examined influence of monetary policy of BoE to the UK equity market. Eksi & Tas (2017) concluded via VAR model that in the Post-Recession period since 2008 with limited MP left but unconventional policy, MPC meetings became seven times more effective than before the Great Recession. All the papers found out that stock markets reacted oppositely during periods of economic booms and crises as the result of lack in investors´ confidence. Feldman (2017) tested an impact of interest rates on the stock price returns in emerging markets via Monte Carlo simulations.

In the world the central banks got the big monetary decision-making power especially in the third millennium, even though their staff is the one never being publically voted, oppositely to the fiscal policy makers.

3 Methodology

All the time series used in this paper were firstly stationarised (de-trended). Generally, time series´ stationarity is a character usual for a low-volatile stocks with relatively stable mean value and nearly no trend representing an autocorrelation of residuals then. Stationarity may be apparently visible from the chart, but from some statistical methods a correlogram and unit root tests can be used – in this paper it is Augmented Dickey-Fuller (ADF) test. Stationarisation of time series not in levels is usually processed by natural logarithmisation or by first order differentiation, or by both parallel. Non-stationary time series may suffer from the spurious regression, and hence stationary variables are a must assumption for VAR model. Nevertheless, the long-run model Vector Error Correction Model (VECM) solving an issue with cointegration does not require stationarity of the data.

Specifically, instead of using US federal fund rates (FFR) changes, the author had decided to use an alternative approach of estimating MP shocks based on changes in target FFR and changes in effective FFR (EFFR).

$$MP\ shock_{1,t} = (FFR_t - FFR_{t-1}) - (EFFR_t - FFR_t) \quad (1)$$

Monetary shock identification is based on the logic, what proportion of target FFR change was predicted one day ahead (difference between target and effective FFR).

Any negative shock is considered to be an expansionary shock as market participants are significantly lowering their interest rate expectations, at least more than been expected previously, enhancing more spending and investing. Oppositely a positive shock is having a restrictive power slowing the economy down.

3.1 VAR model

Vector autoregressive (VAR) model is an example of econometric model used for analysing, simulating and predicting linear interdependencies among several time series. Each of the variable is treated equally once as an exogenous, and once as endogenous variable. VAR as a model with many equations is due to potential cointegration among them considered to be the short-run model.

A *p*-order VAR model, described as a VAR(*p*) is a set of *k* variables and their *p* time-lagged values. The specific model for endogenous variable explained by its own time lags and one another variable could be as follows:

$$y_t = A_1 y_{t-1} + \cdots + A_p y_{t-p} + Bx_t + \varepsilon_t$$

where y_t are *k* observed endogenous variables, *A* are the parameters´ vectors of endogenous variables parameters and their time lagged values, *B* are the exogenous variables parameters and possibly their own time lagged values and ε_t are normally distributed standard errors and *p* is the maximum time lag. The optimum maximum time of lags to be used is set up mainly by several information criteria.

Tests of distribution, homoscedasticity and autocorrelation of residuals were also run.

3.2 VEC model

Some of the explanatory power of time series models is misled due to the spurious regression as most of financial variables keep drifting together side by side. But even non-stationary financial time series may be still linear. These cointegrating relationships of time series are so called to be cointegrated and their long-run linear relationship may be estimated in the cointegrating equation of VEC model through ordinary least squares (OLS). Cuthbertson et al. (1993, p. 129) concluded that VECM maked economic models based on theories to be economically more correct when estimating whether variables are indeed influencing / cointegrating each other. VEC model is effective with all the variables non-stationary in levels, but also integrated of higher orders – taking the highest one *I(max(x1, x2, ...))*. Once the cointegration test is run, and time series identified to be cointegrated, there may be an evidence of any long-run relationship among the variables that could be translated as sort of potential short-term shock oscillating around and converging to the long-run equilibrium.

$$\Delta y_t = \prod (y_{t-1} + X_{t-1}) + \sum_{i=1}^{p-1} \Gamma_i (\Delta y_{t-i} + \Delta X_{t-i}) + \varepsilon_t$$

where Δy_t are the differences of non-stationary endogenous variables, x_t are non-stationary deterministic variables and ε_t are normally distributed standard errors and *p* as a maximum time lag. VECM is summarized by adjustment coefficient of cointegrating vector and its parameters based on lagged raw time series, then of short-run coefficients linked to the lagged differenced variables, error and finally by a possible intercept.

$$\prod = \sum_{i=1}^{p} A_i - I \qquad \Gamma_i = -\sum_{j=i+1}^{p} A_j$$

where Π is a matrix of parameters being a result of multiplication of two *k x r* matrices α and β^T under condition of $r < k$ and $\beta^T y_t$ being stationary in levels, where *r* represents the reduced cointegration rank and β cointegrating vector. This coefficient matrix, as well as reduced rank, are both tested by the Johansen method from an unrestricted VAR model.

Even if any deterministic trend is evident, it is natural that just as time series may have means and trends, cointegrating equations may have their intercepts and deterministic trends too. Accordingly, there are two types of columns – cointegrating column and outside column depending on whether the deterministic variables do or do not appear inside the cointegrating relations.

As trends are typical for financial time series, only deterministic trends cases with linear trends (case 3 and 4) of five Johansen´s were considered.

Case 3 for linear trend of the level data only:

$$H_1(r): \prod y_{t-1} + \mathrm{B}x_t = \alpha\left(\beta' y_{t-1} + \rho_0\right) + \alpha_{\perp}\gamma_0$$

Case 4 for linear trends of both the level data and the cointegrating equations:

$$H^*(r): \prod y_{t-1} + \mathrm{B}x_t = \alpha\left(\beta' y_{t-1} + \rho_0 + \rho_1 t\right) + \alpha_{\perp}\gamma_0$$

Where $\alpha_{\perp}$ represents the deterministic term outside of the cointegrating relation (rank).

Mostly known Engle-Granger cointegration test is used for VEC models with one cointegrating equation model while another Johansen cointegration test is suitable even for those with several equations. Based on their amount and statistical significance, there are two types of test statistics to be run: *trace* statistics and *maximum eigenvalues* statistics. Estimation of cointegrating relations *r* in their first columns is run in the order from *0* up to *k–1* until the rejection of the null hypothesis. The second columns involve the estimates of the ordered eigenvalues of the matrix Π. Trace statistics for testing the null hypothesis of *r* cointegrating ranks is calculated as below:

$$LR_{tr}(r|k) = -T\sum_{i=r+1}^{k} \ln(1-\widehat{\lambda_i})$$

where $\widehat{\lambda_i}$ represents *i*-th largest eigenvalue of the matrix. The equation for setting up maximum eigenvalues statistic for testing alternative hypothesis of *r+1* cointegrating relations can be found below. Estimation of the cointegrating vector β coefficients and its adjustment coefficients follows. Identification of the cointegrating vector is based on the normalisation $\beta S_{11}\beta^T = I$

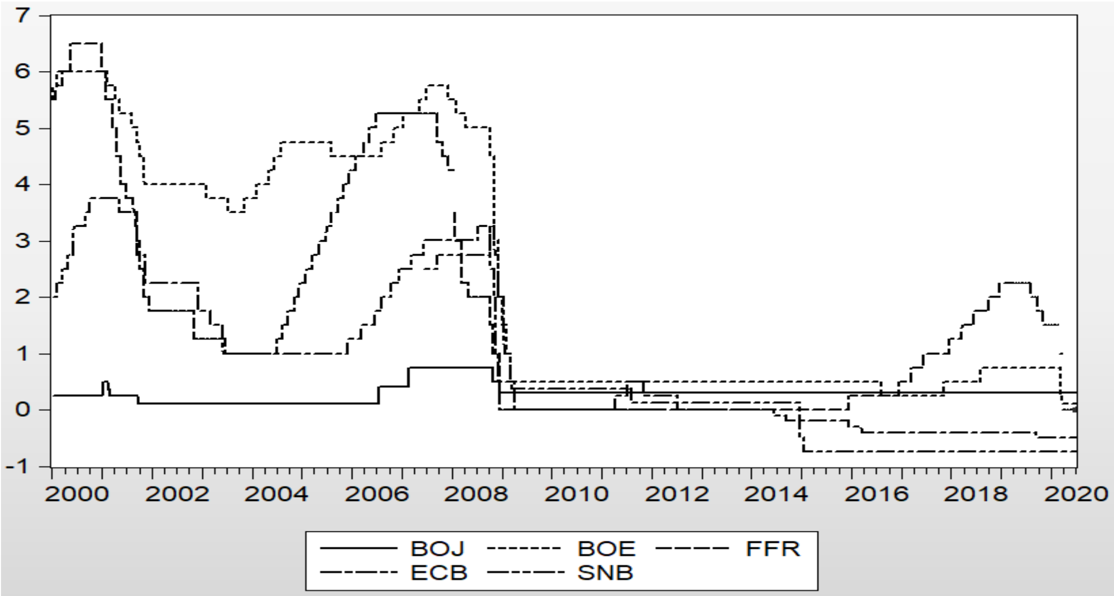
$$LR_{tr}(r|r+1) = -T\ln(1-\widehat{\lambda_{r+1}}) = LR_{tr}(r|k) - LR_{tr}(r+1|k)$$

For more details about residual tests see Branzovsky (2018).

4 Data

The author´s research was observed over the daily data from 3 January 2000 to 30 June 2020 representing over 5 300 observations per each time serie. As exogenous variables were selected interest rate changes of ECB, BoE, BoJ, SNB and as for the US Fed, the author made his own calculation of expansive and restrictive monetary policy shocks on the daily basis.

Chart 1: Basic discount rates in top five central banks globally



Source: in percentage points [eViews]

All interest rates were first – differenced (e.g. *D_ECB*).

As for endogenous variables were picked nineteen randomly selected stock price indices of which most of them are market capitalisation-weighted (except of DJIA and Nikkei).

Table 1: Stock indices selected

Variable	Variable´ name	Currency	Target country	Stock Exchange	# of stocks
ASX	S&P/ASX 200 Index	USD	Australia	Australian SE	200
BOVESPA	Ibovespa Brasil Sao Paulo SE	BRL	Latin America	Sao Paulo SE	50
BSE	BSE SENSEX	INR	India	Bombay SE	30
CAC	CAC 40 Index	EUR	France	Euronext Paris	40
DAX	DAX 30 Index	EUR	Germany	Deutsche Boerse	30
DJIA	Dow Jones Industrial Average	USD	USA	NYSE	30
Euronext	Euronext 100 Index	EUR	EU	Euronext Paris	100
FTSE	FTSE 250 Index	GBP	UK	London SE	250
HSI	Hang Seng Index	HKD	Hong Kong	Hong Kong SE	51
IPC	S&P/BMV IPC Index	MXN	Mexico	Mexican SE	35
ISEQ	iSeq All Share	EUR	Ireland	Euronext Dublin	All
JCI	Jakarta Composite Index	IDR	Indonesia	Indonesia SE	All
KOSPI	KOSPI Composite Index	KRW	South Korea	Korea SE	776
MIB	Italy FTSE MIB Index	EUR	Italy	Borsa Italiana	40
Nikkei	Nikkei 225 Index	JPY	Japan	Borsa Italiana	225
PX	PX index	CZK	Czech Republic	Prague SE	13
RTS	RTS Index	RUB	Russia	Moscow SE	45
SPX	S&P 500 Index	USD	USA	NYSE	505
SSE	SSE Composite Index	CNY	Shanghai	Shanghai SE	All

Source: own table [Excel]

The historical data was downloaded from the Finance Yahoo agency.

Stock market indices were natural – logarithmed into stationary series (e.g. *L_PX*) and differenced at first levels as well into obtaining their daily returns (e.g. *DL_PX*). For VAR model, all time series must be stationary at *I(1)* and hence not only naturally logarithmed, but first – differenced too. Impact of different foreign currencies, as well as of inflation, over the stated period of time was ignored.

We involved also a DUMMY variable of economic conditions – zero´s except of the ones representing business cycles of crises (as pronounced by NBER, 2020). The models were as follows:

$$DL\_INDEX_t = A_1\, DL\_INDEX_{t-1} + A_2\, MP\_shock_{(-1;0;1)} + A_3\, D\_ECB_{(-1;0)} + A_4\, D\_BOE_{(-1;0)} + A_5\, D\_BOJ_{(-1;0)} + A_6\, D\_SNB_{(-1;0)} + DUMMY + \varepsilon_t$$

The research focuses on how particular interest rate policies of top central banks have an impact on the particular stock indices returns. Economically, interest rates should be negatively affecting the stock returns.

5 Results

ADF test was run in order to address non-stationarity at 10% significance level[3]. All stock markets were recognised as trendy, non-stationary, having a unit root.

Regarding to VEC models, the raw non-stationary variables *X* were used, only being naturally logarithmed into *L_X*. And as for VAR models, all variables were stationarised by first differentiation of naturally logarithms (*DL_X*) as time series are required to be stationarity of the first order *I(1)*. See short-run VAR models initially, and then let be verified the potential cointegration among regressors.

To specifically point out, the focus of this research was not on how discount rates (representing sort of risk free rates or money market yields) have been affecting stock returns. This study was aimed at how changes in base interest rates do influence stock returns. The reason behind is that due to the fact that all five interest rates in developed countries are pretty much correlated which would cause the spurious regression due to their multicollinearity as these "independent" variables are truly dependent on each other.

Regarding to the **lag order** for models, there was one time lag selected for central banks changes in interest rates pointing to how stock markets react today and tomorrow post the monetary intervention. US Fed´ serie was a bit more progressive as it was not an interest rate jump once in time, but it could be understood more as monetary shock calculated from both market rate and limit fed fund rate in the USA. Author considered involving not only spot effect and one time-lagged effect, but also MP shock on the following day as there is market participants´ anticipation involved too.

Estimation of the **VAR** parameters was done via OLS method. All the parameters were rounded to two decimals. Only statistically significant parameters are mentioned, the others were omitted, with no intercepts included.

[3] This paper works with * at 10%, ** at 5% and *** at 1% statistical significance level.

Table 2: Short-term VAR models parameters estimation

Index	Name	Adj R^2	$Index_{-1}$	D_Fed $shock_{-1}$	D_Fed $shock_0$	D_Fed $shock_1$	D_ECB_{-1}	D_ECB_0	D_BoE_{-1}	D_BoE_0	D_BoJ_{-1}	D_BoJ_0	D_SNB_{-1}	D_SNB_0	Dummy
ASX	S&P/ASX 200 Index	2%	-0,06***		0,01***			-0,01*	0,02***						
BOVESPA	Ibovespa Brasil Sao Paulo	1%	-0,04**		0,02***										
BSE	BSE SENSEX	1%	0,03*	0,01***	0,01***				0,02**		-0,04*	-0,07***	0,03***		
CAC	CAC 40 Index	1%	-0,03**	0,01*	0,01***	0,01**			0,03***						
DAX	DAX 30 Index	1%			0,01***	0,01**			0,02***						
DJIA	Dow Jones Industrial Ave	2%	-0,11***		0,01***				0,01**	0,01*					
Euronext	Euronext 100 Index	1%		0,01**	0,01***	0,01**			0,02***						
FTSE	FTSE 250 Index	2%	0,11***		0,01***	0,01***									
HSI	Hang Seng Index	1%		0,01***	0,01***		-0,01*		0,02***			0,05**			
IPC	S&P/BMV IPC Index	1%	0,09***		0,01***	0,01***	-0,01**		0,01*						
ISEQ	iSeq All Share	2%	0,04***	0,01***	0,02***	0,01***			0,03***			-0,07***			
JCI	Jakarta Composite Index	2%	0,11***	0,01***	0,01***				0,02***		-0,04**	-0,07***			
KOSPI	KOSPI Composite	1%		0,01***	0,01**		-0,02***		0,01**						
MIB	Italy FTSE MIB Index	1%	-0,05***	0,01***	0,01***				0,03***						
Nikkei	Nikkei 225 Index	0%		0,01***	0,01**		-0,01*		0,01**						
PX	PX index	1%	0,05***	0,01***	0,01**				0,02***						
RTS	RTS Index	2%	0,07***	0,02***	0,03***	0,02***			0,03***			0,07*			
SPX	S&P 500 Index	2%	-0,11***		0,01***				0,01*	0,01**					
SSE	SSE Composite Index	1%		0,01***			-0,01*	-0,01***	0,01*						

Source: own table [Excel]

All VAR(1) models were **statistically verified** and found **significant**, nevertheless their explanatory power was pretty much insignificant. Their adjusted coefficients of determination R^2 are around 1 – 2 %. From the Table 2 it is apparent that stock returns on the previous day do generally impact their own results on the following day, in some cases positively, some negatively, but mostly with a low magnitude of less than 0,1 % on the next day. MP shocks one day ahead, on the spot date as well as one day later based on market participants´ expectations in the USA all do positively impact stock price returns (positive MP shocks represent more restrictive policies than being generally anticipated). BoE is also statistically significant important, just like the US´ Fed, with similar results. Based on VAR results, ECB and BoJ play less meaningful role in the global equity markets, having mostly positive impact on stock returns. Both, ECB and BoJ, do influence mainly Asian stocks. SNB was not identified to be a strategically important one.

In terms of unrestricted VARs´ **residual econometric verification**, the models suffer from the residual autocorrelations based on LM and Portmanteau Autocorrelation tests at selected level of statistical significance. Additionally, White tests of residual heteroscedasticity with/without cross terms are also lower than 10% significance level rejecting null hypothesis of homoscedasticity rather than accepting it – hence VAR models are heteroscedastic[4] too. Moreover, their residuals do not come from the normal distribution[5] as these are skewed and kurtosed. These negative results are mitigated by generally weak models´ coefficients of determination.

[4] Heteroscedasticity may be negatively influencing the accuracy of the OLS estimated parameters, but the trends should be still alright.

[5] Normal distribution is not a necessary requirement for suitable BLUE OLS estimated parameters. This issue gets even less important with greater observed time series sample.

Table 3: Long-term and short-term VEC models parameters estimation

Index	Name	Adj R^2	VECM	$Index_{-2}$	$Index_{-1}$	D_Fed $shock_{-1}$	D_Fed $shock_0$	D_ECB_{-1}	D_ECB_0	D_BoE_{-1}	D_BoE_0	D_BoJ_{-1}	D_BoJ_0	D_SNB_{-1}	D_SNB_0	Dummy
ASX	S&P/ASX 200 Index	3%	no		-0,06***		0,01***			0,02***	0,02***	-0,08**			0,03***	
BOVESPA	Ibovespa Brasil Sao Paulo St	1%	no		-0,07***		0,01***	-0,02*		0,02**	0,02**			0,03*		
BSE	BSE SENSEX	**3%**	**yes***			0,01***	0,02***	-0,03***	-0,02*			-0,12**	-0,21***	-0,04***	0,02**	
CAC	CAC 40 Index	2%	no		-0,04**	0,01**		-0,03***		0,02***	0,03***			0,02*	0,03**	
DAX	DAX 30 Index	1%	no			0,01**		-0,03***		0,02**	0,03***				0,03**	
DJIA	Dow Jones Industrial Averag	4%	no		-0,15***		0,01***	-0,02*		0,02**	0,03***			-0,02***	0,04***	
Euronext	Euronext 100 Index	2%	no			0,01***		-0,03***		0,02***	0,02***			0,02*	0,03***	
FTSE	FTSE 250 Index	2%	no		0,11***		0,01**	-0,03***			0,02***					
HSI	Hang Seng Index	2%	no			0,01**	0,01***	-0,02*		-0,01*	0,03***		0,11**		0,03***	
IPC	S&P/BMV IPC Index	2%	no	-0,06***	0,09***		0,01**	-0,03***			0,02***					
ISEQ	iSeq All Share	3%	no	-0,03*	0,04**	0,01***	0,02***	-0,02**			0,03***		-0,17***		0,04***	
JCI	Jakarta Composite Index	4%	no		0,10***	0,01***	0,01***				0,02***	-0,16***	-0,15***		0,02*	
KOSPI	KOSPI Composite	3%	no			0,01***		-0,01*		-0,01*	0,03***		-0,07*	-0,02**	0,05***	
MIB	Italy FTSE MIB Index	2%	no		-0,05***	0,02***	0,01*	-0,04***		0,03***	0,02***					
Nikkei	Nikkei 225 Index	**2%**	**yes***					-0,02*		0,02***	0,03***		0,16***		0,05***	
PX	PX index	2%	no	-0,06***	0,08***	0,01***	0,01*	-0,04***				-0,09**			0,05***	
RTS	RTS Index	3%	no		0,09***	0,02***	0,02***	-0,05***	0,06***		0,02*	-0,17**			0,04***	
SPX	S&P 500 Index	4%	no		-0,15***		0,01***	-0,02**		0,01**	0,02***			-0,02**	0,05***	
SSE	SSE Composite Index	**1%**	**yes*****			0,01***									0,02**	

Source: own table [Excel]

VEC models were as follows

$$DL\_INDEX_t = \gamma_1(L\_INDEX_{t-1} - \lambda_1 MPSHOCK_{t-1} - \lambda_2 ECB_{t-1} - \lambda_3 BOE_{t-1} - \lambda_4 BOJ_{t-1} - \lambda_5 SNB_{t-1} + c_1) + \sum_{i=1}^{2} \delta_{1,i}\, DL\_INDEX_{t-i} + \sum_{i=0}^{1} \delta_{2,i}\, D\_MPSHOCK_{t-i} + \sum_{i=0}^{1} \delta_{3,i}\, D\_ECB_{t-i} + \sum_{i=0}^{1} \delta_{4,i}\, D\_BOE_{t-i} + \sum_{i=0}^{1} \delta_{4,i}\, D\_BOJ_{t-i} + \sum_{i=0}^{1} \delta_{4,i}\, D\_SNB_{t-i} + DUMMY + \varepsilon_t$$

where γ_1 is adjustment coefficient and λ_i are coefficients of the cointegration vector, and δ are coefficients of the short-run models with intercepts excluded. Running VECM led to only three statistically significant results at 10% level with long-run cointegrations identified, in the Japanese, Indian and Shanghai markets. Table 3 represents estimation of coefficients for the other markets in short run too as in this case the reader may be indifferent between usage of VEC and VAR models.

Based on long-term VECM results, BSE and SSE indices are positively influenced by restrictive MP of the Federal Reserve system while not impacted by BoE at all. Spot change of SNB positively impacted all three markets, ECB had negative effect on BSE and Nikkei. Impact of BoJ on the Japanese Nikkei 225 was opposite for ultra-loosen MP lasting for over three decades there, if even more negative rates might been signalling even deeper financial structural issues for Japanese economy. This would support the theory that magnitude of monetary policy during economic downturns is usually more massive than during economic booms, and often with opposite relationship as ultraloosen MP is a sign of lack of trust.

Short-term results of VEC differ from those of VAR models in terms of other central banks been found statistically significant as well. Generally spoken, regarding to the ECB and BoJ, this negative relationship may be the case for their ultra-loosen monetary policy. Once zero rate policy is implemented, any change in interest rate makes the market participants reacting more. Dummy variables were generally statistically insignificant in both models having a low impact.

Economically controversial data of positive co-movement between interest rates and stock prices in Table 2 may be pointing out different economic environment in the third millennium generally impacted by the Great Recession as most of the interventions occurred during this period. These results may be supported by several recent studies showing that under contractual business conditions extra-loosen monetary policy resulted in negative stock yields, and vice versa.

6 Conclusion

This research was focused on the long-run and short-run relationships between changes of interest rates and stock prices indices in 19 selected countries on the daily data since 3 January 2000 up to 30 June 2020. Research was specifically aimed to the impact of interest rates´ changes on stock indices´ returns. The model was based on dependent variables (time lagged stock index itself & interest rate changes by top five reserve central banks) and one independent variable (business cycle dummy variable).

For all the countries, both VEC and VAR models had been run. Variables were tested in *t-1* and *t0*, except of US MP shocks being additionally tested even in *t+1*.

The explanatory power of all the models has been regrettably low. Generally, the stock returns were impacted by their own time-lagged returns. Dummy variables were statistically insignificant in both models with a very limited impact.

VAR model identified US Fed and BoE as two statistically significant central banks with positive relationship between their interest rates and stock markets´ returns. It was mainly spot reaction of the markets on the US MP on that specific date, less one day ahead, and even less a day after. As for BoE, the most important day is one day following after its intervention. US MP shocks do positively impact stock price returns (positive MP shocks represent more restrictive policies than being generally anticipated). Via VAR models ECB and BoJ did both influence mainly Asian stocks.

VEC models identified three long-run cointegrations in India, Shanghai and Japan, while in short-term all central banks were found statistically significant. Based on VECM, there is an apparent difference in between the countries having positively interest rates and those countries having zero interest rate policies (or even negative rates) in long-term. Japanese equity market´ reaction to its own state-dependent BoJ was different from elsewhere. As most of expansionary monetary interventions occurred globally during the Great Recession with very few restrictive ones going onward, this research has sort of supported the theory that extra-loosen policy during economic downturns resulted in negative stock yields, and vice versa during economic booms.

Models residuals suffered from autocorrelation and heteroscedasticity. More data of interest rate changes would be desired, or at least any daily anticipations of market participants, just like the case for the USA. Findings were comparable and compatible to those of author´s older ones, run on EU data in 2019, as well as on the US market in 2018. For once more, monetary variables seem to be somewhat exaggerated, next to the real macroeconomical indicators.

References

[1] Allan, P., Andrews, M., Cuthbertson, K., Hall, S.G., Hempstead, H., Taylor, M.P., (1993). Applied econometric techniques. ISBN 0-86003-084-9. *Journal of Applied Econometrics* [online]. **8**(4), 423-424 [cit. 2019-08-15]. DOI: 10.1002/jae.3950080410. ISSN 08837252.

[2] Balafas, N., Florackis, C., Kostakis, A. (2018). Monetary policy shocks and financially constrained stock returns: The effects of the financial crisis. *International Review of Financial Analysis* [online]. **58**, 69-90 [cit. 2019-08-15]. DOI: 10.1016/j.irfa.2018.05.001. ISSN 10575219.

[3] Branzovsky, J. (2018). *Relationship of the selected macroeconomic indicators and the stock market index.* Proceedings of MMFR conference. September 2018.

[4] Branzovsky, J. (2019). *Relationships between monetary variables and the stock price returns: Case for Czechia, Hungary, Poland and the UK*. Proceedings of FMFFI conference. September 2019.

[5] EViews: *VEC model* [online]. [cit. 2019-07-02]. Available: www.eviews.com

[6] Fausch, J., Sigonius, M. (2018). The impact of ECB monetary policy surprises on the German stock market. *Journal of Macroeconomics* [online]. **55**, 46-63 [cit. 2019-08-15]. DOI: 10.1016/j.jmacro.2017.09.001. ISSN 01640704.

[7] Feldman, T. (2017). Unwinding ZIRP: A simulation analysis. *Finance Research Letters* [online]. **24**, 278-288 [cit. 2019-08-15]. DOI: 10.1016/j.frl.2017.09.024. ISSN 15446123.

[8] Finance Yahoo: *Historical Data* [online]. New York, USA [cit. 2019-04-16]. Available: www.finance.yahoo.com

[9] Haitsma, R., Unalmis, D., De Haan, J. (2016). The impact of the ECB's conventional and unconventional monetary policies on stock markets. *Journal of Macroeconomics* [online]. **48**, 101-116 [cit. 2019-08-15]. DOI: 10.1016/j.jmacro.2016.02.004. ISSN 01640704.

[10] Hanclova, J. (2012). *Ekonometricke modelovani: klasicke pristupy s aplikacemi*. 1. vyd. Praha: Professional Publishing. ISBN 978-80-7431-088-1.

[11] Johansen, S. (1991). *Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models.* Econometrica, 59, 1551–1580.

[12] Johansen, S. (1995). *Likelihood-based Inference in Cointegrated Vector Autoregressive Models.* Oxford: Oxford University Press.

[13] Kulhanek, L., Matuszek, S. (2006). *Penezni zasoba a vyvoj akciovych trhu v Ceske republice, Slovenske republice a ve vybranych zemich*. Ekonomicka univerzita v Bratislave.

[14] NBER: *US Business Cycle Expansions and Contractions* [online]. Cambridge, USA: The National Bureau of Economic Research [cit. 2019-05-14]. Dostupne z: https: //www.nber.org/cycles.html

Analysis of non-accelerating inflation rate of unemployment in the Czech Republic

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Abstract

The paper focuses on the development of the non-accelerating inflation rate of unemployment in the Czech Republic according to the methodology of the Czech Statistical Office and the Ministry of Labour and Social Affairs. Using an econometric model and own calculations, this paper analyses the non-accelerating inflation rate of unemployment from the establishment of the Czech Republic to the present and examines both how total unemployment manifests itself in the national economy and between the sexes, as well as how it changed during economic recessions and booms. Based on this analysis, it predicts future developments on the labour market in the near future.

Keywords

NAIRU, Hodrick-Prescott filter, stationarity, general unemployment rate, share of unemployed persons, unemployment gap

JEL Classification: G0

1. Introduction

This paper deals with the development of the Non-Accelerating Inflation Rate of Unemployment (NAIRU) and the unemployment gap (UG). Data from the Czech Statistical Office (CZSO) and the Ministry of Labour and Social Affairs (MoLSA) were used to determine data on the long-term unemployment rate. The values themselves were then estimated using the Gretl econometric program. The aim of this paper is to evaluate the development of the unemployment rate in the Czech Republic using econometric tools, because low unemployment also affects the inflation rate, which is higher than the European average. Both macroeconomic indicators contribute significantly to the overall economic cycle of the country. Based on data from previous years, part of the aim of the paper is to analyse the development of total unemployment from two sources and explain the various phases of the unemployment rate in the Czech Republic.

In the subchapter *Background*, the paper first examines both the impact of analysis indicators on unemployment and the use of methods that were used to process this paper, including an explanation of their advantages and disadvantages. To understand this issue, the basic econometric concepts related to the topic were precisely defined. Furthermore, the paper contains an overview of the findings of theoretical and empirical research. In the conclusion, first the time series that were used for testing were explained, then there is information about the database used and the characteristics of the data used, and finally an overview of econometric tests. The empirical part is divided into two chapters. The first one analyses the original data from the CZSO and the MoLSA, on which tests for stationarity were performed for individual time series. In the second chapter, an HP filter was applied, which was used to

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determine the development of NAIRU and UG. The final part presents the results and clarification of the empirical part, it contains an interpretation of the results of the analysis of the development of the unemployment rate and a prediction of the future development of these labour market indicators.

2. Background

2.1 NAIRU

The basic indicator of this paper is the NAIRU, which is methodically related to the original Phillips substitution curve. The NAIRU is an unemployment rate that does not change the rate of inflation and represents the balance of the labour market. This is another expression of the natural rate of unemployment (M. Yglesias, 2014). However, determining the NAIRU is not easy. There are three possible NAIRU estimates, e.g. (Richardson, 2000).

The short-term NAIRU is the unemployment rate at which the current inflation rate can be fixed even for the following period. However, this means that such an unemployment rate, and thus the NAIRU, is relatively variable, as it is influenced by all supply shocks and the inflation inertia. What is usually understood as NAIRU is the medium-term NAIRU. This is the equilibrium value to which the real unemployment rate is heading when the temporary supply shocks subside, and the real inflation rate adjusts to its expected value. Long-term NAIRU is the equilibrium unemployment rate to which the system is heading after all supply (and other) shocks have subsided, including those that have a long-term impact on the economy. In addition to (Pošta, 2008), this includes e.g. (Richardson, 2000) and (Szeto, Guy, 2004).

Economists present this unobservable variable as the "equilibrium" or "expected" values of the variables they are trying to estimate (L. Boone, 2000). The deviation of unemployment from the natural rate is a cornerstone of monetary policy (A. Estrada, I. Hernando and J. D. López-Salido, 2000). In the course of their paper, the authors found that alternative tools provide different NAIRU point estimates and that these estimates are characterised by a high degree of uncertainty. P. McAdam and K. McMorrow (1999) consider the applicability of the NAIRU concept in macroeconomic policy discussions to be somewhat limited.

When calculating the NAIRU, we use five variants (E. Jašová, 2010), which provides us with an alternative view of the development of the NAIRU and the unemployment gap. The first variant of the analysis of the relationship between the unemployment rate and inflation is the *One-equation Model*, which estimates only one NAIRU value for the entire reference period. This method defines the NAIRU as the level of unemployment that is consistent with the Phillips curve extended by stable expectations. The most widespread variant is Gordon's "*Triangle Model*" (R. J. Gordon, 1997), assuming that the inflation rate depends on three basic factors, namely (and as expected) inflation, demand conditions approximated by the unemployment gap and supply-side shocks. The second variant of NAIRU identification is the HP filter (S. Fabiani and R. Mestre, 2000). The use of the HP (Hodrick-Prescott) filter is based on the idea of substituting real unemployment with the levels of the smoothed trend by choosing the value of the coefficient λ. The other three variants are the NAIRU calculation using the Break model (S. Fabiani and R. Mestre, 2000), and the NAIRU estimation variant according to the Kalman filter (P. Richardson, L. Boone, C. Giorno, M. Meacci, D. Rae and D. Turner, 2000), and finally the fifth variant, where the averaging of NAIRU values estimated by the whole set of methods was used.

2.2 Hodrick-Prescott filter

The basic task of the HP filter is to divide the time series into a trend component and a cyclical component. The HP filter is based on minimising the following expression:

$$\{y_t^g\}_{t=0}^{T+1} = \arg min \sum_{t=1}^{T} \left(y - y_t^g\right)^2 + \lambda.\left[\left(y_{t+1}^g - y_t^g\right) - \left(y_t^g - y_{t-1}^g\right)\right]^2, f \tag{1}$$

where, λ is the filter parameter and ytg is a member of the filtered series. In essence, this is about minimising the differences in moving averages between the actual development of the time series and the calculated trend component, λ reduces the variability of the trend component, which means that the filter output will have a smoother course. The ideal value used for the decomposition of macroeconomic time series is usually 1600 for quarters, 100 for annual cycles and 700 for half-yearly cycles (M. Hájek and V. Bezděk, 2000). The authors V. Bezděk, A. Dybczak and A. Krejdl (2003) then used the value 480 to smooth the quarterly data due to structural breaks affecting the previous and subsequent observations. This value is closest to the ideal values of the ratio of the squares of the variance of the cyclical component and the variance of the growth rate of the trend component (D. Tráge, 2011).

The NAIRU estimation in this paper is always performed using the HP filter. When using the above filter, it is not necessary to use other factors (e.g. consumer and import prices), which are not statistically supported for the analysed sectors, sex and age categories (E. Jašová, 2016).

According to this author, the HP filter has the following advantages:

- Low data demand factor
- Simplicity and transparency
- Simplicity of interpretation

The disadvantages of the HP filter according to M. Hájek and V. Bezděk (2000) are:

- Absence of a universally applicable λ value
- Possibility of estimates deviation at the beginning or end of the series
- The HP filter does not capture structural changes in the economy

Purely statistical models usually paper only with the actual unemployment rate and, using the chosen filter (the often-used HP filter), estimate the trend that is considered to be sought by NAIRU. The advantages are the ease and speed of calculation. Disadvantages include subjectivity regarding the parameters of the filter that need to be chosen and filters that slowly absorb significant changes in unemployment, which may be structural in nature. In general, this approach makes it impossible to use any economic considerations (J. Hráčková, 2015).

2.3 Data definition

Before applying the tools to estimate the unobservable variables, it is necessary to verify the stationarity of the initial time series. The stationarity of published unemployment rates means that its probability distribution is constant and independent over time, and at the same time relevant within the econometric model.

Two sources of unemployment rate data were used to prepare this paper:

- Czech Statistical Office (CZSO) – which publishes the general unemployment rate (GUR);
- Ministry of Labour and Social Affairs (MoLSA) – which publishes the share of unemployed persons (SUP).

The general unemployment rate of persons aged 15 to 64 represents the share of the unemployed in the labour force, i.e. the sum of the employed and the unemployed. On the contrary, the share of unemployed persons indicates the ratio of the number of available job seekers aged 15-64 and the total number of inhabitants in the same age category (CZSO, 2018).

3. Empirical part

3.1 Original data and stationarity tests

According to the CZSO, the GUR time series begins on 1 January 1993 and provides data on the unemployment rate as a percentage for each quarter until Q2 2019. In each quarter, the database also contains data on the distribution of the unemployment rate by sex. The development of SUP begins in Q1 1997 and ends in Q3 2019. As with the GUR, the data are measured in relative numbers. This source offer data on the distribution of the unemployment rate by sex only since 2004. Unfortunately, older data from the MoLSA website are not available. On the contrary, compared to the CZSO, the MoLSA reports data on the unemployment rate on a monthly basis, but for the harmonisation and comparison of both methods, our database also contains quarterly data.

The GUR peaked in Q1 2000, after which it gradually declined slightly until the economic crisis began in 2008. However, even during the biggest economic recession, the GUR did not reach double-digit levels. The state of relatively high GUR ended after 2013, when the economy recovered from the recession and boomed. The situation on the labour market is currently record-breaking – the official value of the GUR for Q2 2019 is 1.9%. The comparison of the GUR in men and women is interesting, because here we can observe a slightly different development over time. On the one hand, the GUR for men in each reference period is lower than for women, and the development of values is slowed down for men compared to women. This phenomenon is investigated in the first part.

The second time series, the development of the SUP, shows the highest value in Q1 2004, when it was 10.8%, which is 4 years later than in the case of the GUR. The development of the decline to values around 5% before the crisis is similar, on the contrary, the increase after the crisis is more pronounced, as it went up to 9.8%. The latest data point is from Q3 2019 with a value of 2.7%. When comparing the development of the SUP in men and women, there is one significant difference as compared to the GUR – in some quarters of 2013, the SUP was higher in men than women.

All of these time series were tested for stationarity. This test was applied using the extended Dickey-Fuller test. In order for the time series to be stationary, i.e. for the time series to not continuously increase or decrease, the p-value must always be less than or equal to 0.10, and this has only been achieved for the total GUR, GUR men and SUP men.

Table 1: Stationarity test

	GUR (in %)	women GUR (in %)	men GUR (in %)	SUP (in %)	women SUP (in %)	men SUP (in %)
p-value	0,08094	0,5145	0,04542	0,7939	0,8603	0,07745
test statistics	-2,66106	-1,53787	-1,65519	-0,884201	-0,610286	-2,6802
estimated value	-0,05015	-0,02820	-0,05115	-0,03042	-0,01828	-0,10410
stationary	yes	no	yes	no	no	yes

Data source: GUR: https://vdb.czso.cz/vdbvo2/faces/cs/index.jsf?page=vystup-objekt&pvo=ZAM01-D; SUP: https://www.mpsv.cz/web/cz/mesicni; Gretl; Own calculation in Excel

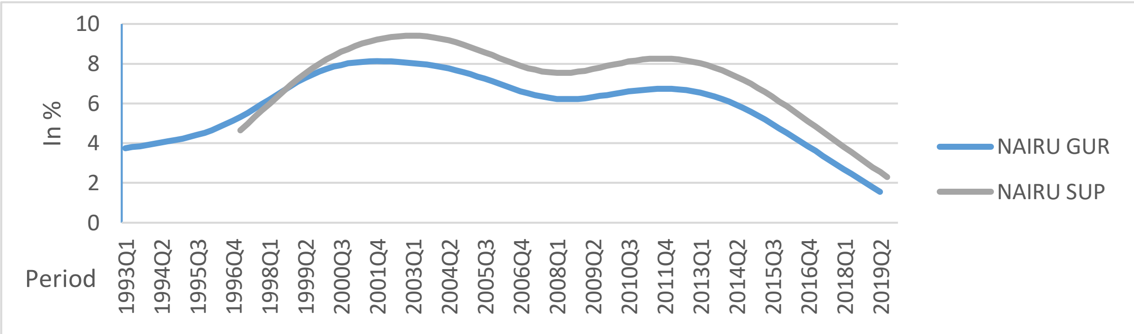
3.2 HP filter

In this part, the HP filter was applied, where lambda is 1600, because the data in time series are quarterly. Using the HP filter to find the NAIRU. According to the GUR, total NAIRU initially grew from less than 3.8% in 1993 to more than double (8.1%) in Q4 2001. This highest peak is delayed by 6 quarters compared to the peak in the time series of the actually measured GUR. Subsequently, the NAIRU declines to 6.2% until the pre-crisis period (Q3 2008). The economic crisis caused a slight increase in the NAIRU, but it was only an increase to 6.7% in

Q4 2011. After this quarter, the NAIRU has been steadily declining to 1.55% at the end of the time series.

According to the SUP, the total NAIRU also begins to grow in 1997 until Q1 2003, when it jumped from 4.6% to 9.4%. The growth rate of total NAIRU was higher for the SUP than for the GUR. Subsequent developments until and during the crisis were the same for the SUP as for the GUR; after Q4 2011, the NAIRU decreased at a more pronounced rate than for the GUR (from 8.3% to 2.2% – see Chart 1).

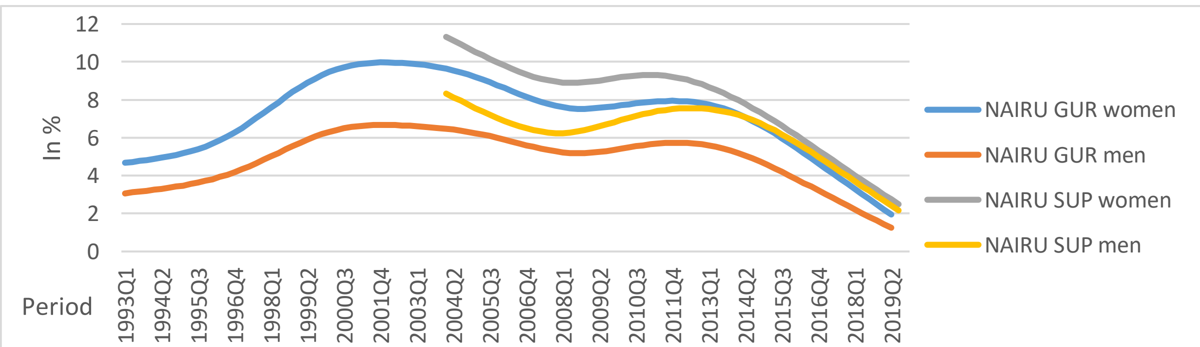
Chart 1: Total NAIRU according to the GUR (from CZSO) and the SUP (from MoLSA)



Data source: own calculation based on the GUR data:
https://vdb.czso.cz/vdbvo2/faces/cs/index.jsf?page=vystup-objekt&pvo=ZAM01-D
SUP: https://www.mpsv.cz/web/cz/mesicni

For the purposes of this paper, the development of NAIRU of men and women according to both methodologies is also essential. Due to the absence of older data by the Ministry of Labour and Social Affairs, the data are based only on the CZSO. At the beginning of the time series in 1993, the NAIRU difference between men and women was 1.6%. The economic slowdown at the turn of the millennium was more pronounced for women than for men in the GUR – in Q4 2001, the gender gap was 3.3%. Subsequent economic growth narrowed this gap once again, but it was still more than 2% for the GUR and 3% for the SUP. However, the economic crisis in 2008-2012 and its increase in the unemployment rate did not affect this observed difference for the two methodologies much. On the contrary, the difference has started to narrow more markedly in recent years – in Q2 2019, the difference was only 0.7% (see Chart 2).

Chart 2: NAIRU of men and women according to the GUR and the SUP



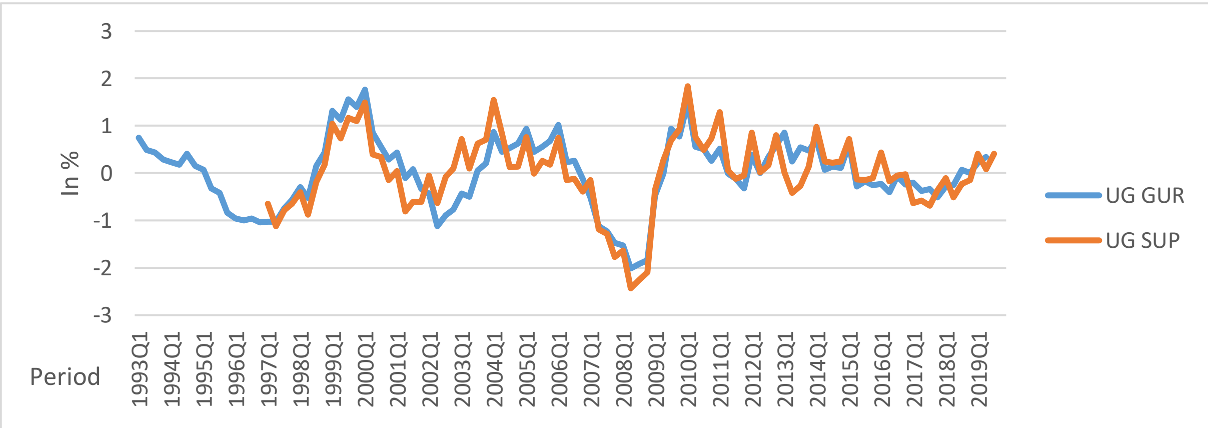
Data source: own calculation based on the GUR data:
https://vdb.czso.cz/vdbvo2/faces/cs/index.jsf?page=vystup-objekt&pvo=ZAM01-D
SUP: https://www.mpsv.cz/web/cz/mesicni

It follows from the above that during economic recessions, women are more involved in raising the unemployment rate than men. On the contrary, in periods when the economy is thriving, this difference between the sexes is minimal. Using the HP filter, the development of the phases of the economic cycle on the labour market is also determined, i.e. in what period there was a positive or negative "gap" or unemployment gap. The UG is expressed in percentage points, again either in total or by gender. The UG is calculated by deducting the NAIRU in a given period from the actual unemployment rate. If the value for a given quarter is negative, the economy was booming and a positive value means a decline in economic activity.

Total UG began to decline from 1993 to 1997, when it showed values around -1, see Chart 3. Within the sexes, this gap was more pronounced in women (-1.4) than in men (-0.8), see Chart 4. After the crisis in 1997, the real increase in the unemployment rate was faster than in the NAIRU, so the gap turned to positive numbers. The UG here was again higher in women (+2.1) than in men (+1.9). Subsequently, there was a slight oscillation on both sides, until the year 2008, when the real unemployment rate decreased regularly, while the NAIRU still takes into account the high values from the previous period. The overall gap was slightly lower for the SUP (-2.4) than for the GUR (-2.0). Post-crisis stabilisation meant that the signs changed for the UG, but the positive values are not as high as those with a minus. This is explained by the fact that during the economic crisis, economic development declines more sharply than it increases during the boom. From Q2 2011 to the present, there have been no significant deviations – the gap has never exceeded +/- 1.

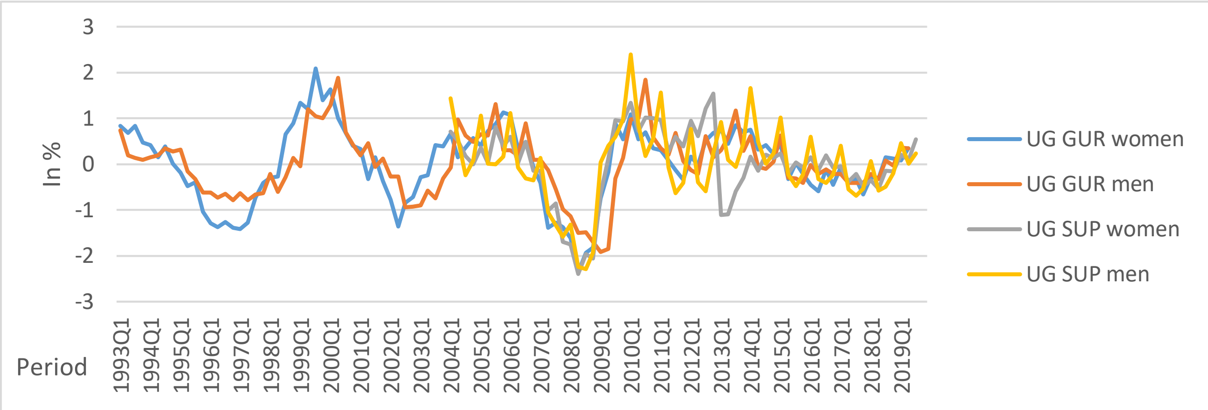
The development of the UG is similar for both methods, and there are no significant differences between genders either. Perhaps it is worth mentioning the fact that the development of the UG in men compared to women is usually delayed by one quarter. This difference can be seen in Chart 4 below, especially during significant fluctuations before and after crises. The UG is always at the lowest in the pre-crisis period and at the highest during the crisis.

Chart 3: Total unemployment gap according to the GUR and the SUP



Data source: own calculation based on the GUR data:
https://vdb.czso.cz/vdbvo2/faces/cs/index.jsf?page=vystup-objekt&pvo=ZAM01-D
SUP: https://www.mpsv.cz/web/cz/mesicni

Chart 4: Unemployment gap of men and women according to the GUR and the SUP



Data source: own calculation based on the GUR data:
https://vdb.czso.cz/vdbvo2/faces/cs/index.jsf?page=vystup-objekt&pvo=ZAM01-D
SUP: https://www.mpsv.cz/web/cz/mesicni

4. Conclusion

The aim of this paper was to evaluate the development of the unemployment rate in the Czech Republic according to the methods of the CZSO and the MoLSA using the NAIRU and the unemployment gap. Based on the analysis of the historical development of the unemployment rate and the results obtained, the conclusion deals with the prediction of the state and development of the labour market in the near future.

In the empirical part, the HP filter was first applied, with which we found out the development of NAIRU in the Czech Republic from 1993 to the present according to the GUR and according to the SUP, and we also distinguished between women and men for both methods. NAIRU was lower in men than in women throughout the reference period. The gender gap has started to narrow significantly in recent years, at a time of record low unemployment. In times of economic recession, women are more involved in raising the unemployment rate than men and the development of the UG in men compared to women is usually delayed by one quarter. We believe the main reason of this fact is that men have more stable jobs, i.e. during the recessions more women lose their jobs than men. UG trend in men is usually delayed because the unemployment rate of men is not fluctuating as that of women. We also found out the development of the UG, which showed the lowest values in the pre-crisis years – 1996, 2002 and especially in 2008. On the contrary, the highest values were measured just after the crises, especially in 1999 and 2010. In the last decade, the development of the UG has stabilised – after the economic crisis in 2009, not even the weak recession in 2012 and 2013, which caused a second smaller bottom in GDP development, did not affect the UG development much and only caused a longer period of positive UG and a short increase in NAIRU.

Nevertheless, a certain trend can be observed in the last year of the observed time series. Until 2018, this development declined slightly, after which it began to rise slightly from the beginning of 2019. Although the UG bottom is not nearly as significant as in 2008, this means that the labour market and related economic activity have already peaked and are gradually weakening. Although the latest fresh data on the development of both monitored unemployment rates are still the lowest in their history, as is also confirmed by the still declining NAIRU according to both methodologies, we can nevertheless expect their gradual increase not only by past experience but also as we experience a global pandemic.

References

[1] BEZDĚK, V.; DYBCZAK, A.; KREJDL, A. (2003). *Czech Fiskal Policy: Introductory Analysis*. Czech National Bank, Working Paper. 2003, no. 7.

[2] BOONE, L. (2000). *Comparing Semi-Structural Methods to Estimate Unobserved Variables*. The HPMV and Kalman filters Approaches OECD. Economics Department Working Papers No. 240.

[3] Český statistický úřad (2018). *Podíl nezaměstnaných osob* [online]. [cit. 7.12.2019]. Available at: https://www.czso.cz/csu/xc/podil-nezamestnanych-osob-v-kraji-v-prosinci-2018-lehce-vzrostl

[4] Český statistický úřad (2019). *Základní charakteristiky ekonomického postavení obyvatelstva ve věku 15 a více let* [online]. [cit. 7.12.2019]. Available at: https://vdb.czso.cz/vdbvo2/faces/cs/index.jsf?page=vystup-objekt&pvo=ZAM01-D

[5] ESTRADA, A.; HERNANDO, I.; LÓPEZ-SALIDO, J. D. (2000). *Measuring the NAIRU in the Spanish Economy*. Documento de Trabajo n° 00009.

[6] FABIANI, S.; MESTRE, R. (2000). *Alternative measures of the NAIRU in the euro area: estimates and assessment*. ECB WP.

[7] Fileplanet. *Gretl*. Available at: https://gretl.fileplanet.com/

[8] GORDON, R. J. (1997). *The Time-Varying NAIRU and its Implications for Economic Policy*. Journal of Economic Perspectives

[9] HÁJEK, M; BEZDĚK V. (2000). *Odhad potenciálního produktu a produkční mezery v ČR*. WP Česká národní banka.

[10] HRÁČKOVÁ, J. (2015). *Mezera výstupu a historie jejího užití*. s.29-30 [online]. Masarykova univerzita. [cit. 7.12.2019]. Available at: https://is.muni.cz/th/r1se3/Diplomova_prace.pdf

[11] Investiční web (2015). *Obecná míra nezaměstnanosti v Česku* [online]. [cit. 7.12.2019]. Available at: https://www.investicniweb.cz/news-2015-1-30-obecna-mira-nezamestnanosti-v-cesku-v-prosinci-klesla-na-59/

[12] JAŠOVÁ, E. (2010). *Podobnosti a rozdíly ve vývoji míry nezaměstnanosti neakcelerující inflaci a hospodářského cyklu ve vybraných středoevropských zemích do roku 2008*. Současná Evropa a Česká republika. Centrum evropských studií VŠE Praha, Praha.

[13] JAŠOVÁ, E. (2016). *Koncept a metody odhadu NAIRU a hospodářského cyklu na trhu práce v zemích* Visegrádské skupiny. s. 106. Národohospodářská fakulta VŠE v Praze.

[14] KŘIVÝ, I. (2012). *Analýza časových řad* [online]. [cit. 7.12.2019]. Available at: http://www1.osu.cz/~bujok/files/ancas.pdf

[15] MCADAM, P.; MCMORROW, K. (1999): *The NAIRU Concept – Measurement uncertainties, hysteresis and economic policy role*.

[16] Ministerstvo práce a sociálních věcí (2019). *Nezaměstnanost – měsíční* [online]. [cit. 7.12.2019]. Available at: https://www.mpsv.cz/web/cz/mesicni

[17] POŠTA, V. (2008). *Odhad NAIRU pro českou ekonomiku* [online]. Ministerstvo financí České republiky [cit. 7.12.2019]. Available at: https://www.mfcr.cz/assets/cs/media/Odborne-vyzkumy_2008_Odhad-NAIRU-pro-ceskou-ekonomiku.pdf

[18] RICHARDSON, P.; BOONE, L.; GIORNO, C.; MEACCI, M.; RAE, D.; TURNER, D. (2000). *The concept, policy use and measurement of structural unemployment: estimating a time varying NAIRU across 21 OECD countries*. OECD WP, 2000.

[19] TRÁGE, D. (2011). *Časově-frekvenční analýza hrubého domácího produktu ČR*. s.6 [online]. Vysoké učení technické v Brně [cit. 7.12.2019]. Available at: https://www.vutbr.cz/www_base/zav_prace_soubor_verejne.php?file_id=36049

[20] YGLESIAS M. (2014). *The NAIRU, explained: why economists don't want unemployment to drop too low*. Vox [online]. [cit. 7.12.2019]. Available at: https://www.vox.com/2014/11/14/7027823/nairu-natural-rate-unemployment

Financial Distress Prediction in the Czech Republic: a CART approach

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Abstract

Financial distress prediction is always an actual issue because it is important to find a suitable tool to assess the future financial development of the company. This paper aims to create the model for predicting potential financial distress situation of companies using decision tree generating algorithm CART. The financial data (balance sheets and profit and loss accounts form 2017 and 2018) of real companies operating in the economic conditions of the Czech Republic were used. The final model achieves high accuracy of 91.8%. Moreover, this model works only with four financial ratios and is simple to interpretation and real implementation.

Key words

Financial distress; Decision tree; CART algorithm; Financial ratios; Prediction model; Czech Republic.

JEL Classification: C31, C53, G33.

1. Introduction

Every company may face the risk of financial distress situation or even business default. Therefore, financial distress prediction has been a very interesting topic over the last decades because of its great importance not only for companies itself but also for interested stakeholders and even for the economy of a country. If this prediction is reliable, managers of companies can initiate remedial measures to avoid the financial distress situation. (Kovacova et al., 2019a; Svabova et al., 2020)

The topic of financial distress prediction fully developed in the 1980s. Since then, many prediction models have emerged. These are either global models or national models that consider the specifics of a given country or even a sector of the economy. These models are then in general of limited use in another country. (Kral, Valjaskova and Janoskova, 2019).

The paper focuses on the presentation of the model of financial distress prediction model of Czech companies. This model is created using CART algorithm that generates binary decision tree. The purpose of the paper is to identify significant determinants of potential financial risks in actual post-crisis conditions of economics in the Czech Republic. The originality of the research lies in the using of a large dataset of financial indicators of more than 41,000 real Czech companies. The model is constructed without regard to any economic sector and any company size. Thus, potential financial risks threatening all companies can be predicted using this model. It can be useful for eliminating potential losses of companies or their stakeholders. (Kovacova et al., 2019b)

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The rest of the paper is divided into four main parts. Literature review briefly describes the theoretical background and most important related works, especially in the Czech Republic. The description of the used dataset and a short description of CART algorithm is described in the section Data and Methods. The results section is focused on the description of the developed model and on short discussion of results. In the last section, some conclusions are provided.

2. Literature review

Since the first univariate prediction models, researchers created many multivariate prediction models. Based on the reviews of existing models (Ravi Kumar and Ravi, 2007; Prusak, 2018; Alaka et al., 2018; Kovacova et al., 2019a), two groups of popular techniques are used in this field, i.e. statistical techniques (especially multiple discriminant analysis (MDA) and logistic regression (LR) or Logit) and artificial intelligence techniques.

The first commonly used prediction model is the Altman Z-score model that was created using MDA technique (Altman, 1968). In 2017, a study was published in which the authors analysed the classification ability of the Altman Z-score on a sample of 31 European and 3 non-European companies. The authors state that this was the first study to provide such a comprehensive international analysis (Altman et al., 2017).

Ohlson O-score model is the first published Logit-based prediction model. This model was published in 1680 and reached a very high prediction ability of up to 96% (Ohlson, 1980). Then, many other logit prediction models have been created around the world, especially in the US but also in Central Europe. In 2005, Virág and Kristóf published one of the first prediction models using Logit and other techniques (Virág and Kristóf, 2005). In Slovakia, several Logit-type models were created (Kovacova and Kliestik, 2017). Many other authors compare existing models created using mentioned statistical techniques (Valaskova et al., 2020; Gregova et al., 2020; Kliestik et al., 2020).

In the second group, the artificial neural network is the most used technique (Kovacova et al., 2019a). Other techniques include decision trees (Liu and Wu, 2017; Zięba, Tomczak and Tomczak, 2016), support vector machines (Alaminos, Castillo and Fernández, 2016), etc. Especially, CART technique is often used to create prediction models (Karas and Reznakova, 2017; Nyitrai and Virag, 2019; Affes and Hentati-Kaffel, 2019).

In the Czech Republic, the model of Neumaierova and Neumaier from 1995 represents probably the first widely used national bankruptcy model (the so-called IN95 model). Then, the same authors several times upgraded this model as IN99, IN01 and IN05 model. Another widely used Czech national model was created by Jakubík and Teplý in 2011 (Jakubík and Teplý, 2011). These models are still used in practice. Karas and Reznakova (2017) built several models using various statistical techniques (MDA and Logit model) and data-mining techniques (e.g. CART decision trees). Typical representative in using artificial neural networks (and other data-mining techniques) in the field of financial distress prediction is Marek Vochozka. In his studies, the highly efficient LOGIT model for transport companies (Vochozka, Straková and Váchal, 2015) and the prediction models for Czech manufacturing and construction companies (Vochozka and Rowland, 2015; Vochozka, 2017) were created. In 2020, Horak et al. (Horak, Vrbka and Suler, 2020) created a model for predicting financial distress situation of Czech companies using Support Vector Machine and artificial neural networks.

3. Data and Methods

The data for the creating of the model were obtained from the database Amadeus, covering the accounting period of the years 2017 and 2018. The original database consists of real data on nearly 100,000 Czech companies. After data preparation (deleting incomplete data, control of logical and economical correctness, etc.), and creating a balanced group (according to (Agrawal and Maheshwari, 2016)) of prosperous and non-prosperous companies, the final dataset consisted of data on 41,692 companies. One half of the data was the group of 20,846 non-prosperous companies and the second half was the group of 20,846 prosperous companies. We used the data of all non-prosperous companies and the selection of prosperous companies was made randomly, while no specifics (economic sector, size, and legal form) were considered. As independent variables and potential predictors, we used the values of 37 financial indicators (see Table 1). Some of them represent the most frequent indicators, but we also used less frequently used ones, which can identify some specificities of the actual conditions of the Czech economy.

Table 1: Financial indicators used as a potential predictors

ID	Method for calculation	ID	Method for calculation
$X1$	Sales / Total Assets	$X20$	Net Income / Sales
$X2$	Current Assets / Current Liabilities	$X21$	Non-current Liabilities / Total Assets
$X3$	Gross Profit / Total Assets	$X22$	Cash & Cash Equivalents / Current Liabilities
$X4$	Net Income / Equity	$X23$	Cash-flow / Current Liabilities
$X5$	EBITDA / Sales	$X24$	Working Capital / Sales
$X6$	Liabilities / EBITDA	$X25$	Current ratio
$X7$	Net Income/ Total Assets	$X26$	Liquidity ratio
$X8$	Working Capital / Total Assets	$X27$	Return on Assets
$X9$	Operating Profit / Total Assets	$X28$	Return on Equity
$X10$	Total Liabilities / Total Assets	$X29$	Shareholder Liquidity Ratio
$X11$	Current Assets / Total assets	$X30$	Solvency ratio (Liability based)
$X12$	Cash & Cash Equivalents / Total Assets	$X31$	Cash-flow / Operating Revenue
$X13$	Cash-flow / Total Assets	$X32$	Net Assets Turnover
$X14$	Cash-flow / Total Liabilities	$X33$	Interest Paid
$X15$	Current Liabilities / Total Assets	$X34$	Gross Margin
$X16$	Current Assets / Sales	$X35$	Profit Margin
$X17$	Operating Profit / Interest Paid	$X36$	Net Current Assets
$X18$	Stock / Sales	$X37$	Working Capital
$X19$	Cash-flow / Sales		

Algorithm CART generates a binary decision tree. In each step, this tree is growing by choosing the independent variable which provides the best separation of the companies (in so-called parental node) into two (or more) child nodes. Each child node contains the largest possible proportion of individuals from the group of prosperous companies and the group of non-prosperous companies. Then, this procedure is repeated until the stop criterion is met. The separation stops when the maximal depth of the tree (five levels of separations) is reached, or if the number of companies in the parental node is less than 100, or if the next best separation would result to child node with less than 50 companies. As a measure of impurity, we use Gini index.

To avoid potential overfitting of the tree, the grown tree is post-pruned after automatically detecting the optimal pruning threshold. We use the 5-fold cross-validation to carry out this detection. The sub-tree giving the lowest error rate in this validation is then considered to be the best tree.

To evaluate the accuracy of the model, we use the analysis of the classification table. The classification table illustrates absolute and relative numbers of correctly and incorrectly classified cases into the pre-defined groups. We also assess the quality of the model using the ROC curve and the value of AUC (Area under the Curve). The closer the value of the AUC criterion is to 1, the better the model is.

4. Results

We used the sample of 41,692 Czech companies with balanced proportions of the prosperous and non-prosperous company to grow the decision tree using CART algorithm. In 2018, the financial distress situation of the companies was considered. In total, the values 37 financial indicators (not only ratios) from 2017 represent the set of possible predictors. In the first step, the CART algorithm has grown maximum decision tree according to the pre-defined stop criterion. In the next step, this tree was pruned using five-fold cross-validation. Quality of the final model (decision tree) was assessed by using the classification table and ROC curve.

The final CART-based model has five levels of nodes, five non-terminal nodes (five splitting conditions), and seven terminal nodes (see Figure 1). During the growing of the maximum tree, the algorithm used values of 21 financial ratios (of all 37). However, the final model works with values of four ratios only. In Figure 1, we can see that the variable $X10$ (Total Liabilities to Total Assets) is the most influencing variable because the value of this variable is used in the first separation (and in another two separations). Except for this variable, the model works with values of another three financial ratios - $X25$ (Current ratio), $X27$ (Return on Assets) and $X28$ (Return on Equity). These ratios belong to the commonly used ratios in prediction models (Svabova and Michalkova, 2020).

Figure 1: Final model (decision tree)

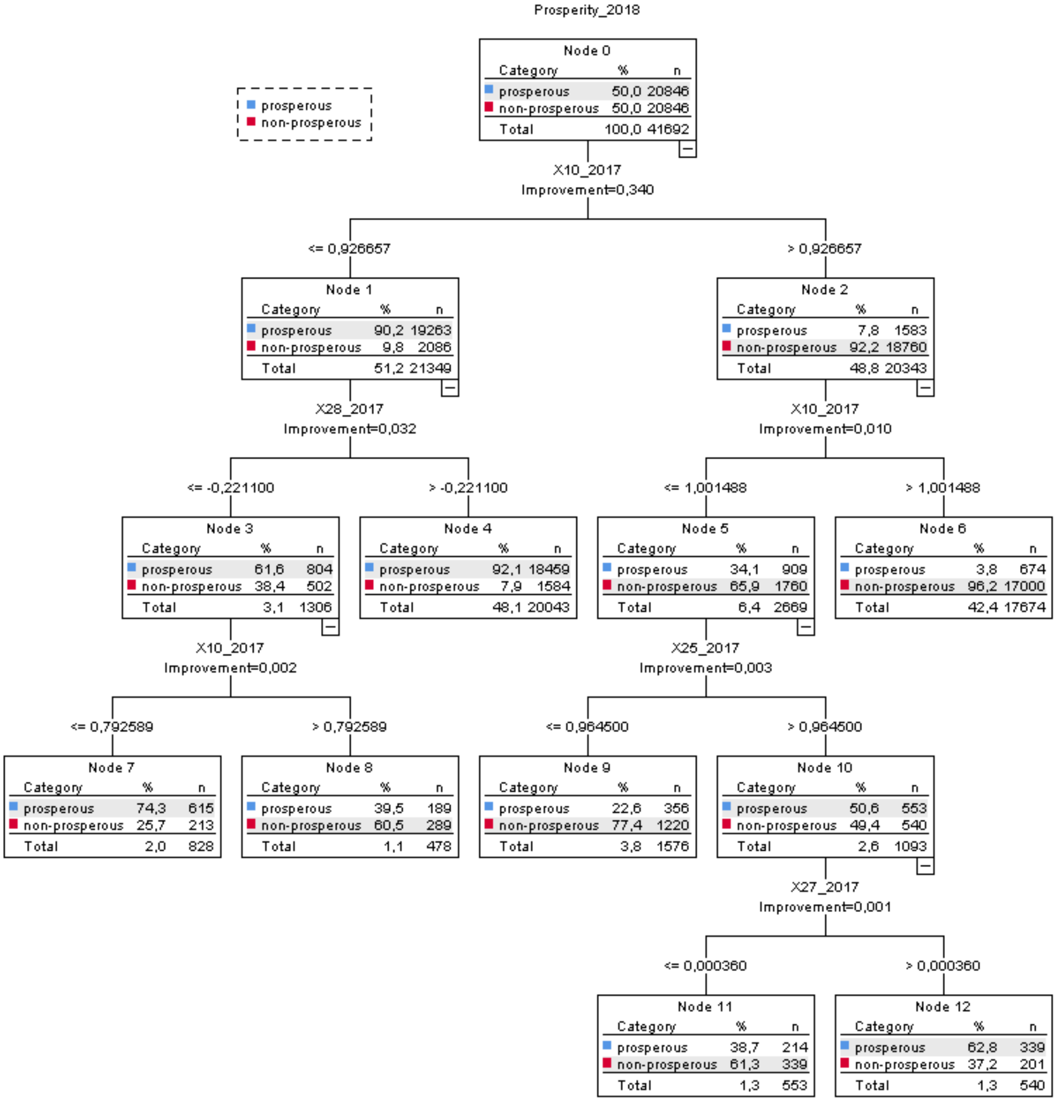


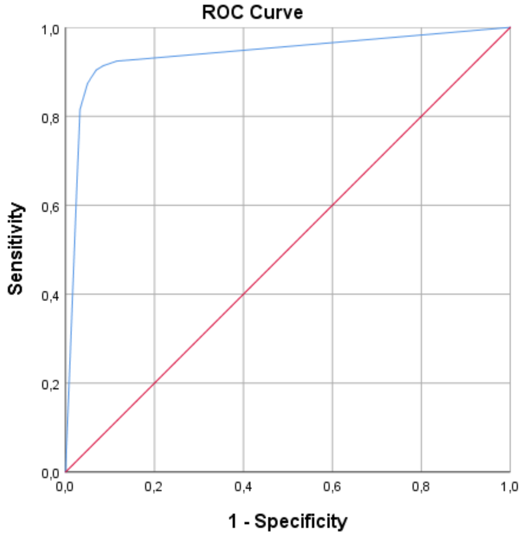
Table 2 illustrates the classification ability of the created model that is not over-estimated. It is because the five-fold cross-validation was used and thus the values in the classification table represent the real quality of the model for the classification of the companies into the groups of prosperous and/or non-prosperous companies. In Table 2, we can see that the classification ability is similar in both prosperous and non-prosperous companies. The overall accuracy is nearly 92%. The model classified the prosperous companies a little bit better, with an accuracy of 93.1%. However, even more than 90% of non-prosperous companies were classified correctly.

Table 2: Classification table

Observed	Predicted		Percent Correct
	prosperous	non-prosperous	
prosperous	19,413	1,433	93.1%
non-prosperous	1,998	18,848	90.4%
Overall Percentage	51.4%	48.6%	91.8%

Figure 2 illustrates the ROC curve of the created model. The shape of this curve indicates the high quality of the model in terms of both specificity and sensitivity. The classification ability of the model is proven by high value 0.939 of the AUC criterion that is very close to the hypothetical ideal value of 1.

Figure 2: ROC curve



All the above-mentioned criteria of quality and prediction ability of the model indicate that it is a quality model with a high prediction ability of potential future financial distress situation of Czech companies. This prediction is also very good in the category of non-prosperous companies (90.4%), which is probably the situation that the managers of companies using the model would like to focus on.

5. Conclusion

Although in recent decades many models to prediction of financial distress situation of the companies were created, so far there is no generally accepted prediction model considering the specific economic conditions and legislation of the Czech Republic. To fill this gap, we create a new prediction model based on the CART algorithm generating the decision tree. Dataset of 37 financial indicators of nearly 42,000 Czech companies calculated using real financial statements from 2017 was used. In 2018, if the financial ratios of the company met the conditions identified by the Czech legislation, financial distress situation of the companies was considered.

The final model achieves a high overall accuracy of nearly 92%. On the other hand, this is proposed in the form of a binary decision tree that is very simple to interpretation and to real implementation in the sense of classification of the new companies. It is because the model consists of very simple if-then rules and works with only four financial ratios. Thus, the model is practically applicable for the prediction of financial distress not only for the Czech companies that have incomplete accounting data.

The further direction of this research lies in verifying and/or modification of this model based on the data from new financial statements of Czech companies with the aim to find the model that can be generally accepted in the real condition of the Czech Republic. Further direction of the research could also lead to the creation of prediction models specific to individual sectors of economic activity of the company or for individual Czech regions.

Acknowledgement

This research was financially supported by the Slovak Research and Development Agency – Grant NO. APVV-14-0841: Comprehensive Prediction Model of the Financial Health of Slovak Companies and Faculty of Operation and Economics of Transport and Communications, University of Zilina, Slovakia – Institutional Grant No. 11/PEDAS/2019: Using of selected data mining methods in business failure prediction.

References

[1] Affes, Z. and Hentati-Kaffel, R., (2019). Forecast bankruptcy using a blend of clustering and MARS model: case of US banks. *Annals of Operations Research*, 281(1–2), pp. 27–64.

[2] Agrawal, K. and Maheshwari, Y., (2016). Predicting financial distress: revisiting the option-based model. *South Asian Journal of Global Business Research*, 5(2), pp. 268–284.

[3] Alaka, H.A., Oyedele, L.O., Owolabi, H.A., Kumar, V., Ajayi, S.O., Akinade, O.O. and Bilal, M., (2018). Systematic review of bankruptcy prediction models: Towards a framework for tool selection. *Expert Systems with Applications*, 94, pp. 164–184.

[4] Alaminos, D., Castillo, A. del and Fernández, M.Á., (2016). A Global Model for Bankruptcy Prediction. *PLOS ONE*, 11(11), e0166693.

[5] Altman, E.I., (1968). Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy. *The Journal of Finance*, 23(4), pp. 589–609.

[6] Altman, E.I., Iwanicz-Drozdowska, M., Laitinen, E.K. and Suvas, A., (2017). Financial Distress Prediction in an International Context: A Review and Empirical Analysis of Altman's Z-Score Model. *Journal of International Financial Management & Accounting*, 28(2), pp. 131–171.

[7] Gregova, E., Valaskova, K., Adamko, P., Tumpach, M. and Jaros, J., (2020). Predicting Financial Distress of Slovak Enterprises: Comparison of Selected Traditional and Learning Algorithms Methods. *Sustainability*, 12(10), 3954.

[8] Horak, J., Vrbka, J. and Suler, P., (2020). Support Vector Machine Methods and Artificial Neural Networks Used for the Development of Bankruptcy Prediction Models and their Comparison. *Journal of Risk and Financial Management*, 13(3), 60.

[9] Jakubík, P. and Teplý, P., (2011). The JT Index as an Indicator of Financial Stability of Corporate Sector. *Prague Economic Papers*, 20(2), pp. 157–176.

[10] Karas, M. and Reznakova, M., (2017). Predicting the Bankruptcy of Construction Companies: A CART-Based Model. *Engineering Economics*, 28(2), pp. 145–154.

[11] Kliestik, T., Valaskova, K., Lazaroiu, G., Kovacova, M. and Vrbka, J., (2020). Remaining Financially Healthy and Competitive: The Role of Financial Predictors. *Journal of Competitiveness*, 12(1), pp. 74–92.

[12] Kovacova, M. and Kliestik, T., (2017). Logit and Probit application for the prediction of bankruptcy in Slovak companies. *Equilibrium. Quarterly Journal of Economics and Economic Policy*, 12(4), pp. 775–791.

[13] Kovacova, M., Kliestik, T., Valaskova, K., Durana, P. and Juhaszova, Z., (2019a). Systematic review of variables applied in bankruptcy prediction models of Visegrad group countries. *Oeconomia Copernicana*, 10(4), pp. 743–772.

[14] Kovacova, M., Valaskova, K., Durana, P. and Kliestikova, J., (2019b). Innovation Management of the Bankruptcy: Case Study of Visegrad Group Countries. *Marketing and Management of Innovations*, (4), pp. 241–251.

[15] Kral, P., Valjaskova, V. and Janoskova, K., (2019). Quantitative approach to project portfolio management: proposal for Slovak companies. *Oeconomia Copernicana*, 10(4), pp. 797–814.

[16] Liu, J. and Wu, C., (2017). A gradient-boosting decision-tree approach for firm failure prediction: an empirical model evaluation of Chinese listed companies. *Journal of Risk Model Validation*, 11(2), pp. 43–64.

[17] Nyitrai, T. and Virag, M., (2019). The effects of handling outliers on the performance of bankruptcy prediction models. *Socio-Economic Planning Sciences*, 67, pp. 34–42.

[18] Ohlson, J.A., (1980). Financial Ratios and the Probabilistic Prediction of Bankruptcy. *Journal of Accounting Research*, 18(1), pp. 109–131.

[19] Prusak, B., (2018). Review of Research into Enterprise Bankruptcy Prediction in Selected Central and Eastern European Countries. *International Journal of Financial Studies*, 6(3), 60.

[20] Ravi Kumar, P. and Ravi, V., (2007). Bankruptcy prediction in banks and firms via statistical and intelligent techniques – A review. *European Journal of Operational Research*, 180(1), pp. 1–28.

[21] Svabova, L. and Michalkova, L., (2020). The impact of Data structure on classification ability of financial failure prediction model. In: T. Kliestik, ed. *19th International Scientific Conference Globalization and Its Socio-Economic Consequences 2019 - Sustainability in the Global-Knowledge Economy*, *SHS Web of Conferences*, 24. EDP Sciences. 05024.

[22] Svabova, L., Valaskova, K., Durana, P. and Kliestik, T., (2020). Dependency Analysis Between Various Profit Measures and Corporate Total Assets for Visegrad Group's Business Entities. *Organizacija*, 53(1), pp. 80–90.

[23] Valaskova, K., Durana, P., Adamko, P. and Jaros, J., (2020). Financial Compass for Slovak Enterprises: Modeling Economic Stability of Agricultural Entities. *Journal of Risk and Financial Management*, 13(5), 92.

[24] Virág, M. and Kristóf, T., (2005). Neural Networks in Bankruptcy Prediction - A Comparative Study on the Basis of the First Hungarian Bankruptcy Model. *Acta Oeconomica*, 55(4), pp. 403–426.

[25] Vochozka, M., (2017). Formation of Complex Company Evaluation Method Through Neural Networks on the Example of Construction Companies' Collection. *AD ALTA: Journal Of Interdisciplinary Research*, 7(2), pp. 232–239.

[26] Vochozka, M. and Rowland, Z., (2015). Prediction of the future development of construction companies by means of artificial neural networks on the basis of data from the Czech Republic. *Математичне моделювання в економіці*, (3), pp. 62–76.

[27] Vochozka, M., Straková, J. and Váchal, J., (2015). Model to Predict Survival of Transportation and Shipping Companies. *Naše more*, 62(SI), pp. 109–113.

[28] Zięba, M., Tomczak, S.K. and Tomczak, J.M., (2016). Ensemble boosted trees with synthetic features generation in application to bankruptcy prediction. *Expert Systems with Applications*, 58, pp. 93–101.

On the Influence of Culture on the Trading Behavior

Hana Dvořáčková [1], Marek Jochec[2]

Abstract

Hofstede (1991) proposed that "*culture is the software of the mind*", which was a significant inspiration for this paper. The goal is to examine the effect of culture, represented by the country of origin of traders, on the disposition effect, a crucial behavioral bias. Motivation to research behavioral biases, specifically the disposition effect, is to help traders to avoid them and make rational decisions when trading. Based on our results obtained on the experimental data set, it is possible to conclude that traders from New Zealand succumbed to the disposition effect differently than traders from other countries.

Key words

Behavioral finance, financial markets, culture, highly liquid markets, experimental finance

JEL Classification: G41, J93

1. Introduction

Modern IT technologies make the world of trading available to a wide range of traders, including those without any professional training. During the past years, several kinds of irrational behaviors, so-called behavioral biases, were described, while the disposition effect, well known as a tendency of investors to hold on losing stocks for too long and sell winning stocks too soon is considered one of them. This tendency contradicts expected utility maximization and conventional wisdom (Douglas and Diltz, 2004) and it is mostly explained based on the prospect theory to investment, proposed by Kahneman and Tversky (1979).

As stated in Dvořáčková (2020), the disposition effect has been described in the stock-investing context as a behavioral tendency of traders to hold on losing stocks for too long and sell winning stocks too early. This is based on the presumption of Kahneman and Tversky (1979) that "*a person who has not made peace with his losses tends to accept risks and gamble, which would not be otherwise acceptable*". Later on, Shefrin and Statman (1985) followed them and published the first work naming this behavioral bias particularly as the disposition effect. One of the most important work from this early period was done by Odean (1998), whereas he used records of 10 000 accounts to prove the disposition effect and concluded that the most obvious explanations, i.e., explanations based on informed trading, rebalancing, or transaction costs, fail to capture the essential features of the data.

This paper aims to examine the influence of culture on the disposition effect and follows from the author's dissertation thesis, Dvořáčková (2020). The idea to examine the culture differences based on the traders' country of origin raised from the GUHA method application, presented in that thesis. Culture can have significant impact on people's attitude to risk and money earning and handling. For example, Wang et al. (2018) compared Chinese and U.S.

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investors, whereas he concluded that female groups in China generate price bubbles statistically identical to those produced by exclusively male groups in both China and the U.S., all of which were significantly larger than the bubbles produced by exclusively female groups in the U.S. Hence their results suggest that gender differences in financial markets may be sensitive to culture. Tan et al. (2019) analyzed the trading history of traders from 21 countries and territories and examined how culture relates to the trading behavior and what are the implications for the liquidity and risk profiles of exchanges around the world. Motivation to research behavioral biases, specifically the disposition effect, is to help traders to avoid them and make rational decisions when trading.

The paper is of standard structure, the introduction is followed by Section 2, focused on the methodology, specifically the statistical methods used for hypothesis testing and experimental data description. Section 3 considers results of the research, and the conclusion is presented at the end of the paper.

2. Methodology & Data Used to Achieve the Goal

This section summarizes the methodology and experimental data used to test the hypothesis about the impact of the culture on the disposition effect, such as statistical methods and experimental data collection procedure.

2.1 Statistical Methods

Firstly, to assess the normality of the distribution of the data set, the Kolmogorov-Smirnov test was used. It is a nonparametric test which quantifies the difference between a cumulative distribution function of the reference distribution (e.g., normal distribution) and the empirical function.

To show the significance of the findings, the standard independent two-sample t-test for the difference between two means is used. The Levene's test is used for testing of equality of variances and based on its results the choice for the t-test is made. For the calculations and results presented in upcoming Section 3, the SPSS software was used, and all tests were done at the significance level 0.05. The statistical methods are not presented in high detail as there are well known. For detailed description see Dvořáčková (2020).

2.2 Experimental Data Collection Procedure

To achieve the goal of the paper, we used an experimental data set, collected by Dvořáčková et al. (2019) from 2009 to 2015 during lectures in several countries (for instance, the USA, New Zealand, Kazakhstan, etc.). In total, over 300 students were trading on the OANDA FX Trade Practice platform. OANDA offers three possible ways of trading: desktop trading platform, mobile trading platform, and Metatrader 4. Students were using usually the desktop trading platform at the beginning of the experiment in 2009 and the mobile trading platform since 2015. OANDA offers a wide range of trading instruments (over 100), including currency pairs, Commodity CFDs, Index CFDs, Bonds CFDs, and Metals.

Besides trading, OANDA provides also a wide range of information about the current global political and economic situation as well as many tools supporting decision-making based on the technical analysis. These include, for example, Volatility Graph, Correlation Table, Candlestick Patterns, or Currency Strength Heatmap. Students were informed about those tools and provided by basic training, which should support them to make better investment decisions.

Initially, students were given \$100 000, meanwhile the trading period was standardized and took three months. As those students did not trade with real money, they were motivated to achieve as good results as possible by a financial reward together with some extra points for the exam on account of the winner (student with the highest account balance at the end of the trading period). One of the learning objectives was to experience the first-hand trading and use the explained technical analysis techniques in practice. At the end of the given trading period, students had to submit in detail the recorded trading history together with their answers to a short questionnaire and demographic information. There were collected students' login information and passwords of their official game account as well, therefore it was not possible to change the account later, reset losses, use more accounts, etc.

The data generation and collection process proceeded as follows. The course was started with a series of lectures and assignments designed to explain the currency trading basics and the use of the trading platform. The game was launched sometime between the second and fourth week of the semester and was running for the rest of the semester (60 to 90 days). Soon the focus shifted to another topics and the game continued in the background. The rules and interference with students were minimal, students were not asked for any specific strategy, neither encouraged nor discouraged to the use of fundamental or technical analysis; there was no ``preferable'' amount, frequency, size, or currency to be traded. The winner was the student with the highest trading account balance at the end of the given time period. The ending profit or loss did not affect the course grade except that the winner (and only the winner) earned a few extra points towards the final course grade, and in some cases a voucher to a bookshop.

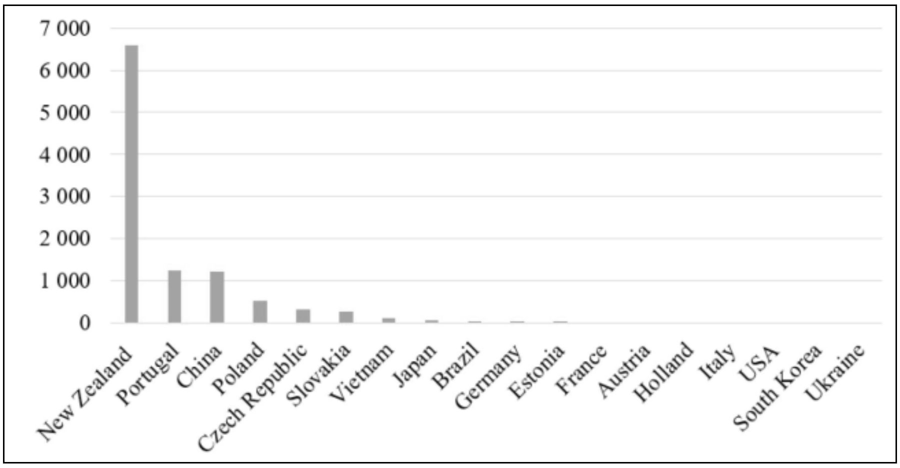
The setting of the game in the currency market is convenient as it is liquid and close to efficient, further explanation to be found in Dvořáčková (2020). It is difficult to make meaningful price predictions and trading is more a matter of luck then skill. Accordingly, the skill component does not distort the picture and the trading patterns and strategies tend to be more behavioral in nature. The experimental setting has one obvious disadvantage, the money is not real, thus the joy of winning (pain of losing) is moderated. This should be slightly counteracted by awarding the winner to support students' motivation to achieve as good results as possible. The counterargument might be that the "winner takes it all" reward scheme is problematic, there is no incentive for scoring second (third, etc.); similarly, scoring low does not bear any penalty. This and the fact that it is hard to predict currency rates even for professional traders means that students were in effect encouraged taking higher-than-normal risk and engaged in "all or nothing" gamble. It was not possible to perfectly rule those problems out, however, there is no indication of a more frequent occurrence of large bets on the last few days of the game, which would point out a tendency towards pure gambling. It can be assumed that students derived some benefits also from simply doing well, even if not the best. This could result from the long-term continuance of the experiment and the psychological benefit (cost) of a favorable (unfavorable) comparison with the peers. It is possible to see a parable with gaming. People around the world are playing several online games which are for free and provide no reward for playing, just a comparison with other gamers within a ranking. According to Morgan Stanley (2012), the social media gaming has around 800 million monthly users around the world, thus 800 million of people are motivated to play a game every month to enjoy a peer pride among people who even do not know. They are even willing to pay for some extra features to achieve a better position, see Gainsbury et al. (2017).

One can identify various advantages of the experimental setting. Firstly, participants do not self-select for roles, thus the sample is less biased. Second advantage of the experimental setting is the homogeneity of objectives. Moreover, in the real life the trader's wealth is given by the sum of different assets, hence a loss in OANDA may be compensated by the gain of another asset and a high net asset value person may trade differently from a low net asset value person. It is significantly different if the traded amount is 10% of the trader's wealth or 100\%. The source of money (gifts, heritage, own business, loans, etc.) matters as well.

3. Results

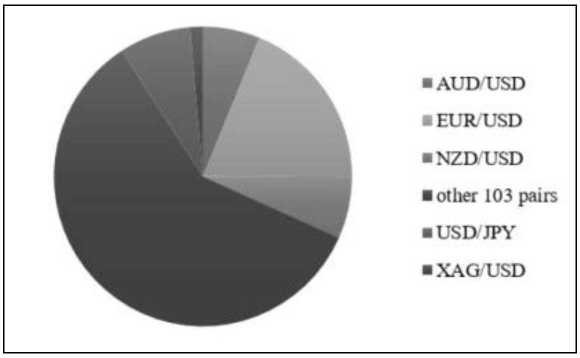
This paper aims to assess the impact of the culture, herein represented by the traders' country of origin. As proposed in Hofstede 1991, culture is the "*software of the mind*". Especially culture can have significant impact on people's attitude to risk and money. Traders of this research became from several countries, which are depicted in Figure 1. This Figure also shows the number of trades made by traders from particular countries, whereas more than half of the trades was done by traders from New Zealand. Therefore, it is examined if there is any difference in length of trades between students from New Zealand and those from other countries

Figure 1: Number of Trades According to the Students' Home Countries



Most trades were done by students from New Zealand, however, looking at the currency pairs used for trading, most often students used the currency pair EUR/USD, a deeper breakdown is presented in Figure 2. Only 5 most often used pairs are shown separately, and the other 103 minor pairs were merged to increase the clarity of this figure. It is interesting that significantly most students were from the New Zealand, but the most often used currency pair was not any combination with NZD. This leads to an idea that students did not succumb to the home bias, which was defined by French et al. (1991) as a tendency of investors to hold nearly all investments in domestic assets. It is mostly focused on stocks or bonds, but in principle it might be applicable also on the FX market.

Figure 2: Currency Pairs Used by Students for Trading



Initially, the normality of distribution of the data set was tested, which is to the normal distribution, which is dependent on *mean* (μ) and standard *deviation* (σ^2). Data of standardized normal distribution has mean equal to zero and standard deviation equal to one. For testing of normality of the data set the Kolmogorov - Smirnov test was used, a non-parametrical test, which was done in SPSS. The null hypothesis states that the distribution function of random data corresponds to the theoretical distribution function. In case of its rejection if Sig < α, where $\alpha = 0.05$, data are not deemed as normally distributed. As the Kolmogorov - Smirnov test for length of trades resulted in Sig = 0.00 and test statistic = 0.287, the null hypothesis has to be rejected and the test result to be that data are not normally distributed. According to Field (2013) it is possible to adjust the non-normal data by, for example, log-transformation or trimming, however, it did not help to achieve the normal distribution. Nevertheless, it was still possible to use parametrical tests for normally distributed data and to rely on the Law of Large Numbers, as stated also in Field (2013).

Looking at the basic characteristics, i.e., mean length of trades, students from New Zealand held trades on average almost 4 days, whereas students from other countries only 2.7 days. This approach to the disposition effect measuring is called duration approach (Eom 2018), and it is more suitable to be used in the high frequency trading environment than the frequency approach, used in Odean (1998). Statistical significance of this information should be tested. Firstly, the F-test at the significance level $\alpha = 0.05$ was done, whereas the null hypothesis was H_0: $\sigma_{nz} = \sigma_{other}$, and the alternative hypothesis H_1: $\sigma_{nz} \neq \sigma_{other}$, where σ_{nz} represents the variance of trades made by students originating from New Zealand and σ_{other} represents the variance of trades made by students from other countries. As $F > F_{crit}$ ($F = 1.95$, $F_{crit} = 1.04$, $p = 2.01E^{-115}$), it is possible to reject the null hypothesis. The test is statistically significant as $p < \alpha$. Based on this result, the two-sample two-tailed t-test for different variances was chosen to test the difference in mean values of the length of trades. The statistical hypotheses were set as follows

H_0: $\mu_{nz} = \mu_{other} \rightarrow$ *origin from New Zealand has no effect on the length of trades,*
H_1: $\mu_{nz} \neq \mu_{other} \rightarrow$ *origin from New Zealand has an effect on the length of trades,*

where μ_{nz} represents the mean length of trades made by students originating from New Zealand and μ_{other} represents the mean length of trades made by students from other countries. Those hypotheses were determined based on the assumption that in case of significantly different lengths of trades within the two groups, those groups should have different tendency to succumb to the disposition effect bias. The difference was found statistically significant, as $t > t_{crit}$ ($t = 8.22$, $t_{crit} = 1.96$, $p = 2.21E^{-16}$).

Subsequently, the difference in holding period within both groups (New Zealand and Others) should be tested. Firstly, trades made by students from New Zealand, recognized according to the profit and loss were considered. The F-test hypotheses were set as H_0: $\sigma_{nzP} = \sigma_{nzL}$ and H_1: $\sigma_{nzP} \neq \sigma_{nzL}$ where σ_{nzP} represents the variance of the length of profitable trades made by students originating from New Zealand and σ_{nzL} represents the variance of loss trades made by students from New Zealand. The F-test resulted in $F < F_{crit}$ ($F = 0.38$, $F_{crit} = 0.94$, $p = 0.00$), therefore it is not possible to reject the null hypothesis and assume a difference in variance. Hypotheses for the one-tailed t-test were set as H_0: $\mu_{nzP} = \mu_{nzL}$ and H_1: $\mu_{nzP} \neq \mu_{nzL}$., while the t-test resulted in the null hypothesis rejection ($t = -7.64$, $t_{crit} = 1.65$, $p = 1.21E^{-14}$) and it is possible to assume that length of loss trades is significantly shorter than length of profitable trades, taking into account only students originating from New Zealand. Regarding the students originating from other countries, the F-test based on the same hypothesis as in the previous case, but assuming variance of length of trades of students from other countries resulted in $F = 0.62$, $F_{crit} = 0.93$, and $p = 0.00$, thus $F < F_{crit}$ and the null hypothesis cannot be rejected. Following one-tailed t-test was not found statistically significant, therefore also the two-tailed t-test was performed, but with the same result, as $p > \alpha$. Detailed results are presented in Table 1.

Moreover, the difference within profitable and loss trades should be tested to see whether there is a statistically significant difference between New Zealand and other countries, considering solely profitable or loss trades. The F-test hypotheses to be given as H_0: $\sigma_{otherP} = \sigma_{nzP}$ and H_1: $\sigma_{otherP} \neq \sigma_{nzP}$. Where σ_{otherP} represents the variance of the length of profitable trades made by students originating from other countries and σ_{nzP} represents the variance of profitable trades made by students from New Zealand. As $F = 1.53$ and $F_{crit} = 1.06$, $F > F_{crit}$, it is possible to reject the null hypothesis and assume the difference in variance. Consequently, the two-tailed t-test for different variances was performed for the null hypothesis H_0: $\mu_{otherP} = \mu_{nzP}$ and the alternative hypothesis H_0: $\mu_{otherP} \neq \mu_{nzP}$, where μ_{otherP} represents the mean of the length of profitable trades made by students originating from other countries and μ_{nzP} represents mean of profitable trades made by students from New Zealand. As $t = 4.54$ and $t_{crit} = 1.96$, thus $t > t_{crit}$ and the null hypothesis can be rejected. The difference in the mean length of profitable trades done by students from New Zealand compared to other countries is assumed to be statistically significant. Testing of this difference for loss trades was done in the same way, while regarding the F-test, $F = 2.52$ and $F_{crit} = 1.08$, thus $F > F_{crit}$. The two-tailed t-test provided us with statistically significant results, whereas $t = 7.28$ and $t_{crit} = 1.96$, hence $t > t_{crit}$ and the difference in mean length of trades can be assumed statistically significant also in this case. Table 1 summarizes the results of all statistical tests.

Table 1: Statistical Tests

	NZ vs. Others	NZ	Others One-tailed	Others Two-tailed	NZ vs. Others Profitable Trades	NZ vs. Others Loss Trades
			F-test			
F	1.95	0.38	0.62	0.62	1.53	2.51
p	$2.01E^{-115}$	0.00	0.00	0.00	$5.99E^{-31}$	$6.63E^{-80}$
F_{crit}	1.04	0.94	0.93	0.93	1.06	1.08
			T-test			
t	8.22	-7.64	-1.30	-1.30	4.54	7.28
p	$2.21E^{-16}$	$1.21E^{-14}$	0.09	0.19	$5.85E^{-6}$	$3.9E^{-13}$
t_{crit}	1.96	1.65	1.65	1.96	1.96	1.96

4. Conclusion

The initial hypothesis of this paper assumes difference in holding periods of profitable and loss trades between students originating from New Zealand and from other countries, due to the general cultural differences. Holding the loss trades longer than profitable trades leads to the assumption of the disposition effect presence[3].

Statistical tests confirmed the difference to be statistically significant, students from New Zealand tend to keep trades significantly longer than other student, whereas their mean holding period was 49% higher. Moreover, one-tailed t-test confirmed that students from New Zealand

[3] Douglas and Diltz (2004), Dvořáčková (2020), Eom (2018), Kahneman and Tversky (1979), Odean (1998), Shefrin and Statman (1985)

held loss trades significantly longer than profitable ones. This was not possible to confirm for students from other countries, which leads us to conclusion, that students from New Zealand succumbed more to the disposition effect, than other students. Final tests to assess solely profitable or loss trades also proved a statistically significant difference between students from New Zealand and other countries.

Acknowledgement

This work has been supported by the Czech Science Foundation (GAČR) under the project GJ20-25660Y and SP2020/11, an SGS research project of VŠB – TU Ostrava, and within RRC/10/2017 support scheme of the Moravian-Silesian region.

References

[1] Douglas, J. and Diltz, J.D. (2004). Day traders and the disposition effect. *Journal of Behavioral Finance,* 5(4), p. 192–200.

[2] Dvořáčková, H. (2020). *Disposition Effect in the Context of Experimental Trading on Highly Liquid Financial Markets.* Dissertation Thesis. VŠB-TUO.

[3] Eom, Y. (2018). The opposite disposition effect: Evidence from the korean stock index futures market. *Finance Research Letters,* 26, p. 261–265.

[4] Field, A. P. (2013). *Discovering statistics using IBM SPSS Statistics: and sex and drugs and rock'n' roll*. 4th ed. London: SAGE.

[5] French, K. R., Poterba, J. M. (1991). Investor Diversification and International Equity Markets. *Working Paper Series.*

[6] Gainsbury, S.M., King, D.L., Russell, A. M., Delfabbro, P., and Hing, N. (2017). Virtual addictions: An examination of problematic social casino game use among at-risk gamblers. *Addictive Behaviors,* 64, p. 334–339.

[7] Hofstede, G. (1991). *Organizations and cultures: Software of the mind.* London/New York: McGrawHill.

[8] Kahneman, D. and Tversky, A. (1979) Prospect theory: An analysis of decision under risk. *Econometrica,* 46, p. 171–185.

[9] Morgan Stanley (2012). Social gambling: click here to play. *Morgan Stanley Research: Global* 23, p. 171–175.

[10] Odean, T. (1998). Are investors reluctant to realize their losses. *Journal of Finance,* 53, p. 1775–1798.

[11] Shefrin, H., and Statman M. (1985) The disposition to sell winners too early and ride losers too long: Theory and evidence. *Journal of Finance* 40, p. 777–790.

[12] Tan, G., Cheong, Ch. S., and Zurbruegg, R. (2019). National culture and individual trading behavior. *Journal of Banking and Finance*. 106, p. 357–370.

[13] Wang, J., Houser, D., and Xu, H. (2018). Culture, gender and asset prices: Experimental evidence from the U.S. and China. *Journal of Economic Behavior Organization*. 155, p. 253–287.

Efficiency of Credit Risk Management of Selected Commercial Banks in The Czech Republic

Xiaoshan Feng [1]

Abstract

This paper focuses on measuring the efficiency of credit risk management of banks in the Czech Republic. We apply the Data Envelopment Analysis (DEA) on 12 commercial banks in the Czech Republic over the period from 2012 to 2018. We find that under different assumptions of returns to scale, the efficiency score of selected banks ranges from 0.28 to 0.43 under CCR model and ranges from 0.61 to 0.71 under BCC model. Moreover, strong evidence from the Malmquist Index showed that the Czech banking sector improved its efficiency during the past 7 years due to innovation in credit risk management. Furthermore, we employ logistic regression to find out that the likelihood of a bank being efficient increases with larger size and higher yearly change of CAR, while decrease with higher GDP growth rate with lag effect under CCR model. And increases with lower profitability under BCC model.

Key words

Data Envelopment Analysis, Malmquist Index, Credit risk management, The Czech Republic, Logistic regression.

JEL Classification: G21, C35, C67, C80, C61, C58

1 Introduction

As a robust growth rate of the volume of bank loans in recent years, commercial banks need to be more cautious about the quality of their assets. Therefore, to investigate the efficiency of credit risk management of commercial banks becomes one of the most important steps to measure the overall soundness of the banking sector.

As for the banking sector in the Czech Republic, there is abundant literature focus on the bank operational efficiency, and it is lack of timeliness. However, it is rare to investigate efficiency from the view of credit risk management of banks. Furthermore, scholars have investigated the determinants of bank efficiency through a dynamic panel data model in previous literature, while it is scarce research to figure out the determinants of credit risk efficiency from external and internal impacts. Therefore, research based on current credit risk management efficiency of the Czech banking sector and its determinants is the exhibited knowledge gap.

Motivated by this gap, the aim of this paper is to evaluate the performance of credit risk management reflected on global technical efficiency (CCR) and pure technical efficiency (BCC), and productivity change of selected commercial banks in the Czech Republic from 2012-2018. Furthermore, based on the efficiency results, we will get insight into the possible determinants of credit risk management efficiency.

We apply Data Envelopment Analysis and the Malmquist index (MI) to measure the efficiency of credit risk management and productivity change of 12 selected banks in the Czech Republic during the period from 2012-2018. Moreover, we employ a regression model to investigate the

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possible internal and external determinants of credit risk management efficiency. This paper is divided into five sections. The first section starts with the introduction and the last one ends with the conclusion. The second section includes the literature review. Section 3 presents a brief description of methodology and data collection. In the fourth section, the empirical results will be discussed.

2 Literature Review

There is rich literature focused on bank efficiency with Data Envelopment Analysis (DEA). Řepková (2012) finds that the efficiency scores from the BCC model have higher values than from the CCR model due to the elimination of deposit management inefficiency. To conduct a DEA estimation, the research employs labour and deposits as inputs, then loans and net interest income as two outputs. The study applied the Malmquist index to estimate the efficiency change in the Czech banks over time from 2001 to 2010. The negative growth on efficiency indicates the industry has lacked innovation or technological progress during the time. Moreover, the catch-up effect has a more accountable result than the frontier-shift effect.

To incorporate risk into bank efficiency has been raised among analysts due to the recent financial crisis. Tsolas and Charles (2015) provide research on the efficiency profile of the Greek banking sector based on the DEA model, in which the financial risk is proxied by credit risk provisions. The study uses a satisficing DEA model that treats loan loss provision as a stochastic controllable input, which corresponding to the intermediation approach.

In real-life applications, undesirable outputs such as non-performing loans may present in the banking sector which needs to be minimized. There is an abundance of research that incorporates undesirable outputs into the analysis. Paradi and Zhu (2013) have surveyed bank branch efficiency and performance research with DEA. The study mentioned three approaches when non-performing loans are incorporated in previous literature. The first is to leave NPL as an output but use the inverse value. The second method is to treat this undesirable output as input, which applied in other studies (Puri and Yadav, 2014; Toloo and Hanclova, 2019). The third one is to treat it as an undesirable output with an assumption of weak disposability, which requires that undesirable outputs can be reduced, but at a cost of fewer desirable outputs produced.

Partovi and Matousek (2019) analyze technical and allocative efficiencies in Turkish banks from 2002 to 2017, under the assumption of constant returns to scale. The study applies a modified version of the DEA approach which employs a directional distance model to provide estimates of efficiency, with a focus on NPLs as an undesirable output.

Gaganis et al. (2009) also examine the efficiency and productivity of a Greek bank's branches from 2000 to 2005. The finding shows that the inclusion of loan loss provisions as an input variable increases the efficiency score, then, fixed and random effects models were used to determine the impact of internal and external factors on the efficiency and productivity scores.

Notwithstanding there are abundant studies focused on the efficiency of the Czech banking sector, it is rare to investigate efficiency from the view of credit risk management of banks. Therefore, we incorporate non-performing loans as a proxy of credit risk to measure the efficiency of credit risk management in the banking sector.

3 Methodology and Data

Based on the previous literature, we apply Data Envelopment Analysis and the Malmquist index to measure the efficiency of credit risk management and productivity change of 12 selected banks in the Czech Republic during the period from 2012-2018. Moreover, we employ a regression model to investigate the possible internal and external determinants of probability of credit risk management efficiency. This section will briefly describe the essence behind each methodology and the collection of data for corresponding analysis.

3.1 Two Classic Models of Data Envelopment Analysis

DEA is a linear programming-based method, which introduced by Charnes, Cooper and Rhodes in 1978. DEA is used to evaluate the relative efficiency of a set of decision-making units (DMUs) with multiple inputs and multiple outputs. Then, Banker, Chames and Cooper has proposed a model in 1984, named the BCC, which is an extended version of the CCR model. The main difference between these two models is different returns to scale. The CCR model assumes all DMUs are operating at an optimal scale, that is, constant returns to scale (CRS); While the BCC model assumes variable returns to scale (VRS).

In DEA models, we measure the efficiency of each DMU. One of the most frequently used methods to measure efficiency is by the ratio. Suppose we have n DMUs in the population, each DMU produces s outputs while consuming m inputs. Consider DMU_j, j represents n DMUs, x_{ri} and y_{ri} are the matrixes of inputs and outputs respectively. The efficiency rate of such a unit can be expressed as:

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \tag{1}$$

The efficiency rate is the ratio of the weighted sum of outputs to weighted sum of inputs. The DEA model assumed inputs and outputs should be non-negative. Let DMU_j to be evaluated on any trial be designated as DMU_o, where $o = (1,2, \dots, n)$.

A ratio of two linear functions can construct the linear-fractional programming model as follows:

$$\max_{v,u} \theta = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \tag{2}$$

$$\text{subject to } \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, (j = 1,2, \dots, n) \tag{3}$$

$$u_1, u_2, \dots, u_r \ge 0, (r = 1,2, \dots, s) \tag{4}$$

$$v_1, v_2, \dots, v_i \ge 0, (i = 1,2, \dots, m) \tag{5}$$

where θ is the technical efficiency of DMU_o to be estimated, v_i $(i = 1,2, \dots, m)$ is the optimized weight of input and the output u_r $(r = 1,2, \dots, s)$. y_{rj} is observed amount of output of the r-th type for the j-th DMU, x_{ij} is observed amount of input of the i-th type for the j-th DMU.

Moreover, we will apply the Malmquist index to deal with our panel data, to evaluate the productivity change of a DMU between two time periods and is an example in comparative statistical analysis. Farrell developed the Malmquist index as a measurement of productive efficiency in 1957, then Fare decomposed MI into two terms in 1994, it can be defined as "Catch-up" and "Frontier-shift" terms. The catch-up term indicates the degree of a DMU improves or worsens its efficiency. The frontier-shift term is used to figure out the change in the efficient frontiers between two time periods. The Malmquist index is computed as the geometric mean of Catch-up and Frontier-shift.

$$MI = (Catch - up) \times (Frontier - shift) \tag{6}$$

when MI larger than 1, it means progress in the total factor productivity of the DMU_o from period 1 to period 2, while MI equals 1 means no change, and MI less than one indicates deterioration in the total factor productivity.

3.2 Data Selection for Measuring the Efficiency by DEA and MI

To make sure the results of applying DEA are accurate, the number of inputs and outputs, and DMUs must support the rule of thumb, which proposed firstly by Golany and Roll (1989), then developed by Bowlin (1998), that is, it should have three times the number of DMUs as there are input and output variables, if this condition will not be met, the results are not reliable (Toloo and Tichy, 2015).

In this paper, we have collected data of 12 commercial banks out of 23 commercial banks which include the large-size bank such as Česká spořitelna, ČSOB; Medium size such as MONETA money bank, Equa bank; Small size such as PPF bank, Air Bank. All data are collected from the annual report of each bank in a consolidated basis. In the data set, twelves banks are observed over seven years (2012-2018). In total, we have a balanced panel of 72 observations.

To select the inputs and outputs, there are basically three approaches when we apply DEA in the banking sector. The first one is production approach, in which, bank plays a role of producer, transfer some physical inputs such as labours into services (Sealey and Lindley, 1977). The next one is intermediation approach, that is, bank acts as an intermediary, transfer funds or deposits from clients to loans and investments (Berg et al., 1993), which is more commonly used in analyzing banking efficiency. Thirdly, the profitability approach measures how efficiently a bank maximizes its profits by decreasing expenses while increasing revenue (Eken et al., 2014).

This study aims to investigate the efficiency of credit risk management in the Czech banking sector. Therefore, we will apply the intermediation approach to measure the efficiency of credit risk management based on DEA model. To assess credit risk modelling in banking industry, Berg et al. (1992) used non-performing loans as a proxy of credit risk in a nonparametric study of the bank production, Altunbas et al. (2000) incorporate loan loss provisions to analyze the efficiency of Japanese bank.

Therefore, based on previous literature, we proxy credit risk by the ratio of non-performing loans to total gross loans, so-called NPL ratio. Then use loan loss provision ratio to represents the ratio of provision and non-performing loan, which is primarily to reflect commercial banks' abilities to compensate for loan losses and to protect against credit risk. Generally, this paper developed two inputs and one output, with 12 DMUs which satisfy the rule of thumb. $Input\ x_1$ is loan loss provision, $Output\ y$ loans and receivables as output, non-performing loan as undesirable output, based on the treatment of undesirable output from previous literature, NPL is transformed as $Input\ x_2$.

3.3 Logistic Regression Model

Furthermore, after we obtain efficiency score based on the CCR model and the BCC model, we can estimate the determinants of banking credit risk management efficiency using regression model, Since the efficiency can be measured as binary outcomes, we can model the conditional probabilities of the response outcome, rather than give a binary result. Therefore, we apply logistic regression model in this paper. The logistic model could be interpreted based on an underlying linear model, shows below:

$$Y_{i,t} = \beta_0 + X'_{i,t}\beta + \epsilon_{i,t}\ , i = 1, \dots, N, t = 1, \dots, T. \tag{7}$$

Where the subscripts i and t denote the cross-sectional (N) and time dimension (T) of the panel data, respectively. There is k $(k = 1, \dots, K)$ regressor in $X_{i,t}$, not including a constant term. $X_{i,t}$ is explanatory variable value for i-th section at t-th dimension; β_0 is the intercept; β is the slope coefficient of a $(k \times 1)$ vector. The variable $\epsilon_{i,t}$, can be called as the error term in the relationship, represents factors other than explanatory variables that affect dependent variables.

Since we have a binary output variable $Y_{i,t}$, and we want to model the conditional probability $p(Y_{i,t} = 1|X'_{i,t} = x_{i,t})$ as a function of $x_{i,t}$:

$$\pi(x_{i,t}) = p(Y_{i,t} = 1|X'_{i,t} = x_{i,t}) \tag{8}$$

The logistic regression model can be constructed as follows:

$$log\,\frac{\pi(x_{i,t})}{1-\pi(x_{i,t})} = \beta_0 + X'_{i,t}\beta\,, i = 1, \dots, N, t = 1, \dots, T. \tag{9}$$

3.4 Data Selection for Logistic Regression Model

The starting point for model construction, we aim to investigate the determinants of efficiency of bank's credit risk management, therefore, our dependent variable is efficiency score obtained from previous mentioned DEA model. The independent variables are, respectively, size of the bank, which measured as the natural logarithm of the value of total assets in Czech koruna; Capital Adequacy Ratio (CAR), measured by dividing a bank's total capital by its risk-weighted assets; Return on average assets (ROAA), it is calculated as the ratio of net income to average total assets; GDP growth rate, which is the year-on-year annual GDP growth rate. The binary outcomes are measured by 1 and 0, which represent *DMU* is efficient and inefficient, respectively.

4 Empirical Results

Based on methodology described in Section 3.1, we can measure the efficiency under the two classic models, which are, the CCR model and the BCC model. To compute the empirical result, we used DEA SolverPro™. Table 1 presents the average scores of efficiencies under the CCR model and the BCC model. We find that under different assumptions of returns to scale, the average global technical efficiency of selected banks ranges from 0.28 to 0.43 and the average of pure technical efficiency ranges from 0.61 to 0.71.

Table 1: Efficiency score under the CCR and the BCC (Industry average)

	2012	2013	2014	2015	2016	2017	2018	Average
CCR	0.33	0.43	0.31	0.34	0.28	0.28	0.31	0.33
BCC	0.71	0.64	0.61	0.65	0.65	0.61	0.71	0.65

The average of efficiency scores under the CCR model is 0.33 for selected banks. It provides the results that if bank can produce its outputs on the efficient frontier, compared to the inputs which currently being used, only 33% of the inputs needed. Put differently, if selected banks in the Czech Republic adopt best practice technology, they can reduce inputs (i.e. more efficient credit risk management, and reduction in loan-loss provisions) at least 69% and still produce the same level of outputs. However, the potential reduction in inputs from adopting best practices varies from bank to bank. The results of BCC model showed a higher efficiency score than results from the CCR model. However, ČSOB still performed well among selected banks.

Table 2: Efficiency score under the CCR and the BCC (Yearly average)

	CS	KB	CSOB	RB	CRED	AIR	PPF	FIO	MMB	UNI	JT	EQUA
CCR	0.89	0.77	1.00	0.23	0.01	0.03	0.15	0.04	0.08	0.50	0.13	0.08
BCC	0.96	0.86	1.00	0.57	0.41	0.46	0.52	0.53	0.34	0.68	0.56	0.97

Based on results, ČSOB has not only global technical efficiency but it also has pure technical efficiency, which forms the reference set for inefficient banks. For a specific DMU, if the technical efficiency scores differ in the CCR model and the BCC model, then the DMU presents an inefficiency of scale. EQUA bank has been one of the most inefficient banks under the CCR model, while it showed inverse results under the BCC model, which presents a large inefficiency of scale.

Next, we applied the Malmquist index (MI) to measure productivity change of a DMU between two time periods, Table 3 showed the overall technological productivity and its decomposition.

Table 3: Malmquist index and its decomposition

	Catch-up	Frontier	Malmquist
2012=>2013	1.79	1.04	1.85
2013=>2014	0.73	1.32	0.97
2014=>2015	1.08	1.08	1.17
2015=>2016	0.92	1.69	1.56
2016=>2017	1.06	1.62	1.72
2017=>2018	1.06	1.25	1.33
Mean	1.07	1.31	1.40

The measurement of the MI reflects the trend of overall productivity of the sample banks during periods. The Malmquist (total factor productivity) exceeded unity except period 2 (2013-2014), which shows that selected banks continuously improved their development in credit risk management and achieved an average 40% increase in the overall efficiency during past 6 periods, although slightly regress in second period.

Furthermore, we can discover the results from decomposition, it can be found that the frontier-shift effect was primarily accountable for the productivity growth rather than the catch-up effect. On average, the technical efficiency was growing at an annual rate of 7%, and the technological progress increased by 31% annually. Therefore, the technological progress (frontier-shift effect) was the main reason for the growth of total factor productivity in selected banks in the Czech Republic.

Under logistic regression, we estimated the likelihood of a DMU is efficient given the applicants SIZE, CAR, ROAA and GDPG. Table 4 presents the results of regression. We found only size and profitability were statistically significant in BCC model. Under CCR model, the probability of a bank being efficient increases with larger size and higher yearly change of CAR, while decrease with higher GDP growth rate with lag effect. And the probability increases with larger size and lower profitability under BCC model.

Table 4: Regression results

	CCR model		BCC model
Variables		Variables	
C	-132.3267*** [39.2733]	C	-16.233*** [5.4123]
SIZE	9.500*** [2.706]	SIZE	1.4002*** [0.4800]
LAGGDPG	-0.2766** [0.1399]	ROAA	-2.290* [0.9449]
DCAR	0.416* [0.238]		

5 Conclusion

In this paper, we aim to measure the efficiency of credit risk management of selected banks in the Czech Republic during the period from 2012-2018. Therefore, we apply two classic DEA models and the Malmquist index to measure efficiency and productivity change of 12 selected commercial banks in the Czech Republic. We find out ČSOB was the most efficient bank on credit risk management under the CCR model and the BCC model. Moreover, the Czech banking sector improved its efficiency during the investigated period mainly due to innovation (frontier-shift effect) in credit risk management. Furthermore, we employ logistic regression to get insight into the possible determinants of the likelihood of a bank managing credit risk efficiently. The probability of bank acting efficient increases with larger size and lower profitability under VRS, while it decreases with lower GDP growth rate with lagged effect in CCR model. Our findings provide strong evidence to explain how ČSOB was the most efficient DMU, that is, the second-largest bank in the Czech Republic and the lowest ROA among large-size banks. These findings also hold for bank efficiency in the Czech Republic within 2001-2012 (Řepková, 2015).

The main contribution of this paper is that we provide a big picture of efficiency on credit risk management of the Czech banking sector and find the main drivers of productivity growth. Furthermore, we investigate the determinants of probability for efficiency on credit risk management to provide more specific insight into the Czech banking sector.

References

[1] Altunbas, Y., Liu, M., Molyneux, P. and Seth, R. (2000). 'Efficiency and risk in Japanese banking', *Journal of Banking & Finance*, Elsevier, vol. 24(10), pp. 1605-1628

[2] Berg S.A., Forsund, F.R and Jansen, E.S. (1992). 'Malmquist Indexes of Productivity Growth During the Deregulation of Norwegian Banking, 1980-89' *The Scandinavian Journal of Economics* pp. S211-S228

[3] Berg, S.A., Forsund, F.R., Hjalmarsson, L. and Suominen, M. (1993). 'Banking efficiency in the Nordic countries', *Journal of Banking and Finance*, 17: pp. 371 – 388.

[4] Bowlin, W.F. (1998) 'Measuring Performance: An Introduction to Data Envelopment Analysis (DEA) '. *Journal of Cost Analysis* 7, pp. 3-27.

[5] Eken, M., Kale, S. and Selimler, H. (2014). 'Analyzing the Efficiency of European Banks: A DEA-Based Risk and Profitability Approach', *Business Science Reference*, pp. 28-55.

[6] Gaganis, C., Liadaki, A., Doumpos, M. and Zopounidis, C. (2009), 'Estimating and analyzing the efficiency and productivity of bank branches: Evidence from Greece', *Managerial Finance*, Vol. 35 No. 2, pp. 202-218. https://doi.org/10.1108/03074350910923518

[7] Golany, B. and Roll, Y. (1989) 'An Application Procedure for DEA', *Omega* 17, pp. 237-250.

[8] Paradi, J. C. and Zhu, H. (2013) 'A survey on bank branch efficiency and performance research with data envelopment analysis', *Omega*. Elsevier, 41(1), pp. 61–79. doi: 10.1016/j.omega.2011.08.010.

[9] Partovi, E. and Matousek, R. (2019) 'Research in International Business and Finance Bank efficiency and non-performing loans : Evidence from Turkey', *Research in International*

Business and Finance. Elsevier, 48(March 2018), pp. 287–309. doi: 10.1016/j.ribaf.2018.12.011.

[10] Puri, J. and Yadav, S. P. (2014) 'Expert Systems with Applications A fuzzy DEA model with undesirable fuzzy outputs and its application to the banking sector in India', *Expert Systems With Applications*. Elsevier Ltd, 41(14), pp. 6419–6432. doi: 10.1016/j.eswa.2014.04.013.

[11] Řepková, I. (2012) 'Measuring the efficiency in the Czech banking industry : Data Envelopment Analysis and Malmquist index', pp. 781–786.

[12] Řepková, I. (2015) 'Banking Efficiency Determinants in the Czech Banking Sector', 23(October 2014), pp. 191–196. doi: 10.1016/S2212-5671(15)00367-6.

[13] Sealey, C. W. and Lindley, J. T. (1977) 'Inputs, outputs, and a theory of production and cost at depository financial institutions', *The Journal of Finance,* 32(4): pp.1251–1266.

[14] Toloo, M., Hančlova J. (2019) 'Multi-valued measures in DEA in the presence of undesirable outputs', *OMEGA,* https://doi.org/10.1016/j.omega.2019.01.010

[15] Toloo, M., Tichy, T. (2015) 'Two alternative approaches for selecting performance measures in data envelopment analysis', *Measurement*, Vol. 65, pp. 29-40.

[16] Tsolas, I. E. and Charles, V. (2015) 'Expert Systems with Applications Incorporating risk into bank efficiency : A satisficing DEA approach to assess the Greek banking crisis', *Expert Systems With Applications*. Elsevier Ltd, 42(7), pp. 3491–3500.

Efficiency Measurement of OECD Insurance Industries with Data Envelopment Analysis

Biwei Guan [1]

Abstract

In recent years, the research of efficiency measurement has been widely concerned, especially in the insurance industry. As a non-parametric method, DEA can be used to calculate the various efficiency values of the insurance industry. As an important organization, OECD is also worth studying. This paper selects 20 insurance markets from OECD member countries as research objects. The life insurance market and non-life insurance market are discussed respectively, with data from 2013 to 2017. By using the BBC model and the CCR model, the corresponding technical efficiency, pure technical efficiency, and scale efficiency are obtained. We found that during the sampling period, Germany's life insurance market and non-life insurance market achieved technical efficiency and scale efficiency. From the results, we also have a better understanding of the efficiency and environment of the OECD insurance market.

Key words

data envelopment analysis, insurance industries, OECD

JEL Classification: C67, G15, G22

1. Introduction and Literature Review

In recent years, efficiency measures related to the insurance industry have been popular and attracted many regulators and investors. For the measurement of frontier efficiency, there are two main methods: Stochastic Frontier Analysis (SFA) and data envelopment analysis (DEA). Kaffash, Azizi, Huang, and Zhu [12] mentioned that recent papers have applied DEA rather than SFA to the evaluation of insurance companies, and more and more interested in evaluating the DEA efficiency of the insurance industry rather than other financial institutions. Therefore, this paper uses the DEA method to evaluate the frontier efficiency of insurance companies.

The purpose of this paper is to evaluate the technical efficiency, pure technical efficiency and scale efficiency of 20 OECD insurance industries through data envelopment analysis, and to understand the reasons and influencing factors of each market not reaching the effective state. This paper is divided into several parts. The first part is the introduction, the second part is the related previous research. The next section includes information about the selected samples, variables, and methods. The fourth part is the empirical results, which will show the results of technical efficiency, pure technical efficiency, and scale efficiency in each country. This part will introduce the findings of this paper, and the last part is the conclusion.

In previous studies, DEA and SFA were most commonly used to assess the efficiency of the insurance industry. Initially, SFA has advantages that DEA does not have. It can analyze influencing factors. However, after much research on improving and innovating DEA models such as Barros et al. [2], Kao and Hwang [13], and Yang and Pollitt [17] have been produced, this advantage has been greatly weakened. There is various research on the efficiency of

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insurance companies. Eling and Luhnen [7] mentioned that there are more than 90 studies on efficiency measurement in the insurance industry. Previous work focused on different documents. Eling and Schaper [8] and Vencappa et al. [16] observed changes in productivity and efficiency; Eling and Jia [6] and Greene and Segal [10] analyzed the relationship between efficiency and profitability; Kader et al. [11] studied the relationship between cost efficiency and board composition. What is missing from previous studies is that they only focus on one type of insurance market, or the samples are limited. Current research focuses on the U.S. market and European countries. In previous studies, many papers also chose OECD member countries as research objects. In Rai [15], more extensive cross-border research involving 11 OECD countries; Donni and Fecher [5] studied the measurement of technical efficiency in 15 OECD insurance industries, etc.

In Farrell [9], a method was created to evaluate the efficiency of modern companies, which mentioned that the efficiency of manufacturers includes two parts: technical efficiency (TE) and allocative efficiency (AE), both of which are included in total cost efficiency (CE). Technical efficiency can be further divided into two parts: pure technical efficiency (PTE) and scale efficiency (SE). PTE reflects the production efficiency of DMU input with the best scale, and the scale return can be changed; SE reflects the gap between the actual scale and the best production scale. This paper uses the CCR model (Charnes et al. [3]) and the BBC model (Banker et al. [1]) to study the technical efficiency, pure technical efficiency, and scale efficiency of insurance companies.

2. Data and Methodology

This paper collected the data from OECD [14], 20 OECD insurance industries were included, and the data was from 2013 to 2017.

2.1 Selected Indicators

The input and output indicators used in this paper are shown in Table 1. In the past, the commonly used input indicators are divided into three categories: labor input, capital input, and other material input. (Irene and Jia [6], Irene and Luen [7], Irene and sharp [8]. When selecting input indicators, this paper refers to these indicators. When considering the output indicators, the non-insurance industry pays more attention to claims, while the life insurance industry pays more attention to income. Therefore, this paper selects different output indicators for them. It should be emphasized here that some previous studies have selected premium income as an output indicator. For example, Irene and Jia [6] use net premium income as one of the output indicators. In Kader et al. [11], the contribution rate of the total premium is chosen as the output indicator, but the actual situation is not ideal. Because premium income is the concept of output multiplied by price, the price here includes not only the component of risk prevention but also the investment factor, cost factor and profit factor of the company. Therefore, the premium income is not appropriate.

Table 1: Summarize of inputs and outputs indicators

	Life insurance market	**Non-life insurance market**
Inputs indicators	1. Number of companies 2. Debt capital 3. Equity capital	1. Number of companies 2. Debt capital 3. Equity capital
Outputs indicators	1. Total investments 2. Net income + Technical provisions	1. Total investments 2. Claims payments + Technical provisions

2.2 Data Envelopment Analysis

Data envelopment analysis (DEA) is suitable for the evaluation of complex multi-output and multi-input problems. One of the characteristics of DEA is that it does not need any weight assumption before an analysis but uses the actual data input by DMU to obtain the best weight. This function enables the DEA to eliminate many subjective factors. Therefore, DEA has strong objectivity. The assumption of the CRR model is that in the production process, the scale return is fixed. When the input changes in proportion, the output should also change in proportion. For the input-oriented CCR model, there are the following constraints:

$$\begin{gathered} min\ \theta \\ s.t. \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_0 \\ \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_0 \\ \lambda_j \geq 0, i = 1,2,\ldots,n; j = 1,2,3,\ldots,n; r = 1,2,\ldots,n \end{gathered} \tag{1}$$

where x_{ij} represents the i-th inputs on the j-th DMU, y_{rj} represents the r-th outputs on the j-th DMU, they are scalar vectors, here are three inputs, two outputs and 20 DMUs; λ_j is a scalar vector, and θ is an input radial measure of technical efficiency. Among them, the optimal solution is θ *, $1\text{-}\theta$ * represents the maximum input that can be reduced without reducing the output level at the current technical level. A larger θ * means a smaller amount of input can be reduced, which means higher efficiency. When $\theta * = 1$, it means that DMU is in a technical effective state currently.

The CCR model's assumptions apply when all manufacturers operate at an optimal scale. But in fact, due to financial constraints, incomplete competition, government regulations, and other factors, DMU is not in the best production state. Therefore, the BBC model is proposed to solve this problem. BBC model has almost the same constraints as the CCR model. The only difference is that in the BBC model, there is also a constraint on λ, which can basically ensure that manufacturers of similar size are compared with manufacturers that are not valid, rather than manufacturers with large gaps. The constraint is as follows:

$$\sum_{j=1}^{n} \lambda_j = 1 \tag{2}$$

Here, in the input-oriented BBC model, technical efficiency does not include scale efficiency, which is called pure technical efficiency. The calculation formula of scale benefit is:

$$SE = \frac{TE}{PTE} \tag{3}$$

3. Empirical Results

The results were calculated by DEA solver lv8. DEA solver lv8 is developed by Kaoru tone and can be applied to many different DEA models, such as BBC, CCR, IRS, DRS, etc. From this software, you can get the results like score, projection, weight, graph, etc. In the following analysis of each countries, the time series is 5 years, this paper analyzes the data of each year separately and then selects their average value as the final result.

3.1 Life Insurance Industry

Inputs are the number of companies, debt capital, and equity capital. The outputs are the sum of total investment, technology supply, and net income. The results of DEA are shown in Table 2.

Table 2: Results of DEA for life insurance industry

	TE	PTE	SE
Australia	0.4313	0.5099	0.7082
Belgium	0.9585	0.9625	0.9959
Denmark	0.9434	0.9467	0.9965
Finland	0.9726	0.9892	0.9833
Germany	1	1	1
Greece	0.7656	0.8206	0.9062
Hungary	0.9663	0.9697	0.9964
Iceland	0	1	0
Ireland	0.9968	0.999	0.9977
Italy	0.981	0.9854	0.9955
Luxembourg	0.975	0.9766	0.9984
Mexico	1	1	1
New Zealand	0.5277	0.5339	0.9889
Norway	1	1	1
Poland	0.9701	0.9718	0.9983
Portugal	0.9728	0.9739	0.9989
Spain	0.9294	0.962	0.9662
Switzerland	0.996	0.9982	0.9978
Turkey	0.1712	0.2279	0.7708
United States	0.9718	1	0.9718

From table 2, we can clearly find the countries that are in technical effective and scale effective, namely Germany, Mexico, and Norway. Among them, Iceland's technical efficiency is shown as "0", because the value of the original data from Iceland is only about 0.1%-0.01% of that of other countries, which leads to such results. Cummins and Weiss [4] mentioned that SFA's efficiency value is higher than DEA's, because DEA regards all deviations from the boundary as invalid, while SFA allows the use of random error terms. In Table 2, we can find that PTE from the BBC model is higher than the TE from the CCR model. The reason is that the BBC model ensures that invalid manufacturers are only compared to similar manufacturers. Whether it is for TE or PTE, Turkey is obviously the least efficient country, the value is only 0.17 and 0.23. This shows that Turkey's life insurance market can reduce its input by more than 77% without reducing its output level. Based on the information obtained from the BBC model, we can infer that for the Turkish life insurance market, if they want to achieve technical efficiency and effective scale, they should expand the scale. Interestingly, the United States, as the leader of insurance, has a long history, a large and mature market, but has not achieved technical efficiency.

3.2 Non-Life Insurance Industry

Input indicators are the same as for life insurance; outputs are the sum of total investment, technical reserves, and claim payments.

Table 3: Results of DEA for non-life insurance industry

	TE	**PTE**	**SE**
Australia	0.8848	0.8905	0.9937
Belgium	0.7162	0.7195	0.9953
Denmark	0.9219	0.9234	0.9983
Finland	0.842	0.8515	0.9888
Germany	1	1	1
Greece	0.7713	0.8365	0.9016
Hungary	0	1	0
Iceland	0.7786	1	0.7786
Ireland	0.8753	0.8827	0.9904
Italy	0.8201	0.8222	0.9974
Luxembourg	0.7753	0.7824	0.9911
Mexico	0.7012	0.7219	0.9712
New Zealand	0.2878	0.2938	0.9798
Norway	0.7641	0.7677	0.9954
Poland	0.8762	0.8937	0.9806
Portugal	0.8959	0.9116	0.9827
Spain	0.865	0.8796	0.9841
Switzerland	0.8729	0.8755	0.997
Turkey	1	1	1
United States	1	1	1

As can be seen from Table 3, for the non-life insurance industry, the efficiency value of Mexico and Norway is no longer "1". Germany still maintains the position of efficiency first, and the United States has achieved pure technical effectiveness and technical effectiveness. While Iceland and Hungary have achieved pure technical effectiveness. The most surprising is that Turkey has the worst performance in the life insurance market, with both technical and pure technical effectiveness in the non-life insurance market. As we mentioned earlier, the BBC model assumes that the scale gains can be changed in the production process, which means that when the input increases proportionally, the output will not necessarily increase proportionally, and the scale may increase or decrease. In the non-life insurance industry, the value of scale efficiency is more than 97% in most countries. Only Iceland, Hungary, and Greece have low scale efficiency values. Hungary has the same problems as Iceland's life insurance industry. Because the relevant data is much lower than in other countries, the technical efficiency value is calculated as "0", which results in the scale efficiency value of "0".

3.3 Summary

We are also interested in the comparison between overall life insurance and non-life insurance, which are composed of 20 OECD member countries. For the comparison between the life insurance industry and the non-life insurance industry, we select the average value of the 20 countries from each insurance industry from each year.

Figure 1: TE of life and non-life insurance industry from 2013 to 2017

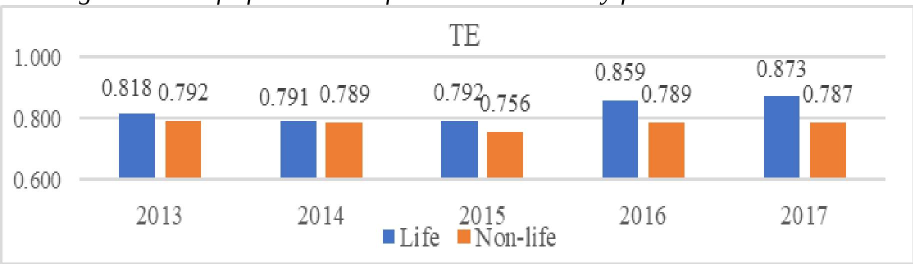


Figure 2: PTE of life and non-life insurance industry from 2013 to 2017

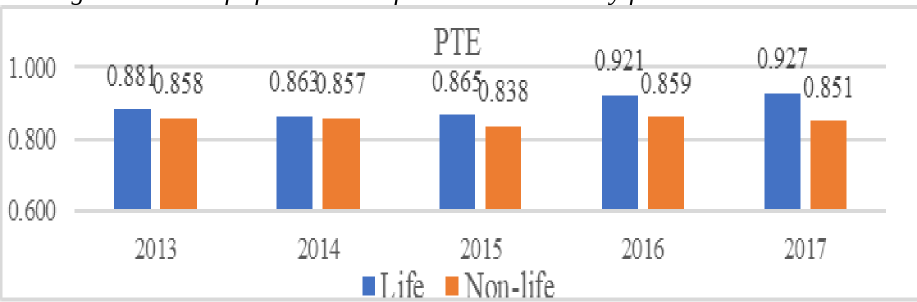
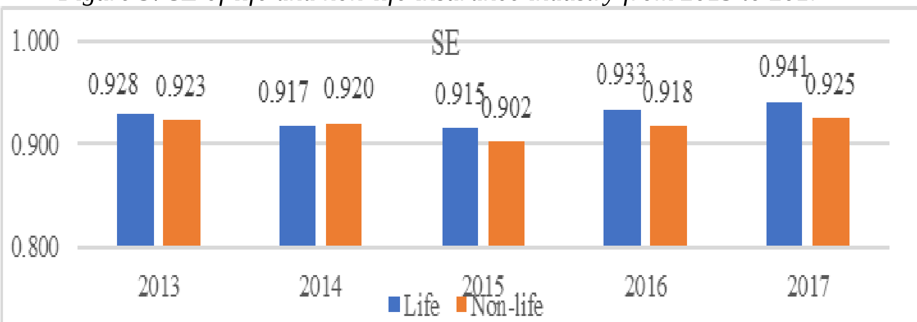


Figure 3: SE of life and non-life insurance industry from 2013 to 2017



For the life insurance industry, we can see from the three charts that the trend of TE, PTE, and SE began to decline in 2013 and began to rise after 2016, and the efficiency value in 2016 was higher than the previous three years. In our opinion, the main reason is that in 2013, many countries/regions were inefficient and shrinking in scale, but in the second year, they did not make appropriate adjustments, resulting in re-duced output and more input, thus making technology more efficient. For the non-life insurance industry, these three efficiency values change relatively little in five years. But in 2015, technical efficiency, pure technical efficiency, and scale efficiency are much smaller than other times. An important reason is that Greece's non-life insurance market experienced a cliff slide in 2015, especially in terms of technical efficiency. If we look at data from Greece over the past five years, we can see that in 2015, its technical efficiency was less than half of its average level.

From these three charts, we can draw a rough conclusion: the efficiency value of the life insurance market is usually higher than that of the non-life insurance market. The only exception in our sample is that in 2014, the scale efficiency of the life insurance market was lower than that of the non-life insurance market. In 2014, six countries in the life insurance market achieved economies of scale. They are Germany, Ireland, Mexico, Norway, Switzerland, and the United States. In the non-life insurance market, only Ger-many, Ireland, Turkey, and the United States have achieved an effective scale. So why is the scale efficiency value of the non-life insurance market still higher than that of the life insurance market? The rea-son is that although more countries in the life insurance market have reached an effective scale, in the non-life insurance market, all countries except Iceland have similar economies of scale, all of which are more than 95 %. In the life insurance market, Australia, Greece, and Turkey are relatively inefficient in scale, which reduces the efficiency value of the whole market.

4. Conclusion

In this paper, the CCR model and BBC model are used to calculate the technical efficiency and pure technical efficiency respectively, so as to get the scale efficiency value. Through these data, we understand the efficiency status of each market and the general market environment.

The sample data are all from the life insurance market and the non-life insurance market of OECD member countries. Considering the lack and inaccuracy of some data, we have selected 20 countries from OECD as the analysis object, not all member countries. The innovation of this paper is that it analyzes the life insurance and non-life insurance market at the same time and compares them.

Through this study, we confirm that the pure technical efficiency calculated by the BBC model is indeed higher than that calculated by the CCR model. Because the former eliminates the efficiency of the scale. Among our research objects, Germany is the best in both markets and has reached an effective state. Turkey has also achieved effective status in the non-life insurance market, but its efficiency value in the life insurance market is at the lowest point. We also have a general understanding of how to adjust a mar-ket to an effective position. In general, most countries have reached an effective position in the life insur-ance market.

Acknowledgements

The author was supported through the Czech Science Foundation (GACR) under project 20-25660Y and moreover by SP2020/11, an SGS research project of VSB-TU Ostrava. The support is greatly acknowledged.

References

[1] Banker R. D., Charnes A., and Cooper W.W. (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. Management Science. 30(9). 1031-1142.

[2] Barros, C. P., Nektarios, M., and Assaf, A. (2010). Efficiency in the Greek insurance industry. European Journal of Operational Research. 205 (2). 431–436.

[3] Charnes A., Cooper W. W., and Rhodes E. (1978). Measuring the efficiency of decision-making units. European Journal of Operational Research. 2 (6). 429–444.

[4] Cummins, J.D., and Weiss, M.A. (2000). Analyzing firm performance in the insurance industry using frontier efficiency methods. In: Dionne, G. (Ed.), Handbook of Insurance Economics. Kluwer Academic Publishers, Boston. 767-829.

[5] Donni O., and Fecher F. (1997). Efficiency and productivity of the insurance industry in the OECD countries. Geneva Papers on Risk and Insurance. 22 (84). 523–535.

[6] Eling M., and Jia R. (2019). Efficiency and Profitability in the global insurance industry. Pacific-Basin Finance Journal. 57 (2019), 101190.

[7] Eling M., and Luhnen M. (2010). Efficiency in the international insurance industry: A cross-country comparison. Journal of Banking & Finance, 34 (7), 1497–1509.

[8] Eling M., and Schaper P. (2017). Under pressure: how the business environment and efficiency of European Life insurance companies. European Journal of Operational Research, 258(2017), 1082-1094.

[9] Farrell M. J. (1957). The measurement of productive efficiency. Journal of the Royal Statistical Society: Series A (Gesneral). 120 (3). 253–281.

[10] Greene W.H., and Segal D. (2004). Profitability and efficiency in the U.S. life insurance industry. Journal of Productivity Analysis. 21 (3), 229–247.

[11] Kader H.A., Adams M., Hardwick P., and Kwon W.J. (2014). Cost efficiency and Board composition under different takaful insurance business models. International Review of Financial Analysis. 32 (2014), 60-70.

[12] Kaffash S., Azizi R., Huang Y., and Zhu J. (2019). A survey of data envelopment analysis applications in the insurance industry 1993-2018. European Journal of Operational Research. Available online 22 July 2019.

[13] Kao, C., and Hwang, S.-N. (2014). Multi-period efficiency and Malmquist productivity index in two-stage production systems. European Journal of Operational Research. 232 (3). 512–521.

[14] OECD. (2014-2018). OECD Insurance Statistics. Database version 2014, 2015, 2016, 2017 and 2018.

[15] Rai A., (1996). Cost efficiency of international insurance firms. Journal of Financial Services Research. 10 (3), 213–233.

[16] Vencappa D., Fenn P., and Diacon S. (2013). Productivity growth in the European insurance industry: Evidence from life and non-life companies. International Journal of the Economics of Business, 20 (2), 281–305.

[17] Yang, H., and Pollitt, M. (2009). Incorporating both undesirable outputs and uncontrollable variables into DEA: The performance of Chinese coal-fired power plants. European Journal of Operational Research. 197 (3). 1095–1105.

Effect of data pre-processing on the accuracy of the algorithm results of the time series forecasting models[1]

Lun Gao, Miroslav Čulík, Gurný Petr [2]

Abstract

Non-stationary time series prediction has always been a problem. Traditional models are often difficult to predict accurately due to too much information hiding in the signal or because of cumulative errors will interfere with the accuracy of prediction. Therefore, it is necessary to preprocess the data for predictive analysis. This article uses Empirical Mode Decomposition (EMD) and Variational Mode Decomposition (VMD) techniques to decompose non-stationary time series into a series of intrinsic mode functions (IMFs) and residual (r). Then, by constructing neural network and traditional ARMA model for each IMF and r, thereby achieving a combined prediction of the original time series. Using the US Gulf Coast Kerosene jet fuel price data from Index Mundi to verify the effectiveness of the method. The results show that the accuracy of prediction results after pre-decomposing the data by mixing EMD and VMD models is significantly better than that of pure neural networks and traditional statistical prediction models.

Key words

Combined Forecast, Empirical Mode Decomposition, Variational Mode Decomposition, Autoregressive moving average model, Neural Networks, Long short-term memory

JEL Classification: C01, C4, C5

1. Introduction

Financial time series signals are the carriers of all information in the market. Signal acquisition and time-frequency analysis are important contents in the field of financial forecasting. However, signals obtained from the market are subject to various influences, and inevitably carry noise with limited accuracy. However, the noise spectrum will inevitably carry a certain amount of information that cannot be discarded. Therefore, the decomposition of time series signals has become the first step in time series analysis and prediction. Empirical mode decomposition, referred to as EMD is an adaptive decomposition method proposed by Huang et al, (1998). In 2014, Dragomiretskiy, K et al. developed a new variational modal decomposition based on Fourier transform after empirical modal decomposition designed. Unlike the principle of empirical mode decomposition, the VMD method determines the center frequency and bandwidth of the modal component by iteratively searching the optimal solution of the variational model. It can decompose complex signals into K amplitude-frequency modulation signals adaptively, effectively suppressing modal aliasing. Both signal

[1] The paper is based on the research activities within the ESF Project No. CZ.1.07/2.3.00/20.0296 and SGS project No. SP2020/124. The authors are grateful to an anonymous referee for many constructive comments and suggestions. Any remaining errors are the author´s responsibility.

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decomposition result consists of several intrinsic mode functions (IMF) arranged in order from high frequency to low frequency. For time-series signals, the noise is concentrated on the high-frequency IMF component, and the IMF can be predicted separately to reduce noise interference.

Fuel cost is one of the airline's biggest costs. For airlines, the price changes of aviation fuel affect airlines' control of costs. Accurately predicting the price trend of aviation fuel will help airlines use financial derivatives in advance to hedge the cost increase caused by price changes and reduce commodity price risk.

Experts and scholars have done a lot of research on time series fluctuations and predictions. For example, Tully and Lucey, (2006) found that the APGARCH model can better describe the price fluctuation of gold than the GARCH model based on normal distribution. However, despite the APGARCH, the ARMA model can describe the long memory of financial time series, the traditional statistical and economic models assume that the data contains a single piece of information and at present, the widely used EMD combination model does not consider whether the data is excessively decomposed, and the excessive interpretation of the data also brings certain errors. It is difficult to capture a variety of information hidden in a time series, and often cannot get accurate prediction results.

In order to overcome the limitations of the traditional model, the article will use EMD-LTSM and VMD-LSTM compare with EMD-ARMA and VMD-ARMA to separately predict the decomposed signal. At the same time, in order to prevent data from being over-decomposed in EMD method, Partitioning Around Medoid clustering and hierarchical clustering will be used to reconstruct the data and use LTSM and ARMA to predict it again. Experimental results show that the prediction accuracy of these methods is better than that of a single traditional model.

The main goal of this paper is to use EMD and VMD method to decompose the original signal, and then predict the decomposed data, and finally obtain the predicted value that reduces the cumulative error. With the help of empirical mode decomposition and variational mode decomposition in the field of digital signal processing and neural network. This paper models and predicts the data series of the U.S. Gulf Coast Kerosene jet fuel daily price from Index Mundi based on EMD and VMD combined with neural network. So far, few scholars have used EMD and VMD combined with neural network to predict the commodity price.

The jet fuel price data is a non-stationary time series because it has trends and periodicity (mean and variance changes with time). The article uses EMD and VMD to decompose the time series into several IMFs and r (trend term), each of which is a time series signal. These time signals have a distinct pattern relative to the original signal that can be mined, using neural network to predict these signal sequences separately. The empirical results of the article show that the model works well.

This paper is divided into four parts. The first part is Introduction, which is a brief introduction to the article; the second part is to describe the model used in the article and list its formula; The third part is the empirical analysis part. The model cited in the second part will be used to predict the U.S. Gulf Coast Kerosene jet fuel daily price data from Index Mundi and the results will be analyzed. Finally, in the fourth part, it is concluded that the composite model based on the signal decomposition is significantly better than the single traditional model.

2. Methodology and Data

2.1 Theory of Empirical Mode Decomposition (EMD)

Empirical Mode Decomposition (EMD) is an adaptive signal decomposition method that can filter the trends of different features existing in the original sequence step by step to obtain the intrinsic modal components with the same feature scale. The idea of EMD decomposition is to decompose the signal into several relatively stationary, uncorrelated intrinsic mode functions (IMFs). An intrinsic mode functions function should satisfy the following two conditions. (Cui et.al 2015): a) in the entire data sequence, the number of extreme points should be equal to the number of zero-crossing points, or at most one difference; b) at any point in time, the local mean defined by the local maxima and local minima of the signal should be zero.

Briefly, IMF is a time series with a mean close to zero and no obvious trend. The nature of IMF is exactly in line with the needs of traditional time series analysis, so non-stationary time series can be decomposed by EMD to obtain a series of IMFs, and then to analyzed. Meanwhile, for non-stationary time series, the residual amount produced by EMD decomposition generally contains a certain trend, so it called trend term. The trend term reflects the overall trend of the original time series, the problem is well-known (Zhu et.al. 2013) which eliminates minor events and short-term effects. The decomposition process is as follows:

A. Search for the extreme point of the original signal $x(t)$, and connect all the maxima and minima points with the cubic spline function to obtain the upper and lower envelopes of $x(t)$,. Expressed as $x_{max}(t)$, and $x_{min}(t)$, respectively.
B. Calculate the mean of the envelope $w_1(t)$,
$$w_1(t) = \frac{[x_{max}(t)+x_{min}(t)]}{2}. \tag{1}$$
C. Calculate the difference between the signal $x(t)$ and the envelope mean $w_1(t)$ get $d(t)$,
$$d(t) = x(t) - w_1(t). \tag{2}$$
D. Determine whether $d(t)$ satisfies the two conditions of IMF. If $d(t)$ satisfies the condition, $d(t)$ is the first component of signal $x(t)$, which is IMF_1; if the condition is not met, then use $d(t)$ as the new original signal and repeat the above steps until the condition is met.
E. Calculate the residual signal r_1,
$$r_1 = (t) - IMF_1. \tag{3}$$
F. Using r_1 as a new signal source, repeat step 1 to 3 and continuously decompose the signal to obtain the IMF_i that satisfies the condition until the residual r_n is a monotonic function and stop the decomposition. At this time, the original signal $x(t)$ can be expressed as the sum of i $IMFs$ and a residual value r_n.
$$x(t) = \sum_{i=1}^{n} IMF_i + r_n. \tag{4}$$

The EMD decomposition is based on three assumptions: 1) any signal can be decomposed into several IMF components; 2) each IMF component could be linear or non-linear, the number of local zero and extreme points are the same, and the upper and lower envelopes are locally symmetric about the time axis; 3) a signal can contain several IMF components. When the sequence extreme points are not obvious, it is necessary to find the extreme points by several differences where Liu et al, (2018) pointed out.

2.2 Theory of Variational Mode Decomposition (VMD)

Variational mode decomposition VMD is a new non-stationary signal adaptive decomposition estimation method proposed by Konstantin Dragomiretskiy in 2014, which can decompose the original signal into certain K frequency band sub-signals. It abandons the recursive solution mode of EMD, and the decomposition result is less affected by noise. The key to Variational mode decomposition lies in solving variational problems. Let the original price series be f, assuming that f can be decomposed into K modal quantities, each modal has a limited bandwidth with a different center frequency, and the constraint to be satisfied is that the sum of each modal is equal to the original signal. The objective function for solving the variational problem is to minimize the sum of the bandwidth estimates of each mode.

Unlike traditional recursive mode decomposition such as EMD and LMD, VMD converts signal decomposition into non-recursive, variational modal decomposition. Its overall framework is a variational problem, which minimizes the sum of the bandwidth of each component after decomposition.

In order to estimate the bandwidth of each modal component, we first need to perform Hilbert transform on each modal function to obtain its single-sided spectrum;

$$\left(\delta(t) + \frac{j}{\pi t}\right) * \mu_k(t) \qquad \delta(t) = \begin{cases} 0, & t \neq 0 \\ \infty, & t = 0 \end{cases} \tag{4}$$

Then, by adding an estimated center frequency $e^{j\omega_k t}$t, the frequency spectrum of each modal component is transformed to baseband;

$$\left[\left(\delta(t) + \frac{j}{\pi t}\right) * \mu_k(t)\right] e^{j\omega_k t} \tag{5}$$

Finally, the square L^2 norm of the analytical signal gradient is calculated to estimate the bandwidth of each modal component.

Assuming that after the VMD decomposition, the original signal is decomposed into k modal components, the variational constraint model is：

$$\begin{cases} \min\limits_{\mu_k, \omega_k} \left\{ \sum_k \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t}\right) * \mu_k(t)\right] e^{-j\omega_k t} \right\|^2 \right\} \\ s.t \sum_k \mu_k = f \end{cases} \tag{6}$$

among them： $\{\mu_k\}$——Collection of modal components, $\{\mu_k\} = \{\mu_1, \mu_2, \dots, \mu_k\}$;

$\{\omega_k\}$——Collection of center frequencies, $\{\omega_k\} = \{\omega_1, \omega_2, \dots, \omega_k\}$;

$\delta(t)$——Unit pulse function.

The VMD algorithm introduces the second penalty term α and Lagrange multiplier L to solve the above variational constraint model, that is:

$$\mathrm{L}(\mu_k, \omega_k, \lambda) = \alpha \sum_k \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t}\right) * \mu_k(t)\right] e^{-j\omega_k t} \right\|_2^2 + \| f(t) - \sum_k \mu_k(t) \|_2^2 + [\lambda(t), f(t) - \sum_k \mu_k(t)] \tag{7}$$

The specific implementation steps of VMD are as follows:

A. Initialize $\{\hat{\mu}_k^1\}$, $\{\omega_k^1\}$, $\{\hat{\lambda}_k^1\}$ and n;

B. Make $n = n + 1$, perform loop process;

C. Make $k = 0, k = k + 1$, update $\{\mu_k\}$, and $\{\omega_k\}$;

$$\hat{\mu}_k^{n+1}(\omega) = \frac{\hat{f}(\omega) - \sum_{i \neq k}(\omega) + \frac{\hat{\lambda}(\omega)}{2}}{1 + 2(\omega - \omega_k)^2} \tag{8}$$

$$\omega_k^{n+1} = \frac{\int_0^\infty \omega |\hat{\mu}_k(\omega)|^2 d\omega}{\int_0^\infty |\hat{\mu}_k(\omega)|^2 d\omega} \tag{9}$$

D. Update λ:

$$\hat{\lambda}^{n+1}(\omega) \leftarrow \hat{\lambda}^n(\omega) + \tau[\hat{f}(\omega) - \sum_k \hat{\mu}_k^{n+1}(\omega)] \tag{10}$$

where τ represents the noise tolerance parameter.

E. Repeat steps 2 to 4, until the condition of:

$$\frac{\sum_k \|\hat{\mu}_k^{n+1} - \hat{\mu}_k^n\|_2^2}{\|\hat{\mu}_k^n\|_2^2} < e \quad (11)$$

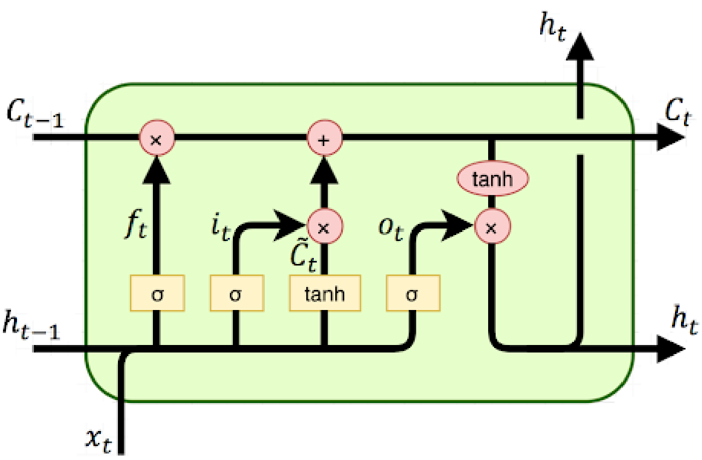
constraint is satisfied, the iteration is stopped.

2.3 Principle of Long Short-Term Memory networks (LSTM)

Each layer of the LSTM network is composed of multiple memory blocks with the same structure. Each memory block contains an "input gate", an "output gate" and a status node. The input gate controls the proportion of input information received by the memory block, the output gate controls the proportion of output information transmitted from the memory block to the outside world, and the state node contains all historical information received by the memory block, forming a "memory" of the input information. With the structure of memory blocks, the LSTM network can memorize information with an interval of more than 1000 time units. Because the state node in the memory block accumulates all historical information, the internal value of the state node grows infinitely over time, and the state node fails due to saturation. Gers et al. solved the problem of node saturation failure by introducing "Forget Gate". By added a "forget gate" structure to the state node, allowing the historical information of the node at this moment to be passed to the next moment at a certain ratio. This modification completes the basic structure of LSTM. Now the LSTM models used by people all contain the structure of "forgotten gate".

The difference between the LSTM network and the standard RNN is that the structure of the hidden unit of the RNN is replaced by a memory block by a long-short-term memory network. The most important structure in the memory block is its three gate structures and a cell structure. Its specific structure is shown in Figure 1.

Figure 1 LSTM neural network (source: GitHub)



Memory blocks are the basic building blocks of long short-term memory networks. Memory blocks, like neurons in the brain, have the function of memorizing information and connecting peripheral neurons.

The memory block is related to time sequence. At time t, the input is x_t, the output is h_t, and the historical information of the memory block "memory" is C_t. Each memory block has three gates, namely the input gate i_t, the forget gate f_t and the output gate o_t. The input gate controls the proportion of input information at time t, the forget gate controls the proportion of historical information stored at time t, and the output gate determines the proportion of information output from the memory block to the next layer.

The memory block has a time series state. C_t contains all input information from time 0 to time t, which is the state of the memory block at time t.

The forget gate f_t is determined by the input information $x_t$and the output information $h_t - 1$ at the last moment of the memory block. After the sigmoid function (the sigmoid function maps the real number interval R to the interval (0,1), generally expressed by the symbol σ, $\sigma(x) = 1/(1+e^{-x})$)transforms, the value of the forget gate is between 0 and 1, as shown in formula (14). f_t indicates how much of the state of the memory block at the previous time C_t is retained until the current time t. When the value of f_t is 0, it means that the state of the previous moment is completely forgotten, and the value of f_t is 1 means that the state of the previous moment is completely remembered.

The input information x_t and the output of the previous moment $h_t - 1$, after being transformed by the tanh function (The tanh function is the hyperbolic tangent function, which maps the real interval R to the interval (-1,1), $tanh(x) = (e^x - e^{-x})/(e^x + e^{-x})$), become the state increment at the current moment $\tilde{C}_t$, as shown in formula (15). The value of the state increment $\tilde{C}_t$ is in the interval (-1,1), which represents the increment size that the input information x_t can cause to the state of the memory block.

Similar to the forget gate, the value of the input gate i_t is determined by the input information x_t and the output information $h_t - 1$ at the previous moment, as shown in equation (16). The input gate controls the proportion of the state increment $\tilde{C}_t$ received by the memory block. If the value of the input gate i_t is 0, then the state increment $\tilde{C}_t$ will be completely ignored; if the input gate i_t value is 1, then $\tilde{C}_t$ will be fully counted into the state C_t.

$$f_t = \sigma(W_f * [h_{t-1}, x_t] + b_f)$$

(14)

$$\tilde{C}_t = \tanh(W_c * [h_{t-1}, x_t] + b_c)$$

(15)

$$i_t = \sigma(W_i * [h_{t-1}, x_t] + b_i)$$

(16)

Now let's take a look at how the status of the memory block is updated. The state value C_t consists of two parts, one part is the state at the previous moment $C_t - 1$, this part is controlled by the forget gate. The other part is the state increment $\tilde{C}_t$. This part is determined by the input gate and determines the proportion of the increment received. The state value $\tilde{C}_t$ update formula with time is shown in (17). It is worth noting that the symbol "*" in formula (17) represents a bitwise multiplication between vectors (Bitwise multiplication means that each dimension of the vector is multiplied, and the result is a vector of the same dimension, for example $(1,2,3) * (4,5,6) = (4,10,18)$), that is, in a memory block, a gate is a kind of scaling of information.

$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

(17)

The output gate o_t is similar to the forget gate and the input gate, and its value is synthesized from the input information x_t and the output information h_{t-1} at the previous moment, as shown in equation (18). The size of the output gate o_t determines the state of the memory block C_t as a probability of being output and captured by other neural network layers.

The symbol h_t represents the output of the memory block at time t. This output is based on the state of the memory block C_t, but it needs to be filtered by the output gate. The state C_t is first transformed by the tanh function to compress the state to the interval (-1, 1). and then the output gate o_t determines the output ratio, as shown in equation (19).

$$o_t = \sigma(W_o * [h_{t-1}, x_t] + b_o) \quad (18)$$

$$h_t = o_t * \tanh(C_t)$$

(19)

The LSTM memory block is a function transformation in time series, which transforms the input time series into another time series. Because the memory block has a built-in state unit and has a special structure of the input gate and the forget gate, the memory block has the ability to associate long interval information.

The characteristics of the price change in the running process can be regarded as a time series. The price change during a period of time has n closing prices, and the price at the current time $t$is related to the k prices before this time, that is, the input vector is:

$$X = \{x_1^{t-1}, x_1^{t-2}, \dots, x_1^{t-k}, x_2^{t-1}, x_2^{t-2}, \dots, x_2^{t-k}, \dots, x_n^{t-1}, x_n^{t-2}, \dots, x_n^{t-k}\}$$

(20)

2.4 The Autoregressive Moving Average model

The ARMA model is one of the most widely used time series prediction models due to its simplicity, feasibility and flexibility. The modeling and forecasting consists of four steps: 1)Sequence smoothing processing, if the sequence is non-stationary, it can be made to satisfy the stationarity condition by differential variation; 2) Model identification, mainly determining the lag order p and q of the model by the autocorrelation coefficient and the partial autocorrelation coefficient; 3) Parameter estimation and model diagnosis, estimating the parameters of the model, and testing (including the significance test of the parameters and the randomness test of the residuals), and then judging whether the model is desirable; 4) Forecasting using a model of the appropriate parameters selected and test.

Autoregressive moving average model $ARMA(p,q)$. If the time series X_n is satisfied:

$$X - \phi_1 X_{n-1} - \cdots - \phi_p X_{n-p} = \varepsilon_n - \theta_1 \varepsilon_{n-1} - \cdots - \theta_q \varepsilon_{n-q}.$$

(21)

Then the time series X_n obeys the $p, q \in (1,2,3\dots)$ order and $\varepsilon_n \sim N(0, \sigma_\varepsilon^2)$ autoregressive moving average model$ARMA(p,q)$where $\phi_1 \cdots \phi_p$ is the autoregressive regression coefficient and $\theta_1 \dots \theta_q$ is the moving average coefficient. When p or q are 0, the model degenerates into MA or AR.

2.5 Data selection and Description

In this paper, data from the U.S. Gulf Coast Kerosene jet fuel price data from Index Mundi will be cited, from 01.01.2010 to 06/04/2020 (total 2578 data), the last close of jet fuel price as the research object. We use the ARMA and LSTM Neural network model to predict the signal, for evaluating our results, we use ARMA and LSTM Neural network model to predict the empirical mode decomposition and variational mode decomposition data and original data. In this paper, all results are calculated by MATLAB.

From the 2578 data, the max data is 3.375, and the minimum data is 0.65 and other statistical indicators seen in table 1, and first, it is necessary to perform Augment Dickey-Fuller (ADF) test see in figure 2 on the data to verify that the data is non-stationarity.

Table 1. U.S. Gulf Coast jet fuel price statistic description (Source: MATLAB calculation)

	MAX	MIN	AVE	SD	MEDIAN	KURT	SKEW
DATA	3.375	0.65	2.185	0.6724	2.061	-1.2431	0.0391

Figure 2. ADF test of signal (Source: Calculate by Matlab)

Null Rejected	P-Value	Test Statistic	Critical Value	Lags	Model	Test Statistic	Significance Level
false	0.3254	-0.8916	-1.9416	0	AR	t1	0.0500
false	0.3254	-0.8916	-2.5688	0	AR	t1	0.0100
false	0.3254	-0.8916	-1.6168	0	AR	t1	0.1000

It can be clearly seen from the figure that the t-statistic is -0.8916 and the P-value is 0.3254 which greater than the significance Level of 5 %. Therefore, the original hypothesis cannot be rejected at 95 % confidence, based on this, the data signal is non-stationary.

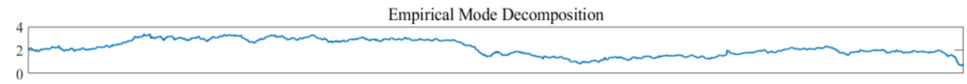
3. Empirical analysis

In this part, the data will be preprocessed and decomposed first. Then predict the decomposition results separately, and finally add up the prediction results to complete the prediction of the original data.

3.1 Empirical mode decomposition

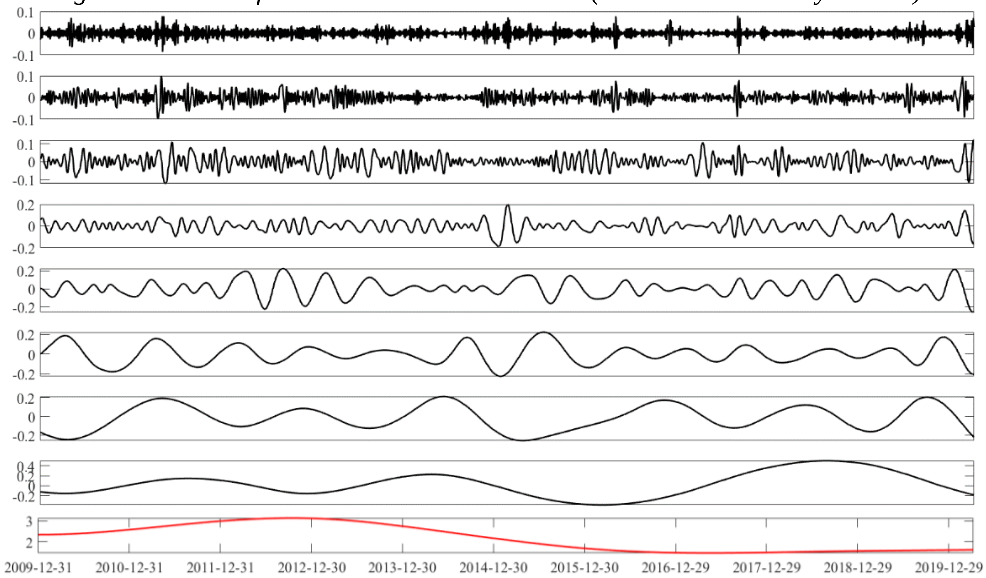
Based on the algorithm's settings, empirical mode decomposition will automatically decompose the signal data and arrange the IMFs from high to low according to frequency. See in figure 4.

Figure 3. Print of original data (Source: Print by Matlab)



From the picture of the original data, the data has obvious fluctuations and trends. The next step is to use the EMD method to decompose the data.

Figure 4. Results of EMD——IMFs and Residual (Source: Calculate by Matlab)



It can be clearly seen from the above figure that as the decomposition progresses, the fluctuation frequency of the IMF is gradually decreasing, and the trend is becoming more and more obvious.

Table 2. Statistic description of EMD result

	PCCs	Var	Max	min
IMF1	0.031715	0.000413	0.080607	-0.09433
IMF2	0.031047	0.000495	0.099576	-0.09572
IMF3	0.030596	0.001277	0.11998	-0.12106
IMF4	0.123715	0.002378	0.197358	-0.18903
IMF5	0.186840	0.007409	0.22283	-0.2584

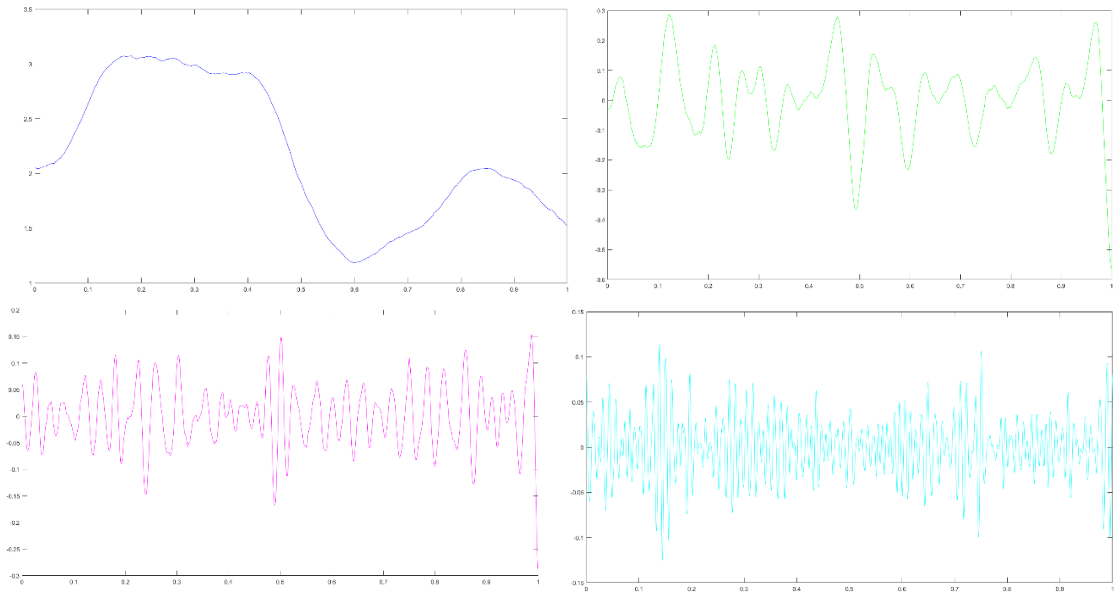
IMF6	0.125477	0.008448	0.221652	-0.22509
IMF7	0.277749	0.016725	0.210875	-0.25649
IMF8	0.301471	0.056186	0.500847	-0.38472
r	0.870421	0.369298	3.126265	1.475747

After EMD decomposition of the data signal, 8 IMFs and a trend term are generated. Each of the IMF component curves exhibits an oscillating form that is substantially symmetric around the zero-mean line, the local maximum and the local minimum. The period of the ten IMF components is from short to long, that is, the frequency is from high to low. The trend term reflects the non-stationarity of the original sequence to some extent and the trend is oscillating, which is consistent with the observation of the original sequence. After the calculation of each components the Pearson correlation coefficient (PCCs), variance, the maximum and the minimum will be got. In the table 3, it can see the results get from calculation. The variance of the trend term contributes the most to the variance of the original sequence, which indicates that the trend term can reflect the basic fluctuations of the original sequence.

3.2 Variational Mode Decomposition

First, the article will decompose the data using the VMD model.

Figure 5. Results of VMD decomposition (Source: Print by Matlab)



In the upper group of graphs, the charts from top to bottom and left to right are: IMF1, IMF2, IMF3, IMF4, evolution of central frequencies Omega, and structural decomposition. Due to the significant difference in calculation method from the EMD method, under the VMD method, the form and quantity of the IMF output are also significantly different. The fluctuation frequency of IMF1 to IMF4 gradually increases, which is completely opposite to the result obtained by the EMD method. Similar to the EMD method, the article will also calculate the Pearson correlation coefficients (PCCs), variance, the maximum and the minimum of each IMF when processing the decomposition results. See in table below.

Table 3. Statistic description of VMD result

	PCCs	Var	Max	min
IMF1	0.970329	0.402696	3.073313	1.185070273

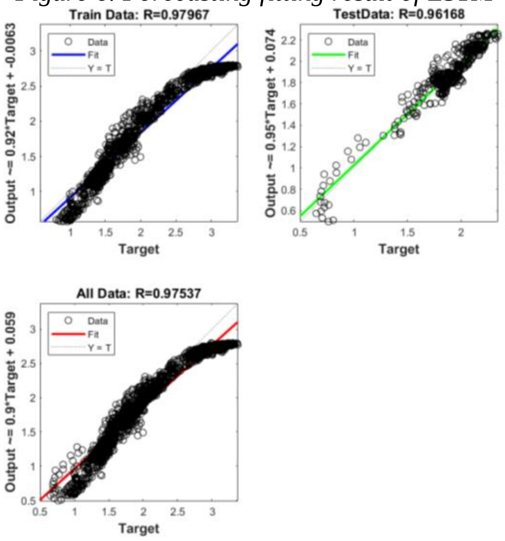
IMF2	0.324355	0.016476	0.287215	-0.56576
IMF3	0.162744	0.0033	0.153077	-0.29027
IMF4	0.080162	0.001112	0.114505	-0.12526

In VMD decomposition, a large input parameter K value (that is, the number of decomposed IMFs) will cause the result signal data to be incoherent. After repeated trials and combining the results of the clustering algorithm above, the final parameter value $K = 4$, namely the number of IMF outputs is 4.

3.3 LSTM Neural Network model

The time series contains a total of 2578 time points. This paper selects the latest 515 of the 2578 time points (about 20 % of the total number of periods) as the target set. The remaining 2063 data will be used as a test set to train and test the LSTM neural network model. There are various criteria for evaluating model predictions, such as mean absolute distance (MAD), sum of squared error (SSE), mean squared error (MSE), and root mean squared error (RMSE). This article uses MSE, RMSE as the standard for model evaluation.

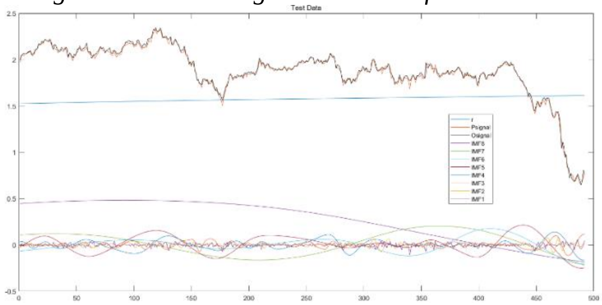
Figure 6. Forecasting fitting result of LSTM



It can be seen from the training results calculated by Matlab in the above figure that the LSTM neural network has a good prediction result for the time series signal. The fitting value R of the test data is 0.96168 and the RMSE of the test data is 0.101. At the same time, the curve of the prediction result is more in line with reality.

After decomposing the original data signal using the EMD method, the predicted result is shown in the figure below. The abscissa of the picture represents the number of days (the length of the test set), the ordinate represents the price, and the curves of IMF1 to IMF8 and r represent the prediction results of the modal function decomposed from the original model. The Osignal curve represents the original model, and the Psignal curve represents the final prediction result. It can be seen from the figure that the prediction results are more consistent with the original signal, and the prediction results are relatively good.

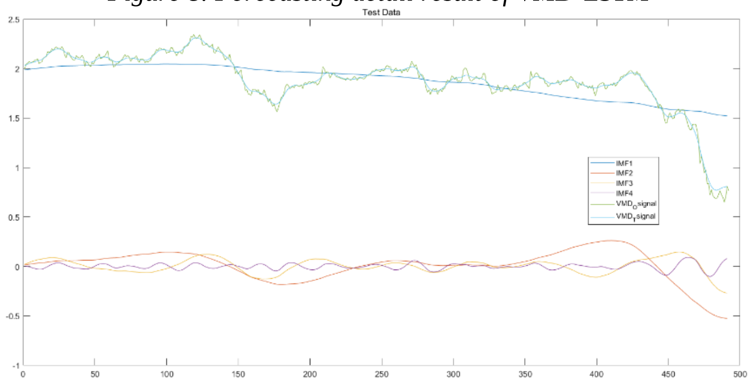
Figure 7. Forecasting detail result of EMD-LSTM



At the same time, after calculation, the RMSE between the test result and the original signal is 0.055, which is a certain improvement compared to the pure LSTM neural network prediction result for the original signal.

Next is the prediction result using variational modal decomposition. The VMD method is based on Fourier transform, and it is considered that the signal is composed of sub-signals dominated by different frequencies. Its purpose is to decompose the signal into sub-signals of different frequencies. The different sub-signals obtained by decomposition represent different market information laws that affect the time series changes. The following figure shows the predicted value curve of the modal function obtained by the variational modal decomposition and the final price forecast curve. The abscissa is the date and the ordinate are the prices. In the figure, VMD_Osignal is the original signal, and its prediction result VMD_Tsignal fits the original model well. After calculation, the root mean square error of the prediction result is 0.365, which is slightly better than the EMD-LSTM model after clustering mentioned above. It can be seen that preprocessing the data using empirical mode decomposition and variational mode decomposition on the basis of the LSTM model can better improve the prediction accuracy of the data.

Figure 8. Forecasting detail result of VMD-LSTM



In order to conveniently compare the prediction results of different models, the article organizes the two indicators of RMSE to measure the prediction error in the following table. It can be seen from the table that after preprocessing the data, the accuracy of prediction has been greatly improved.

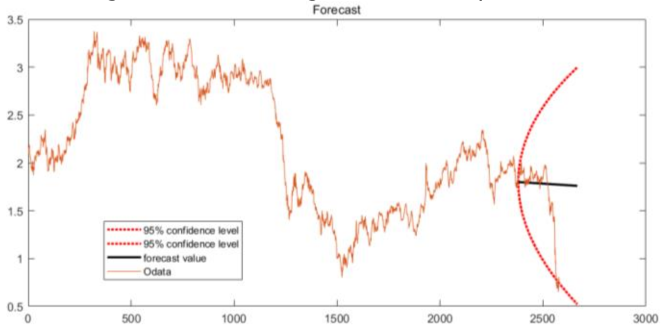
Table 4. Comparison of prediction errors of different models.

	LSTM	EMD-LSTM	VMD-LSTM
RMSE	0.101	0.0556	0.0365

3.4 Autoregressive moving average model

Because the ARMA model can only predict the approximate trend of the signal, the accuracy of long-term prediction is not good. The article will use 80% of data for training, and the last 20% of data will be used to test the prediction results. The prediction results of the original data are shown in the following figure. It can be clearly seen that the curve trend is more consistent for short-term predictions, but as the prediction time step increases, the error continues to expand.

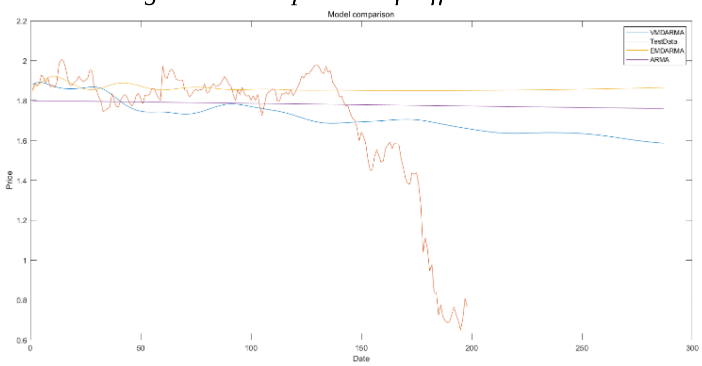
Figure 9. Forecasting detail result of ARIMA



After using the AIt can also be seen from the figure that the ARMA model has a poor prediction effect for a long time. Judging from the prediction results, without abnormal fluctuations, the accuracy of the trend prediction in the next 100 days is relatively good. RIMA model to make predictions on the original data, we select the predicted fitting value and calculate the prediction error of the prediction model, whose RMSE is 0.3729.

Considering the excessive decomposition of the EMD model detected above and the presence of white noise in the modal functions after IMF5, the EMD-ARIMA model prediction will use the results after the clustering for calculation. After the calculation of the model is completed, we compare the prediction results of the mixed model with the prediction results of the single model in a graph. In the figure below we see that there is a certain difference in prediction results between the mixed prediction model and the single model for data preprocessing.

Figure 10. Comparison of different models



It can be seen from the figure that in the prediction process around 30 days, the fluctuation trend of the prediction results of the mixed model of EMD-ARIMA and VMD-ARIMA is better than the single ARIMA model. This can also be seen from the RMSE of the predicted fitted value. The predicted fit value of the 35th day under the EMD-ARIMA model is 0.057, and the predicted fit value of the 35th day under the VMD-ARIMA model is 0.062. However, in the single ARIMA model without preprocessing the RMSE of the 35-day predicted fitted value is 0.109. It can be seen that in the traditional statistical prediction model of econometrics, preprocessing the data first greatly improves the prediction results.

4. Conclusion

In the actual financial market price forecast, the analysis and accurate prediction of price quantity are of great significance. This paper applies empirical mode decomposition and variational mode decomposition for signal preprocessing then use ARIMA and LSTM neural network to forecast the aviation kerosene prediction. The analysis of the example and comparison with hierarchical clustering and k-medio clustering show that the empirical mode decomposition and variational mode decomposition can be used to separate the high frequency fluctuation components and low frequency trend components implicit in the time series. This fully reduces the non-stationarity of the signal sequence and constructs a good external platform for the ARIMA and LSTM neural network model. Compared with a single prediction model, the new combined data preprocessing algorithm provides better prediction accuracy to a certain extent, whether in short-term or long-term prediction. Therefore, the new price prediction algorithm based on empirical mode decomposition and variational mode decomposition for data preprocessing has certain application value in aviation kerosene price prediction.

At the same time, in this paper, the physical signal processing models preprocessing data have achieved good precision in the prediction of financial time series. In the field of financial forecasting, because of the rapid uncertainty of the market, there is great uncertainty. Combining the rigorous science and technology model with high accuracy requirements can effectively improve the accuracy of prediction and reduce unnecessary risks.

References

[1] Anderberg, M.R. (1973). 'Cluster analysis for applications', Academic Press.

[2] Arnold, T. (2014). 'A Pragmatic guide to Real Options' Palgrave Macmillan Inc. ISBN 971137391162, pp.186.

[3] Brockwell, P. J., and Davis, R. A. (2009). 'Time series: theory and methods. Springer Science & Business Media.

[4] Cui, B. B., Chen, X. Y. and Song, R. (2015). 'Application of EMD Threshold Filtering for Fiber Optical Gyro Drift Signal De-Noising'. Acta Optica Sinica 2, pp. 61-66.

[5] Cryer, J. D. and Chan, K. S. (2008). 'Time Series Analysis with Applications in R (Second edition)'. Springer.

[6] Dragomiretskiy, K. and Dominique, Z. (2014) 'Variational Mode Decomposition.' IEEE Transactions on Signal Processing. Vol. 62, pp. 531–534.

[7] D. R. Fuhrmann, J. P. Browning and M. Rangaswamy (2010) "Signaling Strategies for the Hybrid MIMO Phased-Array Radar," in IEEE Journal of Selected Topics in Signal Processing, vol. 4, no. 1, pp. 66-78, Feb. 2010.

[8] Ding, F.; Shi, Y.; and Chen, T. 2006. 'Performance analysis of estimation algorithms of nonstationary arma processes' Signal Processing, IEEE Transactions on 54(3):1041–1053.

[9] Barocio, B. C. Pal, N. F. Thornhill and A. R. Messina, (2015)"A Dynamic Mode Decomposition Framework for Global Power System Oscillation Analysis," in IEEE Transactions on Power Systems, vol. 30, no. 6, pp. 2902-2912, Nov. 2015.

[10] El-Hamdouchi, A., Willett, P. (1989). 'Comparison of hierarchic agglomerative clustering methods for document retrieval', The Computer Journal, 32, (3).

[11] van Keuk and S. S. Blackman, (1993)"On phased-array radar tracking and parameter control," in IEEE Transactions on Aerospace and Electronic Systems, vol. 29, no. 1, pp. 186-194.

[12] Hochreiter, S., and J. Schmidhuber (1997). 'Long short-term memory.' Neural computation. Vol. 9, Number 8, pp.1735-1780.

[13] Huang, NE. et.al. (1998). ‘The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis’ Proceedings of the Royal Society A, pp. 903-993.

[14] Liu, H. J., Liu, S. G. and Li, M. (2018). ‘EMD and Wavelet Threshold Denoising Method of Gear-shift Acceleration Signals’ Noise and vibration control 38(20), pp.198-203.

[15] Park, H. S and Jun CH (2009).'A simple and fast algorithm for K-medoids clustering' Expert Systems with Applications, 36(2-2), pp. 3336-3341.

[16] Rao, Y. Z., Wang, L., Rao, R., Shao, Y. J. and J. Liu (2015). ‘A method for blasting vibration signal denoising based on empircal mode decomposition wavelet threshold’ Journal of Fuzhou University (Natural Science Edition), 43(2), pp. 271-277.

[17] Seong-Taek Park and Jang Gyu Lee, (2001) "Improved Kalman filter design for three-dimensional radar tracking,"IEEE Transactions on Aerospace and Electronic Systems, vol. 37, no. 2, pp. 727-739.

[18] S. Mehrmolaei and M. R. Keyvanpour (2016) "Time series forecasting using improved ARIMA," 2016 Artificial Intelligence and Robotics (IRANOPEN), Qazvin, pp. 92-97.

[19] Tully, E. and Lucey, B.M. (2007). ‘A Power GARCH Examination of the Gold Market’ Research in International Business and Finance 21(2), pp.316-325.

[20] YANG Liang, LIU Chunhong, GUO Yuchen, DENG He, LI Daoliang, DUAN Qingling. (2019).'Prediction of Ammonia Concentration in Fattening Piggery Based on EMD-LSTM.'Journal of Agricultural Machinery (5), pp. 353-360.

[21] Zhang, X., Lai, K. K. and Y. W. Shou, (2008). ‘A New Approach for Crude oil Price Analysis Based on Empirical Mod Decomposition’ Energy Economics, 30(3), pp. 905-918.

[22] Zhao, J. P. and J. Zhou (2017) 'State Prognosis of Rotary Machines Based on Long Short-Term Memory Recurrent Neural Network. 'NOISE AND VIBRATION CONTROL, 37(4), pp. 155-159.

[23] Zhu, J. P., Zhang, N. X. and W. Z. Zhu (2013).‘EMD combined prediction of unstable time series and its application.’ Statistics & Decision (15), pp. 4-8.

[24] Zhou, Z. H. (2016).'Machine learning' Tsinghua University Press.

Analyzing the Impact of Buybacks in China Equity Market

Haochen Guo[1]

Abstract

This paper contributes to the effective analysis of share repurchases in the China equity market. Potential buybacks restrictions would likely have five implications for the equity market: slow growth in earning per share; boosting cash spending on dividends, merger and acquisition, and debt payouts; widening trading ranges; decreasing demand for shares; and lower corporate valuations. In addition to suggesting confidence in a company's performance, buybacks are one way for companies to reward shareholders. Listed companies on China's A-share market conducted a surging number of shares repurchase in 2018, it believes showed companies' strong intention of bottom fishing and would sup-port stock value.

Keywords

Buybacks, Share Repurchases, China Equity Market

JEL Classification: G10, G14, C5

1 Introduction

Buybacks, also called share repurchase, the repurchase of outstanding shares by a company in order to reduce the number of shares on the market. Free cash flow (FCF) is the amount of cash a company has after expenses, debt service, capital expenditures, and dividends. The higher the FCF, the stronger the company's balance sheet. The buybacks tide showed companies believed their share prices were already at a low level. Buybacks is the signal of confidence in a company's performance, something the Chinese government has been keen to encourage.

From the impact of share repurchases on stock prices, the potential buybacks restrictions would likely have five implications for the equity market: slow growth in Earning per Share (EPS); boosting cash spending on dividends, Merger and Acquisition (M&A), and debt payouts; widening trading ranges; decreasing demand for shares; and lower corporate valuations. Generally, share purchases indicate good cash flow, corporate governance, and the potential for future buybacks, the circumstances may be different in China. Hence, the aim of research is in unifying knowledge and terminology in the area of China equity market buybacks within background-theoretical and modeling analysis. Under the background that the financing costs of China's corporate sector are still high, and the management pressure of operating and financing cash flow is still relatively large, it believes that from the perspective of a single individual of a listed company, it is less likely to have a large-scale share repurchase.

For the view of timing of stock repurchase, the choice of share repurchase timing mainly considers two factors: First, having enough cash flow can ensure that share repurchases have enough funding sources. The company itself has enough monetary funds, good financing channels, low debt ratio, and strong cash creation ability in business activities. Second, the company's stock price is relatively low, which can reduce the cost of repurchase, ensure that the repurchase funds can maximize the benefits, and help protect the interests of small and medium investors.

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2 Literature Review

Buybacks as a payout method exist in the US since 1950. But share repurchase as a tool of corporate finance gradually spread to the UK in 1980 and accepted worldwide during the 1990s. In this period, share repurchase started in China (1992), France and Germany (1998), Japan (1995), Malaysia (1997), Singapore and Hong Kong (1998), Taiwan (2000). But the significant surge in the repurchase activity started from 1980 to 2000. The extensive literature on share repurchase proposed many hypotheses (signaling hypothesis, excess capital or cash flow hypothesis, optimum leverage hypothesis, stock option hypothesis, takeover deterrence hypothesis, substitution hypothesis, liquidity hypothesis, corporate governance hypothesis), those drives companies for undertaking share buyback.

Buybacks are like a dividend on cash allocation to investors, they were the legal owners of the business they retain. Shareholders spent their money in the company's share in the ex-anticipated return on their assets earned. The dividend, however, is pervasive in that context; businesses have been paying periodic money dividends since the founding of joint stock companies about three hundred years ago. Initially, it is the only way to pay the shareholders the excess. Researcher paper (Miller & Franco, 1961) first time suggests that the company value remains the same before and after payment of the dividend. So, in commercial finance, dividend payment is regarded as a mystery. The paper (Barclay & Smith, 1988) reported for the first time that liquidity plays significant role in choosing between cash dividend and open market repurchase as a payout method to share-holders in the US. Researcher paper (Bartov, Givoly, & Hayn, 2002) reported that after controlling the overall earnings performance of the quarter, firms those can match the analysts' expectations earn 3% more than their peer who failed to do so. To save the companies from great economic loss, managers always try to achieve the earnings expected by analysts. But when the actual earning is not enough to meet or beat analyst forecast, then manager take help of earnings management to achieve the target. There are two types of earnings management viz. accrual-based earnings management (AM) and real earnings management (RM). Changing methods of depreciation of fixed assets and changing the provisions of doubtful debts are two examples of AM. In contrast, RM is accomplished by changing the company's underlying operations (Roychowdhury, 2006).

Buybacks' primary aim was to allow companies to use their gain in two respects. In the form of dividends or buybacks, one portion of the earnings can be transferred to investors. Retained profits are laid aside for additional investments in the company's potential development. Researcher (Guo, Review and analysis of buyback in US equity market, 2019) (Guo, Analyzing the Impact of Corporate Stock Buybacks in China and the US Equity Markets, 2020) presents the buyback of share activities in US and China equity markets. Paper (Andriosopoulos & Hoque, 2013) reported that company size, money dividend, and concentration of property have an important effect on businesses ' buybacks decisions in all of these nations (UK, Germany and France). The findings indicate that big and commonly owned businesses are more probable to announce share buybacks and discovered a supplementary connection in the UK and Germany between dividend payment and share buybacks. Several variables affecting Australia's open business repurchases were examined in (Yarram, 2014). The research promotes the agency's hypotheses of signaling and leveraging and does not promote surplus money flow and hypotheses of replacement. This study's distinctive input is to mix corporate management factors with other company-specific parameters to examine the effect of corporate governance on Australia's repurchase choices. The findings indicate that the autonomy of the board has a beneficial impact on the choices of repurchase.

3 Background-Theoretical Analysis

Buying away its stocks from an investor by a business that puts up venture assets for the company's data. The stocks are purchased back at a cost that satisfies the shareholder, the amount that the company is prepared to pay for its autonomy. If the company is openly listed or passed over, the buy-back may happen. A corporation's buying back of its shares, particularly in the US, to reduce the number on the market, either to increase the return on those still available shares or to remove threatening shareholders. Action by the govt of a developing country to decrease some or all its debt to foreign banks by purchasing back that debt at the exchange cost or at a significant discount. The bank's appeal is the suppression of a harmful and adverse loan that may have already been made available in its balance sheet. A return to creditworthiness and the chance of obtaining fresh credits is the benefit for the nation in debt.

The motivation of buybacks

Comprehensive stock repurchases cases, it believes there are four main reasons for the listed companies' motivation and indication to buy back shares:

1) Transfer the signal that the stock price of the firm is undervalued into the market, raising morale among investors.

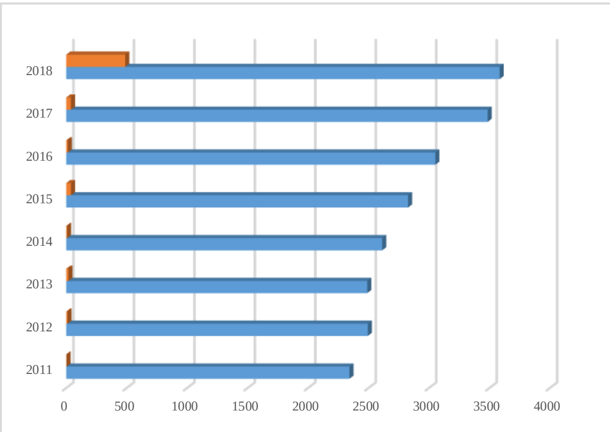
2) One of the essential anti-acquisition steps may be decreased by repurchasing stock, the value of the shares in the secondary market and the stock price of the business will be raised, thereby decreasing the likelihood of the firm being purchased to some degree.

3) Alternatives to cash dividends can indirectly distribute cash to shareholders through share buybacks. On the one hand it can help shareholders reasonably avoid taxation, on the other hand it can also prevent listed companies from avoiding the wasting of redundant self-owned funds in poor return on investment. Reduce future department expenses on the proposal.

4) The shares repurchased from the secondary market was allo-cated to the company's workers through the equity compensation program. We may, on the one side, inspire the workers and, on the other, will not harm the values of the initial shareholders.

There are 38 listed companies from 2011-2014. But, 39 listed companies in 2015, 13 listed companies in 2016, 39 listed companies in 2017. Buyback boost in 2018, buyback helped shares of some Chinese companies beat the benchmark, there are 488 listed companies operated buybacks. 161 listed companies in 2019 (till 19th June 2019). From the time point of view, the number of shares repurchase of listed companies distributes as shown in Figure 1.

Figure 1The Number of Shares Repurchase of Listed Companies Distribution



Source: author summazied

Source of funds for buybacks

There are two sources of funds for buybacks, namely, own funds and debt funds. According to empirical research on the relationship between US buybacks and corporate cash flow changes, sufficient cash is one of the main factors for enterprises to decide to buyback stocks. Share repurchase is closely related to the level of corporate cash flow, from repurchasing corporate cash flow. Looking at the changes in the components, with the implementation of the share repurchase program, the net working capital flow of the company decreased, the net operating flow decreased, and the net investment flow increased. This shows that the source of funds for corporate stock repurchase mainly depends on own funds and less on external financing. Judging from the situation of China's securities market, the source of funds for buybacks must be determined by factors such as the company's own funds, financing capacity and debt ratio, maintaining the financial stability of the company and safeguarding the interests of small and medium shareholders and creditors. The repurchase of state-owned shares should be based on its own funds, and the repurchase of tradable shares should use its own funds. The total amount of funds for share repurchase shall not exceed the sum of undistributed profits and capital reserve of the enterprise. The undistributed profits and capital accumulation fund of the enterprise are the result of long-term accumulation of the enterprise. When the enterprise operates well and the stock price fails to fully reflect its value, the company's stock price undervalued, at this time, the company's buybacks of its own shares will increase the value of the company's external tradable shares and enhance shareholders' equity. From the perspective of protecting the interests of small and medium investors, share repurchasing enterprises should be listed companies with good operating performance and good development prospects. For listed companies with poor performance, the company's undistributed profits and capital accumulation fund are low, and the stock value should be improved by improving economic efficiency. It is not appropriate to carry out share repurchase.

4 Buybacks pretender

For investors, is it worth investing in a listed company that buybacks? In fact, among the listed companies that announced and implemented the share repurchase, there is no shortage of them.

Generally to say, there are two types of buybacks pretender:

1) The amount or the proportion is very low, not really repurchased

For example, Yonghui Supermarket issued an announcement on October 20, 2018, and spent 3.11 million yuan repurchased shares for equity incentives. This amount is low. Suzhou High-tech, the company's board of directors passed the pre-plan, repurchased shares worth 5.03 million yuan, accounting for only 0.07% of the share capital, both in terms of amount and proportion are almost negligible. UFIDA, the company spent 10.59 million yuan to repurchase 442,100 shares, accounting for only 0.02% of the share capital. The proportion of shares repurchased is almost negligible. Han's Laser, which has invested more than 490 million repurchases since the announcement of the repurchase of shares, but only 0.09% of the share capital.

2) Retiring after issuing a repurchase announcement

In 2018, four listed companies - such as Huaye Capital, Hengtai Aipu, Yi Yatong, and Racing Wheel Tire, reelected the repurchase for various reasons after the announcement of the repurchase announcement.

5 Case Study

Illustration 1 – the impact of buybacks on the listed company of Baosteel

Baosteel Co., Ltd. is the largest and most technologically advanced steel conglomerate in China. It was listed on the Shanghai Stock Exchange on December 12, 2000. It is a central state-owned enterprise whose actual controller is the State-owned Assets Supervision and Administration Commission of the State Council. As of September 1, 2012, Baosteel's largest shareholder is Baosteel Group Co., Ltd., with a shareholding ratio of 74.97%. Baosteel is a typical state-owned enterprise with high concentration of equity.

Baosteel issued a share repurchase announcement on September 21, 2012. This is the first blue chip stock to be repurchased in the A-share market since 2008. In the announcement, Baosteel stated that "in order to safeguard the interests of the majority of shareholders, enhance investor confidence and maintain the company's stock price".

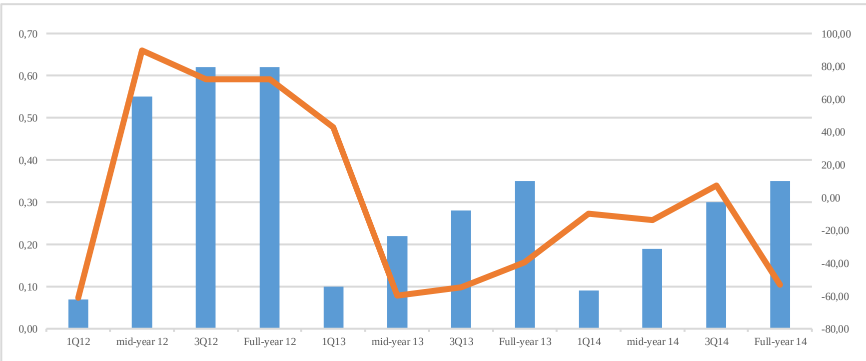
The company will repurchase the company's shares at a price of no more than 5.00 yuan per share. The total amount of repurchase will not exceed 5 billion yuan. Among them: the repurchase method is the centralized bidding transaction method of the Shanghai Stock Exchange; the purpose of the repurchase of shares is to cancel the transaction and reduce the registered capital repurchase price to not more than 5.00 yuan per share, that is, to buy back the shares at a price of 5.00 yuan per share or lower; the repurchase funds are self-owned funds, and the total amount of funds is not more than RMB 5 billion; Huabao Securities Co., Ltd. is the sole brokerage broker for this repurchase.

After this announcement, from September 21, 2012 to May 21, 2013, Baosteel repurchased more than one billion shares in 14 times, accounting for 5.9% of the total share capital before repurchase, and the actual use of funds was 5 billion. The proportion of shares held by Baosteel Group rose from 74.97% before repurchasing to 79.71% after the repurchase.

EPS impact - On September 21, 2013, Baosteel released a repurchase announcement and implemented the first repurchase. The yield increased significantly, exceeding the market yield of 0.99%, but on the second trading day, the stock yield dropped sharply, after deducting the market. After the rate of return, it was -2.9%. In the repurchase on October 15, Baosteel shares fell sharply on the repurchase date and the next trading day, with a cumulative decline of 2.25%. In the repurchase on February 4, 2013, there was a 1.6% yield on the repurchase date, and the cumulative excess return rate during the T [0,1] period was 1.98%.

Overall, the repurchase caused a positive excess return, with an average return rate of 0.14% per repo, but during the T [0,1] period, the cumulative excess return rate excluding the market yield was -0.45%. In the short term, the market does not recognize Baosteel's repurchase to improve the image of the stock.

Figure 2 EPS Impact



Source: own calculation

Volatility impact - From the perspective of volatility, from September 2012, Baosteel Co., Ltd. issued a repurchase announcement and implemented the first repurchase. From September 2012 to May 2013, the share price standard deviation was about 37% lower than the pre-repurchases share price standard deviation.

On May 21, 2013, the last stock repurchase ended. On June 7, 2013, Baosteel issued the "2012 Baoshan Iron and Steel Co., Ltd. Profit Distribution Plan Implementation Announcement", which announced a dividend of 0.13832 yuan per share, and the ex-dividend date was

June 18, 2013. This is the reason why Baosteel's share price standard deviation in May 2013 has risen sharply. Regardless of the impact of June 2013, after the repurchase period, from July 2013 to February 2014, the stock price volatility increased by 89% compared to the share price volatility during the repurchase period.

Table 1 Volatility Impact

	2014	2013	2012
Dividend per share (before tax)	0.18	0.10	0.14
Dividend per share (after tax)	0.17	0.1	0.13
Dividend yield (%)	2.55	2.62	2.81
Benchmark equity (shares)	16,471,026,024	16,471,724,924	16,471,724,924

Source: own calculation

Illustration 2 – Analyzing Inner Mongolia Yili Industrial Group's buybacks

Inner Mongolia Yili Industrial Group Co., Ltd. is the leading company in the global dairy industry, and is the first in the dairy industry in China. It is also the largest dairy company in China with the largest product category. In 2019,Yili issued an announcement to use self-use funds, no more than 12 months from the date of approval by the board of directors, and repurchase shares at a price not exceeding 35 yuan/share, accounting for 2.5%-5%, which will be used to implement equity motivation, this move highlights the company's confidence in its performance and stock price.

Buybacks overview - The large-scale repurchase incentives are sufficient, and the new regulations are fully enjoyed. The number of shares repurchased is not less than 151,953,191 shares and does not exceed 303,906,380 shares, accounting for 2.5%-5% of the total share capital, according to the limit price limit of 5.3-106 billion yuan - the amount of this repurchase is large, leaving a margin for future incentives, and the next round of incentives or expansion of incentives. In the short term, it shows development confidence and releases positive signals. In the long run, because the repurchase is used for equity incentives, it will help strengthen the company's cohesiveness and benefit long-term development. In addition, the company's implementation of share repurchase, can enjoy the new policy of repurchase, may involve: literary stock holding time from 1 year to 3 years (2020-2022): for equity incentives can be written off or transferred within three years; the repurchase amount is included in the cash dividend amount: the new regulation treats the repurchased shares as cash dividends, and the amount of share repurchase that has been implemented in the current year is calculated by the cash dividend amount, which is included in the concept of cash dividends; green channel for financing: according to the updated regualtion, the listed company applies for refinancing after the share repurchase, and the financing scale does not exceed ten times the total amount of share repurchase in the last 12 months. The resolution of the board of directors is not limited by the financing interval on the date of the previous fundraising. The approval of such refinancing applications is given priority.

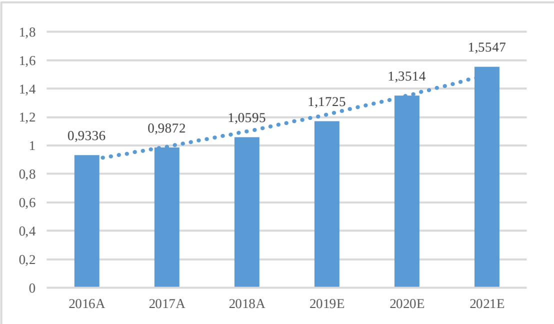
Buybacks outlook - Yili launched a share repurchase in early April 2019 and made its first repurchase in early May 2019. In May 2019, Yili has spent nearly 2 billion yuan to repurchase more than 1% of the shares. In the May 2019, The repurchase amount accounted for 5.7% of the transaction volume. The buybacks in 2015, the repurchase program was announced in July 2015. In early November 2015, the repurchase amount exceeded 1%, and the repurchase limit reached 1 billion yuan. By comparison, during the repurchase process, as the trade war brought the overall stock market volatility, the company's stock price volatility brought more low-priced chips. The repurchase has made rapid progress and has been heavily absorbed before the interim report, which fully reflects the undervaluation of the value of Yili and the long-term confidence of the company's management in the future development of the company. Moreover, the shareholding incentives following the share repurchase will be expected to drive the company's future performance to continue to improve organically.

Table 2 EPS Forecast

Units (yuan)	Dec-2018	Jan-2019	Feb-2019	Mar-2019	Apr-2019	May-2019	Jun-2019	2019E	2020E	2021E
Average Value	1.2078	1.2009	1.1794	1.1708	1.1798	1.1716	1.1725	1.1725	1.3514	1.5547
Median	1.2096	1.2096	1.178	1.1705	1.17	1.1615	1.1656	1.1656	1.325	1.5085
Maximum	1.3223	1.32	1.32	1.32	1.3201	1.3161	1.316	1.316	1.6301	1.9597
Minimum	1.11	1.0957	1.08	1.08	1.0694	1.0662	1.0661	1.0661	1.2068	1.3334
Standard Deviation	0.0531	0.0523	0.0556	0.0489	0.0539	0.0501	0.0498	0.0498	0.1002	0.1464

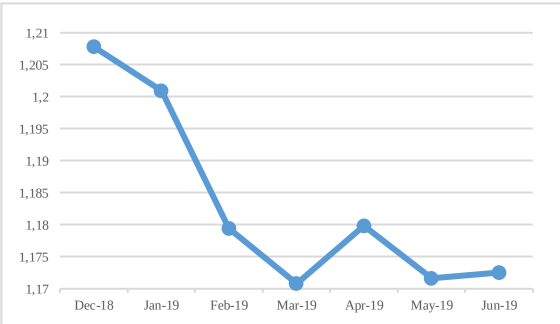
Source:own calculation

Figure 3 EPS Impact



Source: own calculation

Figure 4 EPS Trend Estimation (monthly impact)



Source: own calculation

6 Outlook of Buyback in China Equity Market

Share buyback is new things with development in China. It should strengthen the legislation of share buyback and make constant improvement and development in practice with enlarging applicable scope of share buyback; clear and specific regulation on share buyback; improving content of information disclosure; enriching way of share buyback; reinforcing Supervision of Law Enforcement; enhancing consciousness of responsibility; enhancing awareness and preventing risks; exploring the rule and grasping the characteristic;firm development of self discpline consciousness, safeguarding rights and interests of all parties.

7 Conclusion

Buybacks can signal company executives believe shares are undervalued, or they can be used for purposes such as employee stock incentive plans. In China, they may also reflect the state encouraging corporations to show support for the market during times of turmoil. Buybacks mostly occur during the A-share downturn. Observing the trend of the Shanghai Composite Index since 2012 and the listed company's number of share repurchase plans and the proposed share repurchase amount limit, it can be seen that most of the share repurchases of listed companies occur during the downturn or down period of A-shares.

During the period of April 2012 – November 2012, January 2013 – June 2013, November 2013 – April 2014, May 2015 – February 2016, the Shanghai Composite Index fell by 17.4%, 17.0%, 8.7% and 41.7% respectively. The number of listed companies repurchased by the listed companies was 31, 51, 90, and 239, the corresponding share repurchase amount caps are 88.5, 12.8, 452.7, 52.90 billion yuan.

Acknowledgements

The paper was supported by SGS project VŠB-TU Ostrava SP 2020/124, SP2019/132 Financial modelling and decision-making of companies under risk, flexibility, interaction.

References

[1] Andriosopoulos, D., & Hoque, H. (2013). The determinants of share repurchases in Europe. *International Review of Financial Analysis, 27*, 65-76.

[2] Baker, K. H., Powell, G. E., & Veit, T. E. (2003). Why companies use open-market repurchases: a managerial perspective. *The Quarterly Review of Economics and Finance, 43*(3), 483-504.

[3] Barclay, M. J., & Smith, C. (1988). Corporate payout policy: cash dividends versus open-market repurchases. *Journal of Financial Economics, 22*(1), 61-82.

[4] Bartov, E., Givoly, D., & Hayn, C. (2002). The rewards to meeting or beating earnings expectations. *Journal of Accounting and Economics, 33*(2), 173-204.

[5] Edmans, A. (2017, September 15). *The case for stock buybacks.* Retrieved from Harvard Business Review: https://hbr.org/2017/09/the-case-for-stock-buybacks

[6] Goldman Sachs. (2019). Top of mind buyback realities. *Global Macro Research*(77).

[7] Guo, H. (2019). Review and analysis of buyback in US equity market. *Financial Management of Firms and Financial Institutions*, (pp. 87-94).

[8] Guo, H. (2020). Analyzing the Impact of Corporate Stock Buybacks in China and the US Equity Markets. *International Journal of Monetary Economics and Finance, 13*(20), 89-110.

[9] Michaely, R., & Grullon, G. (2002). Dividends, share repurchases, and the substitution hypothesis. *The Journal of Finance, 54*(4), 1649-1684.

[10] Miller, M. H., & Franco, M. (1961). Dividend policy, growth, and the valuation of shares. *Journal of Business, 34*(4), 411-433.

[11] Roychowdhury, S. (2006). Earnings management through real activities manipulation. *Journal of accounting and economics, 42*(3), 335-370.

[12] Vermaelen, T. (1981). Common stock repurchases and market signalling: an empirical study. *Journal of Financial Economics, 9*(2), 139-183.

[13] Yarram, S. (2014). Factors influencing on-market share repurchase decisions in Australia. *Studies in Economics and Finance, 31*(3), 255-271.

Financial Risks in The European Union Agriculture

Michala Inkábová [1]

Abstract

The economic situation of agricultural sector is influenced by various risks. The aim of this paper is to identify and quantify the impact of determinants on the financial risks, which was measured through the coefficient of variation based on net income item. The database of European Union Farm Accountancy Data Network, years 2008-2017 were examined through a multiple regression analysis. Two hypotheses were defined in the paper. The first hypothesis analysed the impact of subsidies on financial risks. The European Commission provides system of subsidies influences the income situation to make it more stable. The second hypothesis examined the impact of taxes on financial risks. The results of the analysis confirmed the statistical significance of total taxes, on the other hand, the impact of the subsidies were not significant. If the volume of total taxes increased by 1 EUR, financial risks would increased by 0.1529 %. The second part of the analysis dealed with the hierarchical clustering and defined 2 clusters.

Key words

financial risks, coefficient of variation, regression analysis, subsidies, taxes

JEL Classification: G32, Q14, Q19

1. Introduction

Farming is generally a risky business. This is due to the combination of environmental conditions, unpredictable economic shocks, and consequently, the financial situation of farm. Additionally, the higher risk increases the financial risk resulting in lack of external funding. Farming systems in Europe face a spectrum of environmental, economic, social and institutional risks.

Price risks - characterised by strong price volatility, uncertainty about future prices and co-movement of prices, these price risks are driven by an imbalance between demand and supply that can be the result of competition, macro-economic, geopolitical, climatic changes, phytosanitary risks. Variability of output quantities and output price fluctuations are generally considered the most important elements. Income risks - characterised by an imbalance between revenues and costs. Income risks do not only refer to income volatility but also to low levels of income. Farmers with a low profit margin will therefore be more sensitive to income risk when input and output prices are fluctuating.

Farmers are ultimately concerned more about their net incomes. Agricultural income is an important indicator as it provides information on the viability of the agricultural sector.

The aim of this article was identification and quantification of statistically significant variables to financial risks across EU Member States. The selected independent variables described financial situation, taxation, output and special agricultural indicators that influenced the financial risks mainly subsidies, depreciation and rents. The financial risks was expressed through the coefficient of variation based on the net income item. The Common Agricultural Policy of the European Union forms conditions and defines targets for agricultural entities operating on its territory. Through the support system created by the I. and II. pillar of the Common Agricultural Policy subsidies agricultural entities. The subsidy policy affects the

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pension situation, business efficiency and rural development. The importance of subsidies was evaluated through the defined hypothesis. The taxation defined the national characteristic and was tested via the second hypothesis. The different levels of financial risks in EU agriculture was identified according the cluster analysis.

2. Literature review

A large volume of literature has been devoted to risk analysis in agricultural economics. The literature can be broadly classified into three categories:
(1) how to measure farmer's risk perceptions,
(2) normative analysis to provide a guideline for the optimal risk management strategies, and
(3) how risk attitudes, assuming that they are properly measured, influence farmer's actual decision -making (Holt and Chavas, 2002).

The risk balancing literature is embedded in farm finance research and has been analyzed predominantly in a US context. Analyzing the farm financial structure and optimal capital structure is a much studied research topic (Featherstone et al., 2005).

The seminal paper by Gabriel and Baker (1980) postulated the concept of strategic balancing behavior of business and financial risk and provided some preliminary US aggregate evidence in favor of the hypothesis with a linear regression model. Using an expected utility mean-variance portfolio model, Collins (1985) provides theoretical support for the risk balancing hypothesis. He also concludes that aside from business risk reducing measures, policies aimed at augmenting expected farm income might also be associated with greater risk - taking behavior for risk - averse producers.

In a simulation-optimization framework, Escalante and Barry (2001) looked at interactions between risk balancing and alternative risk management strategies. The authors show that the preference of farmers for diversified risk management strategies might downplay the importance of the risk balancing effect.

De Mey et al. (2014) used the original risk balancing framework developed by Gabriel and Baker (1980), where risk is defined in terms of the variability of outcomes. The risk measure used is the coefficient of variation, alternatively one could use standard deviations (Barry, 1981) or variances (Collins, 1985). Business risk - the inherent risk on a farm independently of the way it is financed - can be reflected in the variability of any operational return parameter such as the rate of return on assets, net cash flow or net operating income.

Several methodologies have been used to analyze the risk balancing hypothesis, such as looking at comparative statics in theoretical models (Collins, 1985), simulation/optimization models (Escalante and Barry, 2001), correlation relationship analysis (Escalante and Barry, 2003) and linear regression analysis (Turvey and Kong, 2009). In both approaches, financial risk is defined as the right - hand factor - the ratio of interest paid over net operating income after interest has been paid. This factor reflects the strategic adjustments that are made in the level of financial risk.

The risk analysis of agriculture, using the Markowitz approach or Single index model, has been applied to the number of studies, however many of them did not have aggregate character. They mainly focused on the certain part of agriculture production, for example, Barry (1980) applied the Capital Asset Prising Model assumptions to estimate beta for U.S. farm real estate market, Peterson and Leuthold (1987) used the portfolio approach to examine the cattle feeding problem, Prattley et al. (2007) applied the portfolio concept to find appropriate allocation of surveillance resources in animal populations, Barkley et al. (2010) estimated optimal crop diversification.

3. Methodology

The main research question was addressed to investigate, using a quantitative approach, financial risks in EU Member States agricultural sector, through the coefficient of variation, based on Farm Accountancy Data Network (FADN), 2008- 2017 dataset. Indentify and quantify the role and function of taxation system and financial subsidies allocated by the Common Agricultural Policy and other variables such as rent, depreciations, asset profitability, cost of debt, net worth and output. Multiple regression analysis was selected as the most suitable method.

Profits from agriculture are generally suffering from long-term downward pressure and shorter - term instability. The coefficient of variation is important in investment selection. From a financial perspective, the financial metric represents the risk-to-reward ratio where the volatility shows the risk of an investment and the mean indicates the reward of an investment. By determining the coefficient of variation of different countries, an investor identifies the risk-to-reward ratio of each country and develops an investment decision. Generally, an investor prefers a lower coefficient of variation because it provides the most optimal risk-to-reward ratio with low volatility but high returns.

3.1 Multiple regression analysis definition and description of variables

Researchers have used this multiple regression analysis as a powerful tool because it allows to model statistically the relationship between dependent variable and a set of independent variables.

The multiple regression equation is as follows:

$$\hat{Y} = b_0 + b_1X_1 + b_2X_2 + \cdots b_pX_p + \varepsilon_t \qquad (1)$$

where $\hat{Y}$ is the dependent variable, X_1through X_p are independent variables, b_0 is the value of Y when all of the independent variables (X_1 through X_p) are equal to zero, and b_1 through b_p are the estimated regression coefficients. Each regression coefficient represents the change in Y relative to a one unit change in the respective independent variable and ε_t is random component.

The next part of the analysis was clustering based on the Ward's Hierarchical Agglomerative Clustering Method. Ward's is the only one among the agglomerative clustering methods that is based on a classical sum-of-squares criterion, producing groups that minimize within-group dispersion at each binary fusion. In addition, Ward's method is interesting because it looks for clusters in multivariate Euclidean space according Murtagh and Legendre (2014).

- dependent variable= financial risks (FR) based on the coefficient of variation, measured as the standard deviation of net income/ the mean of net income [%]
- independent variables= cost of debt (COD) measured as interest paid over total outstanding debt; asset profitability (AP) the ratio of net incomes over total assets, net worth (NW) defined as total assets - liabilities; depreciation (DEP); rent (RENT), total taxes (TAX); direct subsidies per hectares (SUBDIR) represent the I. pillar of the Common Agricultural Policy; rural development subsidies per hectares (SUBRD) represent the II. pillar of the Common Agricultural Policy ; output per hectares (OUT)
- b_0 through b_9= regression coefficients
- ε_t = random component

Using these variables, the following model of financial risks was defined as:

$$FR=b_0 + b_1\,COD + b_2\,AP + b_3\,NW + b_4 DEP + b_5\,RENT + b_6\,TAX + b_7\,SUBRD + b_8\,SUBDIR + b_9\,OUT + \varepsilon_t \quad (2)$$

The selection of variables was influenced by the previous research mentioned in the literature review namely De Mey et al. (2014), Escalante and Barry (2003), linear regression analysis realised by Turvey and Kong (2009) and Jensen and Langemeier (1996).

The European Union has significantly reformed its Common Agricultural Policy by introducing direct payments, as key instrument of I. pillar, to farmers and reducing price support levels. The first pillar finances direct aid and market measures, and is fully covered by the EU. While the European agricultural prices become more volatile, all economic models assessing these reforms remain static and ignore the risk dimensions (Zheng and Gohin, 2020). The European Commission proposes the creation of a risk management toolkit under the Rural Development measures of II. pillar, where Member States could, if they felt like it, establish their own national schemes, within framework conditions set by the EU, and co-financed between Brussels and the national budgets (Tangermann, 2011). The aim of subsidies in the first and in the second pillar of the Common Agricultural Policy is to promote the income situation in the agricultural sector. The impact of subsidies on farmer's income and profitability of farms is apparent and many farms without subsidies would generate a loss. Subsidies alleviate credit constraints farms and reduce risk aversion, which could have a positive impact on the productivity of farms (Rizov et al., 2013).

Financial risk refers to the risks associated with how the farm is financed and is defined as the additional variability of the farm's operating cash flow due to the fixed financial obligations inherent in the use of credit. Some sources of financial risk include changes in interest rates or credit availability, or changes in credit conditions (Komarek et al., 2020).

The article defined two hypotheses H1 and H2 :
H1: Financial risks were dependent on subsidies.
H2: Financial risks were dependent on taxes.

The first step of analysis was verification of the statistical significance of individual variables. The model of financial risks had the character of linear model. In this case, the statistical significance of each variable was tested via the summary command. As non-significant variables were identified: rural development subsidies (SUBRD) and output per hectares (OUT). Then, the statistically significant model was subsequently tested.

Statistics Durbin - Watson did not detect the presence of autocorrelation, its value was DW = 1.9973 and p-value > α (0.4857> 0.05). Multicollinearity was tested via the vif command and was not identified. The statistically significant model was burdened by the problem of heteroscedasticity. The next step of the analysis was differentiation of the originally defined model of financial risks in equation (2) and the process of identification the statistically significant variables was repeated.

$$diff\,(FR) = b_0 + b_1\,diff\,(COD) + b_2 diff(\,AP) + b_3 diff(\,NW) + b_4 diff(DEP) + b_5\,diff(RENT) + b_6\,\,diff(TAX) + b_7\,diff(SUBRD) + b_8 diff(\,SUBDIR) + b_9 diff(\,OUT) + \varepsilon_t \quad (3)$$

4. Results

Every country that considers agriculture as strategically important economic sector strives for effective risk management in agriculture. Although risk can ultimately be measured through the losses or gains in income, from a risk management point of view grouping risks offers an effective approach toward identifying similar risks and thus allows one to apply more targeted risk management tools and strategies. In terms of agriculture, the major risks are a business risks and a financial risks.

4.1 Multiple regression analysis results

The final model of financial risks had the character of differentiated model. As non-significant variables were identified in equation (3): asset profitability (AP), direct subsidies per hectares (SUBDIR), rural development subsidies per hectares (SUBRD), output per hectares (OUT), rent (RENT).

The differentiated model of financial risks contained these significant variables: cost of debt (COD), total taxes (TAX), depreciation (DEP) and net worth (NW).
The regression analysis results are in the following table 1.

Table 1. Multiple regression analysis results

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	3.051e+01	4.707e+01	0.648	0.52351	
diff(COD)	1.135e+02	3.501e+01	3.242	0.00374	**
diff(TAX)	1.529e-01	4.382e-02	3.489	0.00208	**
diff(DEP)	7.849e-03	4.206e-03	1.866	0.07539	.
diff(NW)	-4.589e-04	1.213e-04	-3.782	0.00102	**
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1					

Source: author's own elaboration
Legend: COD- cost of debt, TAX- total taxes, DEP-depreciation, NW- net worth.

If we looked at F-statistic we would see that p-value < α (2.063e-08 <0.05), the model was statistically significant. According to the coefficient of determination R^2 is stated that the model explained 81% of the total variability. In this case, 81 % correctly explained the explanatory variable FR and the rest 19 % was a random component. Jarque - Bera Normality Test confirmed the normality, p-value > α (0.572 > 0.05). According studentized Breusch-Pagan test result p-value > α (0.1881 > 0.05) the problem of heteroscedasticity was removed. Vif command confirmed that the model was not burdened by the problem of multicollinearity.

The first statistically significant determinant was cost of debt. The coefficient belonging to this determinant was 113.5 that represented a positive impact on financial risks. If COD would be 1 unit higher financial risks would increase by 113.5 %, with a probability of 99 %. If the volume of total taxes increased by 1 EUR, financial risks would increased by 0.1529 %, with a probability of 99 %. The coefficient belonging to depreciation was 0.007849 that represented a positive impact on financial risks. If the volume of depreciation would be 1 EUR higher financial risks would increase by 0.007849 %, with a probability of 90%.
If net worth increased by 1 EUR, financial risks would declined by 0.0004589 %, with a probability of 99 %. Important role of taxes was confirmed by the results of the analysis and evaluated the hypothesis H2 as true.

On the contrary, De Mey et al. (2014) correlation relationship analysis across 15 EU Member States identified that just over half of the farm observations were risk balancers whereas the other (smaller) half were not. The coefficient in fixed effects regression suggested that a 1% increase in business risk reduced financial risk by 0.043% and had a low standard error. The results rejected evidence of strong-form risk balancing - inverse trade - offs between financial risk and business risk keeping total risk constant - but cannot reject weak-form risk balancing - inverse trade - offs between financial risk and business risk with some observed changes in total risk.

Escalante and Barry (2003) used correlation approach to measure the strength and determinants of risk balancing behavior. Using longitudinal data and cross - sectional time series for the US, they reported that over 50% of the 80 studied farms showed risk balancing behavior. Factors found to significantly influence risk balancing behavior include the amount of crop insurance coverage, the farm tenure position and crop diversification.

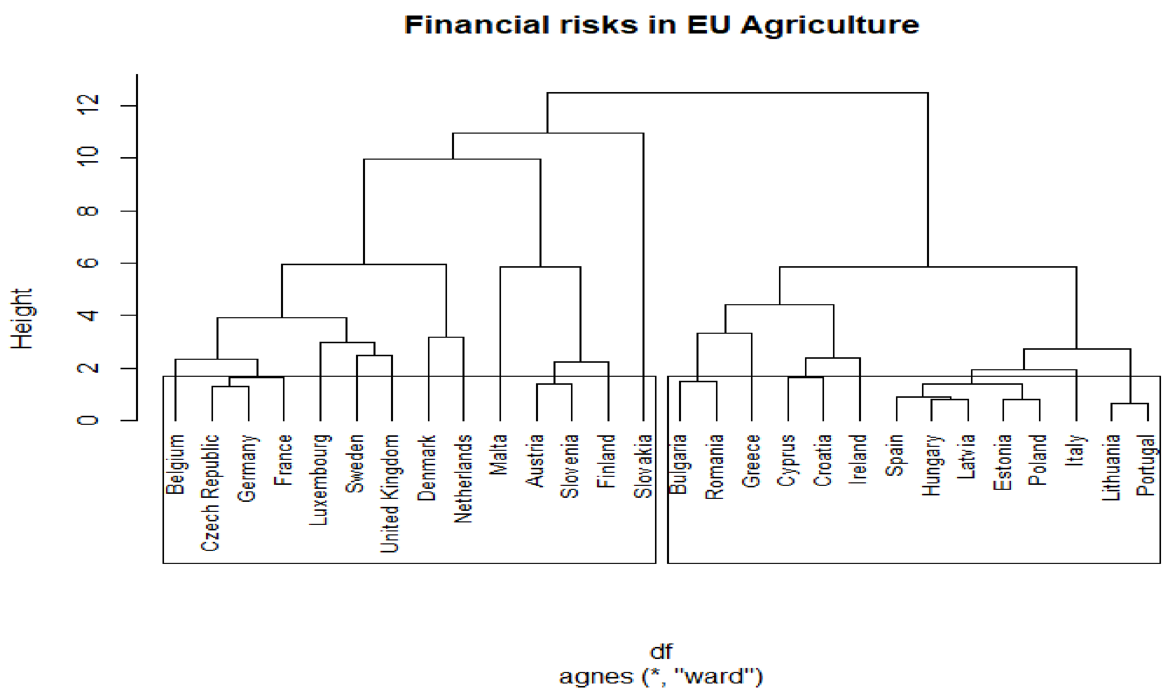
The research provided by Turvey and Kong (2009) in four linear regression models looking at rural credit use, the authors found strong evidence of risk balancing behavior by Chinese rural farm households.

Jensen and Langemeier (1996) empirically tested their unconstrained utility maximization model by estimating a Tobit regression on US panel data. Their results pointed out that leverage was affected by business risk - as measured by the variance in real operating profits - beside other factors such as profitability, tax policy, and the growth rate in the value of assets.

4.2 Hierarchical agglomerative cluster analysis

In this method, in the first stage of clustering, each statistical object – country is considered as individual cluster and subsequently, these objects are grouped to superior cluster, which are grouped again based on the distance between them while the objects with the smallest distance between are grouped together. After, on the highest level of clustering, all the statistical objects are joined into one cluster.

Figure 1: Dendrogram of financial risks in EU Member States



Source: author's own elaboration from R-program

Hierarchical agglomerative cluster analysis identified 2 clusters of EU Member States. Both clusters had 14 members. Cluster 1 represented more stable countries and mainly old members of EU. Countries like Germany, France, United Kingdom, Luxembourg had financial risks above 23 %. The most stable situation based on income stability was identified in Belgium and Finland, with financial risks 14% and 16 %.

The average volume of financial risks in cluster 2 was 28 %. The financial situation across countries in cluster 2 was different, influenced by national characteristics. Bulgaria, Estonia and Croatia reached financial risks more than 67 %. For the contrary, countries like Italy, Lithuania, Latvia, Poland, Cyprus had financial risks around 17 %.

To sum it up, according regression and hierarchical agglomerative cluster analysis results based on 10 years FADN dataset were identified and quantified statistically significant determinants of financial risks and different levels of financial risks. The literature has focused less on institutional and financial risks, compared to production and market risks. Many of the studies that examined multiple types of risk applied quantitative methods such as: Turvey and Kong (2009), de Mey et al. (2014), Escalante and Barry (2003), Jensen and Langemeier (1996), Gabriel and Baker (1980).

5. Conclusion

Results of the analysis pointed to the statistically significant determinants of financial risks and by multiple regression analysis of the European Union agricultural sector was quantified impact of independent variables on the dependent variable, which was measured through the coefficient of variation based on net income item. The analysis was produced according harmonised database of EU FADN, years 2008-2017. It consists of an annual survey carried out by the Member States of the European Union. The article defined two hypotheses. The first hypothesis dealt with the impact of subsidies on financial risks. The European Commission provides systems of subsidies, in order to secure agricultural income and make this specific sector more stable. The first hypothesis was not confirmed. The second hypothesis examined the impact of taxes on financial risks. If the volume of total taxes increased by 1 EUR, financial risks would inceased by 0.1529 %, with a probability of 99 %. The final defined model contained these significant variables: cost of debt, taxes, depreciation, net worth.

Next part of the analysis was hierarchical cluster analysis. According to the Ward's Hierarchical Agglomerative Clustering Method were defined 2 clusters. Cluster 1 represented mainly old members of EU. Countries like Germany, France, United Kingdom, Luxembourg had financial risks above 23 %. The financial situation in cluster 2 was different, influenced by national characteristics. Bulgaria, Estonia and Croatia reached financial risks more than 67 %. On the other hand, countries like Italy, Poland, Cyprus had financial risks around 17 %.

Acknowledgment: This research was supported by VEGA project No. 1/0430/19 Investment decision-making of investors in the context of effective corporate taxation.

References

[1] Barkley, A. P., Peterson, H. H., & Shroyer, J. (2010). Wheat variety selection to maximize returns and minimize risk: an application of portfolio theory. *Journal of Agricultural and Applied Economics*, *42*(1379-2016-113582), 39-55.

[2] Barry, P. J. (1980). Capital asset pricing and farm real estate. *American Journal of Agricultural Economics*, *62*(3), 549-553.

[3] Collins, R. A. (1985). Expected utility, debt-equity structure, and risk balancing. *American Journal of Agricultural Economics*, *67*(3), 627-629.

[4] de Mey, Y., Van Winsen, F., Wauters, E., Vancauteren, M., Lauwers, L., & Van Passel, S. (2014). *Farm-level evidence on risk balancing behavior in the EU-15.* Agricultural Finance Review.

[5] Escalante, C. L., & Barry, P. J. (2001). Risk balancing in an integrated farm risk management plan. *Journal of Agricultural and Applied Economics*, *33*(1379-2016-113055), 413-429.

[6] Escalante, C. L., & Barry, P. J. (2003). Determinants of the strength of strategic adjustments in farm capital structure. *Journal of Agricultural and Applied Economics*, *35*(1), 67-78.

[7] Featherstone, A. M., Ibendahl, G. A., Winter, J. R., & Spaulding, A. (2005). Farm financial structure. *Agricultural Finance Review*, *65*(2), 97.

[8] Gabriel, S. C., & Baker, C. B. (1980). Concepts of business and financial risk. *American journal of agricultural economics*, *62*(3), 560-564.

[9] Holt, M. T., & Chavas, J. P. (2002). The econometrics of risk. In *A comprehensive assessment of the role of risk in US agriculture* (pp. 213-241). Springer, Boston, MA.

[10] Jensen, F. E., & Langemeier, L. N. (1996). Optimal leverage with risk aversion: empirical evidence. *Agricultural Finance Review*, *56*(1), 85-97.

[11] Komarek, A. M., De Pinto, A., & Smith, V. H. (2020). *A review of types of risks in agriculture: What we know and what we need to know.* Agricultural Systems, 178, 102738.

[12] Murtagh, F., & Legendre, P. (2014). *Ward's hierarchical agglomerative clustering method: which algorithms implement Ward's criterion?.* Journal of classification, 31(3), 274-295.

[13] Peterson, P. E., & Leuthold, R. M. (1987). A portfolio approach to optimal hedging for a commercial cattle feedlot. *The Journal of Futures Markets (1986-1998)*, *7*(4), 443.

[14] Prattley, D. J., Morris, R. S., Stevenson, M. A., & Thornton, R. (2007). Application of portfolio theory to risk-based allocation of surveillance resources in animal populations. *Preventive veterinary medicine*, *81*(1-3), 56-69.

[15] Rizov, M., Pokrivcak, J., & Ciaian, P. (2013). CAP subsidies and productivity of the EU farms. *Journal of Agricultural Economics*, *64*(3), 537-557.

[16] Tangermann, S. (2011). *Risk management in agriculture and the future of the EU's Common Agricultural Policy.* International Centre for Trade and Sustainable Development. Issue Paper, 34, 34.

[17] Turvey, C. G., & Kong, R. (2009). Business and financial risks of small farm households in China. *China Agricultural Economic Review.*

[18] Zheng, Y., & Gohin, A. (2020). *Reforming the European Common Agricultural Policy: from price & income support to risk management*. Journal of Policy Modeling.

Exploratory data analysis as a tool for risk management of accommodation services Airbnb New York City

Andrea Kolková [1]

Abstract

Risk management in companies begins with a thorough analysis of data. One of the basic tools for data analysis is exploratory data analysis (here after EDA). The aim of this article is to point out the importance of the EDA in risk management in companies. Based on EDA, Airbnb New York City data quantifies characteristics that will enable further management and leadership of this company. The article points out the possibilities of EDA calculation in statistical programs in Excel, SPSS and R. The resulting calculation is compared in these three programs in terms of their accuracy, time and knowledge of the researcher's knowledge.

Key words

Exploratory data analysis, risk management, graphical analysis, descriptive statistic

JEL Classification: Here write the JEL classification code(s)

1 Introduction

In 1977, John Turkey published a book called Exploratory Data Analysis (Turkey, 1977) (here after EDA). The EDA designation has been used for this analysis, and although many years have passed since then and the abbreviation EDA has been added and refined by a number of publications, it has been used and is still used today.

The aim of the article was to highlight the importance of EDA in risk management in companies and compare options and difficulty in quantifying the EDA statistical programs Excel, SPSS, and R. The data Airbnb New York City are based EDA quantified characteristics that allow more control and management of the company. Subsequently, the individual statistical programs are compared and, on the basis of a multi-criteria assessment, their suitability for the calculation of the EDA is evaluated.

The importance of the EDA for businesses is evidenced by many studies, for example (Huang, 2012), (Gavurova, Suhanyi, L, & Rigelsk, M, 2020), (Gavurova, Ivankova, V, Rigelsky, M, & Kmecova, I, 2020). Almost all researchers use the EDA as the basis for any research (Kolková A. , 2020), (Navrátil & Kolková, 2019), (Marček, 2013), (Macurová, P., Klabusayová, N., & Tvrdoň, L., 2018), (Papastefanopoulos, Linardatos, & Kotsiantis, 2020), (Folvarčná et al., 2020).

2 Data and methodology

The EDA as the primary risk management tool in this article will be presented at Airbnb data in New York City. The researched data set contains activities within this business platform for

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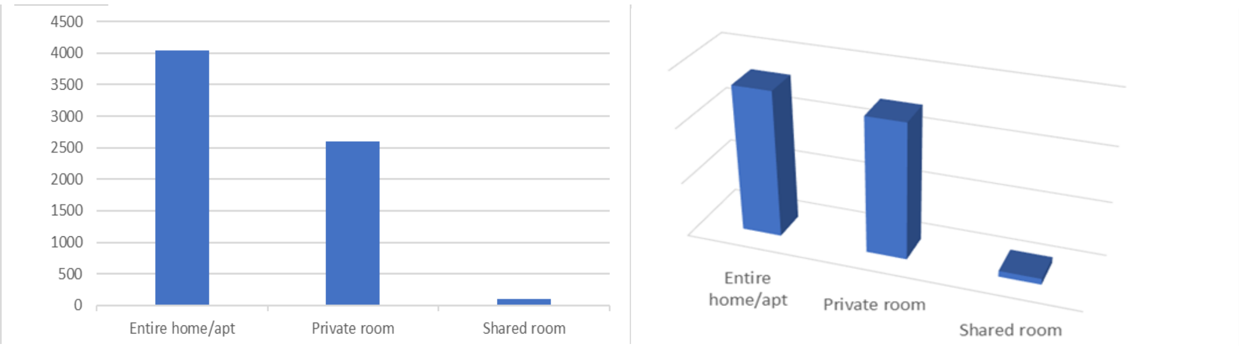
the year 2019. There is information about the name of accommodation and first name of the accommodated person, accommodation area, longitude and latitude, room type, its price in dollars, minimum number of nights spent here, number of reviews the accommodation, number of days when listing is available for booking. Data are obtained from the portal kaggle (www.kaggle.com, 2019). Total is analyzed 47,905 accommodation spaces, 44% of them, located in Manhattan, Brooklyn 41%, 15% in other parts of New York City. It is about entire home/apt (52%), private room (46%) or shared room (2%).

2.1 Graphical analysis of the data set

EDA usually begins with graphical analysis. Graphic data analysis consists of data visualization using various types of graphs (Lishmanová, 2012). Graphical analysis of time series usually begins with a line graph. Another graph used is a bar graph. In this type it is possible to choose a number of different graphic adjustments. Often, especially in popular science articles, a three-dimensional graph is chosen. This, however, must be used very carefully, often causes a lot of distortion.

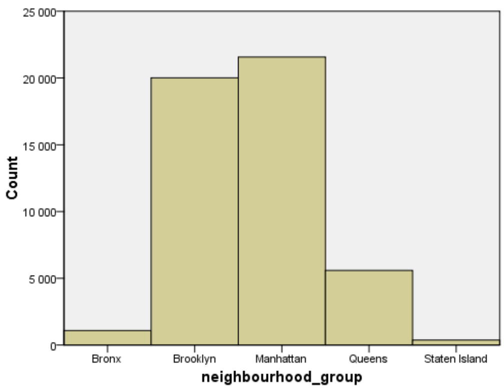
Figure 1: Different forms of bar graph of the number of accommodation in Airbnb New York City by type in excel.



The bar graph, a non-spatial graph, without effects, seems to be the most suitable of bar graphs in business economics. Figure 1 shows the number of different types of accommodation. It is clear from the graph that most people rented whole houses or whole apartments through AIRBN in New York City, private rooms are also used quite a lot from the point of view of this chart, with only a small difference compared to whole houses and apartments. Finally, shared rooms, where it is possible to stay with strangers together, were significantly lower. If we choose a three-dimensional graph, such a sloping view will cause us to feel that almost the same amount of houses and apartments has been rented. As can be seen at first glance, visualization when using graphically attractive shapes can lead to distorting statements. These effects can, of course, be used to manipulate the public and the opinions of interest groups, but in solid research these manipulations have no place. Data manipulation is completely unethical in research.

Statistical programs such as SPSS already recommend graphs according to the most suitable visual style. Figure 2 shows the bar graph of accommodation in Airbnb New York City according to the district in which the accommodation is offered.

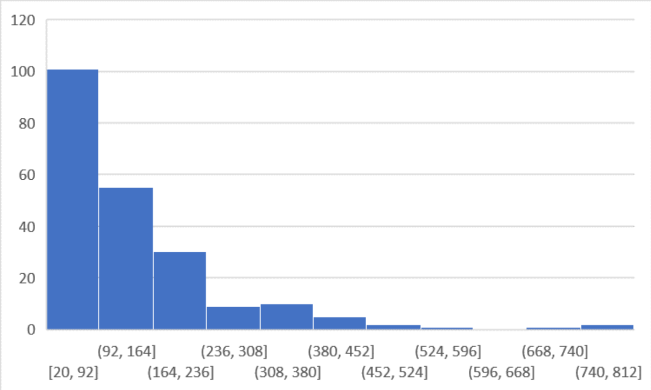
Figure 2: Bar chart of accommodation location in SPSS



This chart clearly shows that Manhattan and Brooklyn are the most popular destinations at Airbnb in New York City. Accommodation in the Bronx and Staten Island are rather marginal.

Especially when processing data for statistical purposes, a histogram is also often chosen. A histogram is a graphical representation of frequencies using a bar graph of a set of values divided into classes. In the following histogram we see again the data of Airbnb New York City, this time the price data, the values are in USD per night.

Figure 2: Histogram of accommodation prices for AIRBN New York city



From the histogram in Figure 3, we can read that the prices of accommodation at Airbnb New York City most often range from 20-92 USD. A specific option of bar graphs, which will provide us with a large amount of information, is a stacked bar graph. This graph shows, for example, the type of accommodation Airbnb in different parts of New York City and shows the share of each type of accommodation in different destinations in New York. In the Bronx, for example, room rentals predominate, while people in Manhattan usually rent whole houses or apartments. The largest number of shared rooms is in Manhattan, but in relation to other accommodation options, this form is the least common in Manhattan. Also in the Bronx, renting whole rooms or apartments is a significantly less common option than renting private rooms, which is not the case in other parts of New York.

The last widely used graph in modern graphical analysis of business data is certainly the box graph. This was first used in 1977 by statistician J.W. Turkey (Lishmanová, 2012). Today it is possible to meet with various designations of this graph, such as a box with a beard, or a bearded box, in the statistical program took the name Boxplot. The construction of this graph is based on 5 basic statistical characteristics, usually the median (or mean), upper and lower quartiles, the minimum and maximum value of the data set, and is supplemented by remote or distant observations.

This graph is already a part of the excel program (from the version of Office 2016), however, due to the fact that its practical use in this program is only marginal for the time being, in the following text preference is given to statistical programs SPSS or R (Pedersen & all., 2020).

2.2 Statistical characteristics of the data set

Another part of the EDA is the quantification of the basic statistical characteristics of the data set. These include, in particular, position measures (mean, minimum, maximum, median, mode, and quartiles), variability measures (variance, standard deviation, coefficient of variation), shape measures (skewness, sharpness). These values can be quantified in all three selected statistical programs with a greater or lesser degree of time for the researcher.

The most laborious and time consuming is the solution and calculation of EDA in Excel. There is no function in Excel that would summarize the entire EDA analysis, so it is necessary to calculate each characteristic with a separate function. The functions used to calculate EDA indicators are summarized in the statistical category, but this also contains a number of other statistical functions, the following table summarizes the functions in Excel most often used to calculate the basic EDA indicators.

Table 1: The most commonly used functions to calculate the EDA in Excel

Statistical characteristics		Functions in Excel
Measures of position	mean	AVERAGEA
	minimum	MIN
	maximum	MAX
	median	MEDIAN
	modus	MODE.SNGL
	quartiles	QUARTIL (minimum=0, first quartile=1, median=2, third quartile=3, maximum=4)
Measures of variability	variance	VAR.VÝBĚR
	standard deviation	SMODCH.VÝBĚR
	Variation coefficient	SMODCH.VÝBĚR/AVERAGEA
Measures of shape	skewness	SKEW
	kurtosis	KURT

Subsequently, it is necessary to insert the data into the text or table and create a user-friendly output. Due to the time-consuming nature of this procedure and it's very little use in practice today. When using the statistical program SPSS, the path to EDA is somewhat shorter and I get the basic characteristics at once using the menu Analyze - Descriptive statistic - Descriptive.

Airbnb data in New York City were again used for the calculation in this article, this time only for the variable Price (price of shared accommodation). The output of the SPSS program is Table 2. From this it can be read that the average price of accommodation is 152.79 USD with a standard error of 1.083 on average. The standard deviation is 238,831. From the values of the skewness, which reaches 19,151, it is evident that the prices are smaller than the average and are positively skewed. From the values of sharpness it can be read that the statistical distribution of the variable price is much skewed.

Table 2: EDA for Airbnb in New York City

Descriptive Statistics

	N	Range	Minimum	Maximum	Mean		Std. Deviation	Variance	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
price	48618	9990	10	10000	152,79	1,083	238,831	57040,302	19,151	,011	591,879	,022
Valid N (listwise)	48618											

EDA in the R program also allows to quantify all characteristics at once. First, we name the variable, for example, price. Further, he then just a simple summary statement and the basic characteristics are evaluated:

summary(price)

```
Min.    :   10.0
 1st Qu.:    69.0
 Median :  107.0
 Mean   :  152.8
 3rd Qu.:  175.0
 Max.   :10000.0
 NA's   :11
```

We can also calculate the standard deviation, variance, or any quantile, according to,

```
sd(price,na.rm = T)
var(price,na.rm = T)
quantile(price,prob = 0.3,na.rm = T)
```

For other statistical quantities such as skewness or sharpness, it is necessary to use another package. The best known package for statistical functions is *dplyr*. The skew and sharpness feature is also part of the *moments* package. The result is then,

```
  skewness(price)
[1] 19.151
  kurtosis(price)
[1] 591.879
```

2.3 Analysis and solution of missing values and outliers

Data in the business economy are very often not what we would like and it often happens that some values are skewed, extreme, or missing altogether. Therefore, it is necessary to perform further analyzes on the basis of the EDA analysis and subsequently solve any problems arising from their existence. The basic data problem in the business economy is missing data and outliers.

Missing values, despite the great efforts of researchers to have complete data, nevertheless sometimes occur in the file. Especially when it comes to question-based research, respondents do not want to answer. But also with the values of sales, prices, or other quantitative corporate variables, it may happen that the given data was incorrectly entered, there was a power failure, system, or human error and the historical data can no longer be ascertained. With small selections and a small amount of data, the replacement of missing data is inappropriate and could greatly affect the results. In such a case, it is only possible to recommend supplementing the data for the research, increasing the number of respondents or observation values. With large selections and a wide data matrix, the possibility of replacing a few missing values can already be considered. Nevertheless, it is necessary to approach the replacement of missing values with caution.

There are several ways to replace the missing value. The easiest way is to replace the missing value with the sample average, or the average or median of nearby points. When knowing the distribution of the investigated quantity, it is possible to use the replacement by a random number based on the parameters of the given distribution. Another option is to use a regression function or linear interpolation. Many of these options are also offered by statistical software, however, it is always appropriate to consider whether it is not more appropriate for research to completely eliminate missing values from the data matrix.

After checking the completeness of the data, it should also be customary to assess their accuracy and consistency. In other words, we are looking for outliers. Values that are skewed in the corporate economy for various non-standard reasons, such as the price deviated by a one-off event, a batch in production changed due to defective material, changes in purchasing preferences due to weather conditions or other non-standard weather.

Finding and resolving missing and outlying values in Excel is already a very time-consuming affair consisting only in filtering data and its subsequent manual replacement. A certain possibility of using Excel is perhaps only a graphical analysis using a box diagram, which allows to omit remote observations or not. The possibilities of Excel are tested again on the data of the price of shared accommodation. Figure 4 shows a comparison of the box diagram when the outer points were not displayed and the second graph of the same values when displaying the external values, of course it is necessary to note the need to change the scale in the second graph.

From both graphs we can read the quartile range and clearly say where 50% of all data is located. Also, 75% of accommodation prices are less than $ 170, and conversely, 75% of them are higher than $ 58.75. From UDS 20 to USD 325, all accommodation prices range from a few remote values, i.e. individual prices exceeding this value.

Figure 3: Box diagram of Airbnb shared accommodation prices in New York City.

Working in the SPSS statistical program seems to be much more efficient. Searching for outdated and missing values is again enabled by one dialog box. When searching for missing and outlying values in SPSS, the values of the price of shared accommodation are again used. The results are given in Table 3 and it is evident that the missing value occurred 11 times in the given data, the least extremely high values reach 765 times.

Table 3: Finding outliers and missing value in Airbnb New York City in SPSS

Univariate Statistics

	N	Mean	Std. Deviation	Missing		No. of Extremes[a]	
				Count	Percent	Low	High
price	48618	152,79	238,831	11	,0	0	765

a. Number of cases outside the range (Mean - 2*SD, Mean + 2*SD).

If we subsequently want to solve the missing values, the procedure is automated again and it is up to the researcher to choose the method of replacing the missing data. In the solution we have the mean value of the series (series mean), the average of the nearest points (mean of nearby points), the median of the nearest points (median of nearby points), linear interpolation (linear interpolation) and the linear trend in the point (linear trend at point).

When detecting missing values in the R language, a simple *is.na (price)* command can be used, and if you want to omit missing values, just enter *na.omit(price).*

3 Results

From the assessment of customer requirements, it was found that they significantly prefer entire home or apartment. The least requested were shared rooms. The highest risk of customer disinterest is therefore associated with offering these shared rooms. The biggest interest is in accommodation in the Manhattan district, followed by the Brooklyn. There is minimal interest in accommodation in the Bronx and Staten Island, and again their offer can be described as the type of accommodation with the greatest business risk. When examining the price, it was found that the median and the mean value differ significantly. And so that the median is significantly lower than the mean. This is probably because several outliers of significantly higher prices for Manhattan and Brookline affected that average. People who offer Airbnb New York City accommodation that is not significantly luxurious should, based on competitively oriented pricing, target their pricing policy by median rather than mean. From the values of the skewness, which reaches 19,151, it is evident that the prices are smaller than the average and are positively skewed. This is also confirmed by the previous fact, so targeting a competitively oriented pricing policy according to mean carries a considerable business risk. From the values of sharpness it can be read that the statistical distribution of the variable price is much skewed.

If we would like to decide which of the used software is the most suitable for solving the EDA problem, it is certainly expedient to support this with multi-criteria decisions. The basic parameters of this decision will certainly be the time required to perform the calculation, the requirements for the researcher's knowledge and, of course, and the accuracy of the result. The results presented here show that the accuracy of the calculation is the same for all software in the EDA analysis. Therefore, this criterion will not be decisive for the choice of software. Thus, it is clear that the choice of software depends on a comparison of the time spent calculating the EDA and the researcher's knowledge requirements. In terms of time, the EDA software R calculates the fastest. Where the whole calculation can be performed at once with a few simple commands. SPSS software requires more computational setup, however, the entire EDA can be handled in minutes. By setting all parameters together, the software calculates the EDA characteristics in summary. Excel software seems inappropriate over time. Each characteristic must be calculated by a separate command, so the whole EDA is a relatively lengthy process. Another possible criterion is the complexity of the researcher's knowledge requirements. To use the R program, knowledge of the basics of programming is required, which may limit the range of researchers who are able to use the software. SPSS and excel, on the other hand, do not require programming knowledge. Of these two programs, Excel can be described as theoretically easier. In our conditions, this program belongs to the basic equipment of most home and school computers, and it can be expected that its knowledge is general. SPSS is a program that students usually encounter at universities, which is why it is only slightly more challenging for researchers.

4 Conclusion

The aim of the article was to point out the importance of EDA in risk management in companies and to compare the possibilities and complexity of quantifying EDA in statistical programs Excel, SPSS and R. On Airbnb New York City data, characteristics were quantified based on EDA. The results show that the EDA can be an interesting primary tool for risk management. And it is a necessary basis for further statistical analysis of any time series examined.

When comparing the individual software, the result was partly subjective. In accuracy, the programs are similar. However, Excel, as a possible statistical program, seemed to be the least advantageous, especially in terms of time, which significantly exceeds the EDA calculation time for other software. R is the most demanding of the researcher's knowledge, as it is based on basic programming knowledge. However, this is now a common part of education, so the final decision depends on the preferences of a particular researcher and, of course, on how he intends to use the data further.

Acknowledgment

This research was supported by the Czech Science Foundation (GACR Project GA18-13951S).

References

[1] Banerjee, N., Morton, A., & Akartunalı, K. (2020). Passenger demand forecasting in scheduled transportation. *European Journal of Operational Research, 286*(3), p. 797-810.

[2] Björn Bokelmann, S. L. (2019). Spurious patterns in Google Trends data - An analysis of the effects on. *Tourism Management (75)*, pp. 1-12.

[3] Bojer, C., & Meldgaard, J. (2020). Learnings from Kaggle’s Forecasting Competitions. *Work.Paper*.

[4] Dostál, P. R. (2005). *Pokročilé metody manažerského rozhodování: konkrétní příklady využití metod v praxi.* Praha: Grada.

[5] Dostál, P., Rais, K., & Sojka, Z. (2005). *Pokročilé metody manažerského rozhodování: pro manažery, specialisty, podnikatele a studenty, konkrétní příklady využití metod v praxi.* Praha: Grada Publishing.

[6] Folvarčná, A., Fialová, V., Kolková, A. (2020). Innovations in family business: succession and its influence on sources of investment. . *8th International Conference Innovation Management, Entrepreneurship and Sustainability.* Prague: VŠE.

[7] Gavurova, B., Ivankova, V, Rigelsky, M, & Kmecova, I. (2020). *Ow Do Gender Inequalities in Health Relate to the Competitiveness of Developed Countries? An Empirical Study*. *Journal of Competitiveness, 12*(3), pp. 98 -118.

[8] Gavurova, B., Suhanyi, L, & Rigelsk, M. (2020). Tourist Spending and Productivity of Economy in Oecd Countries -Research on Perspectives of Sustainable Tourism. *Entrepreneurship and Sustainability Issues, 8*(1), pp. 983-1000.

[9] He, YX, Wang, MY, Guang, FT, & Zhao, WB. (2020). Research on the method of electricity demand analysis and forecasting: the case of China. *Electric Power Systems Research*.

[10] Huang, C. (2012). To model, or not to model: Forecasting for customer prioritization. *International journal of forecasting., 2*(28), pp. 497-506.

[11] Hyndman, R., & Athanasopoulos, G. (2018). *Forecasting, principles and practise.* Middletown: Otext.

[12] Kolková, A. (2018). Measuring the accuracy of quantitative prognostic methods and methods based on technical indicators in the field of tourism. *Journal Acta Oeconomica Universitatis Selye, 7(1),* pp. 58-70.

[13] Kolková, A. (2020). Forecast accurqacy: application to forecasting sales in services to using increase business competetiveness. *Journal of Competitiveness*.

[14] Kolková, A. (2020). The Application of Forecasting Sales of Services to Increase Business Competitiveness. *Journal of Competitiveness*(12(2)), pp. 90–105.

[15] Lishmanová, M. (2012). *Úvod do statistiky.* Ostrava: VŠB- TU Ostrava.

[16] Macurová, P., Klabusayová, N., & Tvrdoň, L. (2018). *Logistika 2*. Ostrava: VŠB-TU Ostrava,.

[17] Marček, D. (2013). *Pravděpodobnostné modelovanie a soft computing v ekonomike.* Ostrava: VŠB - TU Ostrava.

[18] Navrátil, M., & Kolková, A. (2019). Decomposition and Forecasting Time Series in the Business Economy Using Prophet Forecasting Model. *Central European Business Review 2019, 8(4),* pp. 26-39.

[19] Papastefanopoulos, V., Linardatos, P., & Kotsiantis, S. (2020). COVID-19: A Comparison of Time Series Methods to Forecast Percentage of Active Cases per Population. *Applied Sciences, 10*(11).

[20] Pedersen, T. L., & all., e. (2020). *Package 'ggplot2'(version 3.3.2)*. Načteno z cloud.r-project.org: ggplot2.tidyverse.org,https://github.com/tidyverse/ggplot2

[21] Turkey, J. W. (1977). *Exploratory Data Analysis.* California: Addison-Wesley Publishing Company.

[22] *www.kaggle.com*. (2019). Available at kaggle: https://www.kaggle.com/dgomonov/new-york-city-airbnb-open-data

[23] Yang, YX, Heppenstall, A, Turner, A, & Comber, A. (2020). Using graph structural information about flows to enhance short-term demand prediction in bike-sharing systems. *Computers Environment and Urban Systems*(83).

Front-End Loads Problem

Kateřina Kořená, Karel Kořený [1]

Abstract

The paper is focused on the description and explanation of the problem which is connected with investing in funds. The authors deal with the payment of the front-end loads.

Key words

Investing in funds, mutual fund, fees, problem, front-end loads, short-term investing, long-term investing.

JEL Classification: G21

1. Introduction

Collective investment has become an important part of the financial markets. More and more investors start invest in the different types of funds. The aim of this paper is to focus on the investing of the retail investors. The authors want to describe and explain the shareholders´ fee problem. The main task is to identify and clarify this important issue.

1.1 Fee Management of the Mutual Funds

Generally, two types of fees are connected with the investing in funds – ongoing fund operating fees and shareholder fees. The ongoing costs can include management fees (the costs paid to fund portfolio managers and investment advisors) and administrative costs (e.g. accounting, legal and custodial expenses). Management fees are calculated as a fixed percentage of the average net asset value of a fund over the year. Net asset value is the fund's total assets minus current liabilities. The shareholder fees are fees connected with buying and selling the shares. They can be purchase and redemption fees (transaction fees) and in case of load funds they are called sales load commissions. The purchase fees are fees assessed by a broker or a bank, for assisting purchase of a security - funds charge this fees when investors buy mutual fund shares. The redemption fees are fee charged to an investor when shares are sold from a fund. These fees are charged by the fund company and they are also known as exit fees, market timing fees, or short-term trading fees.

2. Front-End Loads

Sales loads are commissions that investors pay when they buy or sell mutual fund shares and they are calculated as a percentage of the amount investors want to invest or have invested in the fund. The fees paid at the time of purchase are front-end loads, while fees paid at the time of sale are back-end loads. The paper is focused on the importance of the front-end loads and on the necessity to explain their functioning.

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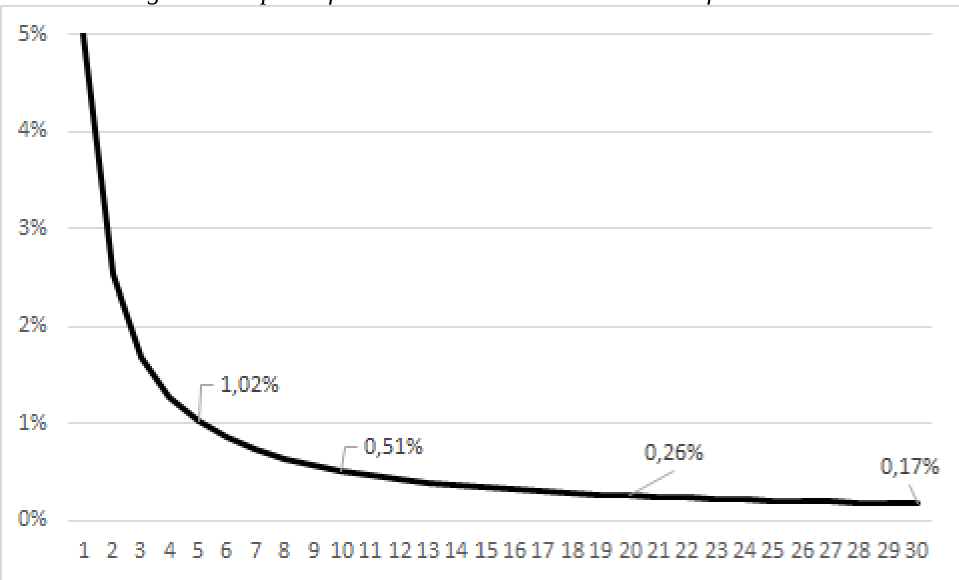
2.1 Impact of Front-End Loads on Total Value of Investment

It is necessary to understand that front-end loads reduce the amount of the investment. In case that you invest 1,000 CZK in a mutual fund with a 5% front-end load you must pay 50 CZK as a commission. This means that 950 CZK remain and this amount of money is in fact invested in the fund. Therefore, the authors want to describe the impact of this 5% commission on the total value of the investment.

Figure 1: Impact of Front-End Load on Total Value of Investment

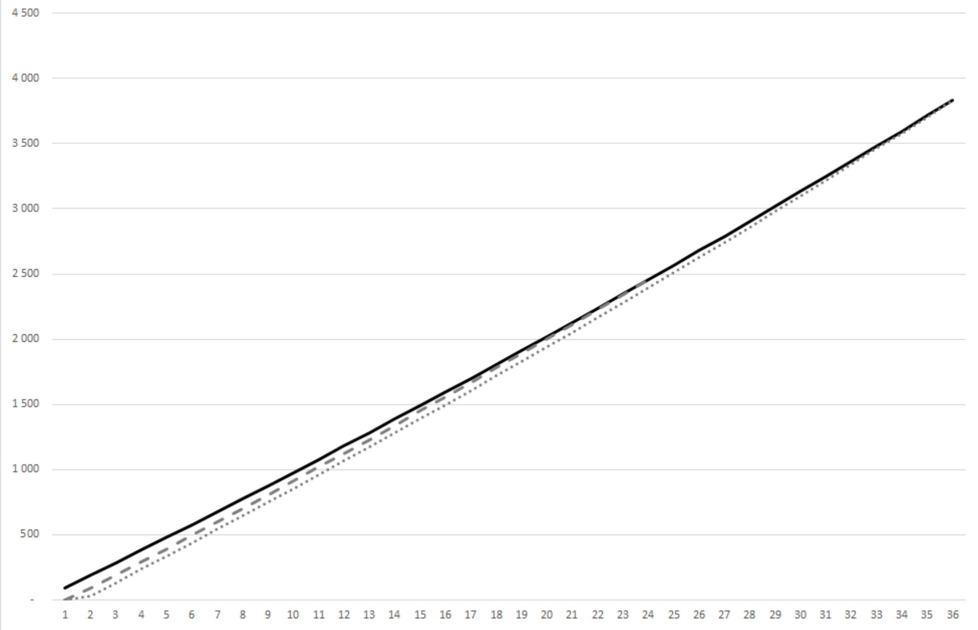


In Figure 1 we can see that there is an important difference in case of short-term and long-term investing. In case of investing for 1 year it is visible that this impact is very high because investor will lose 5% from his total investment. If investor invests for 5 years this impact is going down to 1,02%. The longer investment horizon, the smaller impact of this commission. Finally, this means that in case that investor invests for 20 years this impact is only 0,26% and for 30 years is even only 0,17%. On that account is possible to claim that the investors must be very careful concerning these front-end loads if they want to invest for shorter time. In such situation is crucial to compare the fees and it is reasonable to try to find the lowest fee. But if they want to invest for more than 5 years the importance of these commissions is diminishing. For long-term investing these commissions are not such important and do not significantly influence the total value of the investment.

2.2 Prepaid Fees Problem

Recently and more and more frequently some financial advisors have started to force their clients to choose the prepaid commissions for long-term investing. It is explained that this version of fee is for them comfortable, advantageous and even more profitable choice. Therefore, the authors would like to aim at these fees and introduce the problem connected with them. They focus on regular investment during time horizon from 1 to 30 years (Figure 2 and Figure 3).

Figure 2: Comparison of Prepaid Front-End Loads and Regular Front-End Loads in 36 Months



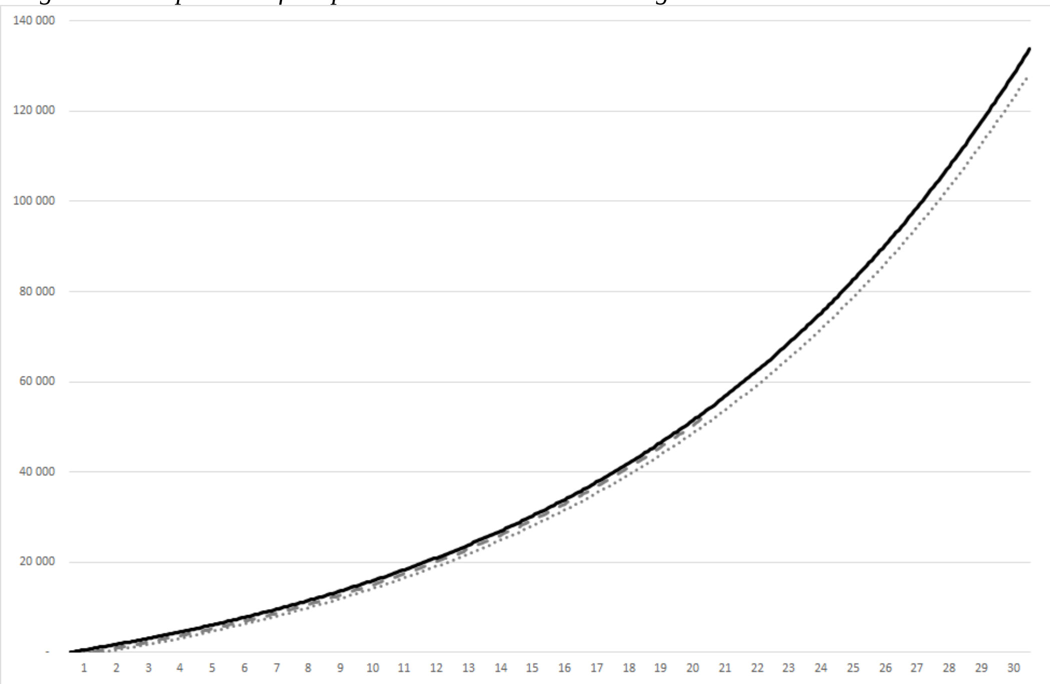
In Figure 2 the axis x is in months (36 months), the yield (return) is 8% (long-term investing in equity instruments). Front-end load is 5% in case of regular payment of these commissions during next 30 years. Prepaid front-end load is reduced to 4,6% from 2 to 9 years, to 4,1% from 10 to 19 years and to 3,6% from 20 to years[2]. The commission is charged from the first investments. In Figure 2 we can see first 3 years, in Figure 3 the whole-time horizon 30 years. The result is that for 2 years are for investors advantageous prepaid front-end loads and for 3 years regular front-end loads. The difference calculated as total value of this investment is very small in both examples – for 2 years + 0,059%, for 3 years + 0,128%.

In Figure 3 is the same example of regular investment but the axis x is in years (30 years). It is visible that if investors invest for longer time horizon than 3 years the difference in total value of investment is increasing and finally is considerably more advantageous not to pay prepaid front-end load. For investors is not possible to catch up the margin that it is in case of prepaid front-end load – they start to invest the whole amount of their investment later because they have to firstly pay this commission. The result is that for 20 years is the difference 1,96% and for total 30 years even 4,06%. This result means that investors' total value of investment is reduced and that in case of prepaid fee they lose their money.

One more disadvantage is connected with the prepaid payments – if investors pay these fees in advance and then they want to finish their investing in this specific fund earlier they do not receive their commissions back. That is why it is necessary to add that liquidity of such investment is limited because is not suitable to withdraw or change this investment in short time period.

[2] According real offer of the investment company in the Czech Republic.

Figure 3: Comparison of Prepaid Front-End Loads and Regular Front-End Loads in 30 Years



3. Conclusion

This paper is focused on the payment of front-end loads in mutual funds. The result from this paper and the main recommendation is following: it is important difference between short-term and long-term investing in funds in case of front-end loads. Concerning the rate of this fee it is necessary to grasp that the importance of this fee is for long-term investing less relevant in comparison to short-term investing. Concerning the advantageousness of prepaid front-end loads, it is real that they are for investors advantageous only in the short time period. Therefore, they are not appropriate for the long time period. If they are offered by financial advisors for long-term investing it is not right for investors. Therefore, for final statement it is possible to express that every investor should primarily understand these facts not to lose his money.

References

[1] Bogle, J.C. (2009). Common Sense on Mutual Funds. 10th ed. Wiley.

[2] De Lavenère Lusan, J. (2012) Investing in Funds. Financial Times/ Prentice Hall.

Run-off analyse as the tool of control the adequacy of reserves in life insurance

Ingrid Krčová, Vladimír Mucha, Katarína Sakálová [1]

Abstract

One of the basic tools of the life insurance business is its technical reserves or technical provisions (TRPP), which is the sum of the technical reserves for claims reported but not settled (RBNS) and the technical reserves for claims incurred but not reported (IBNR). For life office is necessary by law to test the sufficiency of technical reserves. To determine whether the calculation of the sufficiency test has been performed correctly, various control mechanisms and control tools are used. One of them is run-off analysis (run-off test). The aim of this article is to show an example of the use of run-off test. The main idea of this test is the calculation of special absolute or ratio coefficients for RBNS, IRBN and TRPP analysis.

Key words

Technical reserves, sufficiency test, RBNS, IBNR, run-off test.

JEL Classification: G22

1. Introduction

The purpose of this article is to deal acquainted with one of the tools for checking the sufficient amount of reserves of an insurance company operating in the field of life insurance. It is clear that insurance companies are currently valuing their reserves using stochastic methods in accordance with the Solvency II Directive. However, various tools are used to determine whether the reserves have been determined by these stochastic methods in a sufficient, and therefore correct, amount. One of them is a deterministic tool, namely run-off analysis, the use of which is regulated by the Directive of the Slovak Society of Actuaries[2]. Despite the fact that this tool is deterministic and simple, life insurance companies operating on the Slovak insurance market use it.

2. Technical reserves in life insurance

One of the basic tools of the life insurance business is its policy values (technical reserves, technical provisions). Technical provisions are accumulated from premiums received. The technical reserves include funds that life insurance companies will need in the next period to cover various types of risks. Without them life insurance company would not be able to function effectively. It is necessary to be aware of the different creation and nature of technical provisions which are connected with characteristics of different kinds of life insurance products. For some types of products (i.e. endowment) reserves are created from the whole of

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[2] Odborná smernica SSA č. 1 v3: The adequacy of the technical reserves in life insurance.

the premiums received after deducting administrative costs. Premiums are accumulated in technical provisions for a longer period of time than for another type of life insurance products (i.e. term assurance). In the case of this type of risk insurance, premiums are almost completely consumed during the given year.

Technical provisions are created as a compensation for fluctuations in claims and are determined to compensate the differences in time between insurance premiums paid and claims paid. The use and creation of reserves for life and non-life insurance is regulated by the Insurance Act. The technical reserves are therefore compulsorily created by the life insurance company. They are on the liabilities side of the balance sheet of the insurance company. Each of the reserves in accounted separately. The insurance company further invest resources from technical reserves in financial market funds. The investment conditions are not only subject to strict limits of law[3], but must fulfil the principles of security, profitability, liquidity and diversification. An insurance company, like any other financial institution (i. e. banks) by the decision how to invest technical reserves in a various kind off assets must ensure, that volatility of its risk portfolio in optimal or as good as possible or smallest.[4]

2.1 TRPP, RBNS and IBNR reserves

Technical claims reserve (in what follows we will use TRPP) for policy claims is an estimate of the total costs that result from the payments of all claims for claims occurring at the end of the reporting period, less payments for already paid for those claims. TRPP also includes costs connected with the settlement of claims. TRPP is the sum of the technical reserves for claims reported but not settled (RBNS - Reported But Not Settled) and the technical reserves for claims incurred but not reported (IBNR - Incurred But Not Reported). RBNS is determined for individual risks by a various actuarial methods separately for individual, (in terms of creation of IBNR), homogeneous groups of insurance risks. The TRPP for insurance benefits in the form of an annuity is also determined by actuarial methods.

The insurance company is adopting adequate arrangements to ensure that it is sufficiently informed about its exposure to risks. However, in view of the uncertainty in the determination of TRPP, probably the final outcome will differ from the original assumption. The amount of claims is particularly sensitive to the level of court rulings and the emergence of a legal precedent in matters of contractual and civil liability. In calculating the estimated cost of unpaid claims (both reported and unreported), a life insurance company can also use the methods based on development triangles of incurred and reported claims. Of course by calculations is used an appropriate safety margin, which takes into account the uncertainty of the future development of such claims.

The procedure of determining RBNS is specified in the relevant internal directives of each insurance company for the liquidation of life insurance claims. In RBNS are also external costs associated with the liquidation of an insured event (expert opinions, etc.). The RBNS determining by the liquidator does not explicitly include reserve for internal liquidation costs (liquidator's work, forms, etc.) and also does not include explicitly so-called regression reserve (negative) specifying expected future regressions or sanctions in respect of the insured event for the benefit of the insurance company. Reason is prudence. At the same time reason is also compensation for any RBNS inadequacy resulting from the explicit non-inclusion of the reserve for internal liquidation costs).

The IBNR is calculated using two chain-ladder methods. In the first one, the chain-ladder procedure applies only to the number of insured events (to limit the impact of accidental distribution of above-average claims in individual time intervals), in the second one, the

[3] Legislation: Zákon č. 39/2015 Z. z. Zákon o poisťovníctve a o zmene a doplnení niektorých zákonov.
[4] Included in [1].

"classic" chain-ladder procedure applies to the sum of insurance. The final IBNR is determined as the weighted average of the resulting IBNR from each method. All related cash flows are considered in the calculation of IBNR, i.e. anticipated costs of liquidation of the insured event, security charge intended to cover possible adverse development of the insured event (than it took into account the input data about the insured event), loss inflation.

3. Testing the sufficiency of technical provisions in Life insurance

Life insurance companies analysing the amount of technical reserves as of the balance sheet date (but can also make it with higher frequency) in terms of whether they are able to meet their obligations under life insurance contracts. Of course, they make it in accordance with IFRS4. When testing the sufficiency of technical reserves, insurance companies use estimates of future cash flows arising from insurance contracts. The sufficiency test of life insurance technical reserves is regulated by the Slovak Actuaries Society (SSA) Directive.[5]

In the case of life insurance provisions for long-term contracts, the discounted cash flow method is used to test the adequacy of provisions. For short-term insurance contracts, cash flows are not discounted because the effect of discounting is negligible. Cash flows for the adequacy test of technical provisions are written premiums, claims (including surrender values) and costs (administrative, acquisition and investment).[6] The portfolio of insurance contracts is divided into homogeneous groups with respect to specified properties, e.g. type of contract, age, duration of a contract, etc. The minimum value of insurance liabilities is then calculated within these groups using the best estimate of expected future development of input assumptions such as mortality, cost of inflation, surrender value, legislative changes and others. The test results in adequacy (sufficiency) or inadequacy of reserve. In the case of inadequacy of the provisions, the insurance company is obliged to complete the provision and also show the total difference in the profit and loss account. The adequacy test of technical provisions must be carried out for each type of life insurance.

4. Run-off analysis

Responsibility for a sufficient amount of technical provisions and for sufficient insurance lies on the actuary of the life insurance company, which determines and approves the amount. This is not only about their sufficiency, but also about ensuring that reserves are not overestimated. At the same time, actuary monitors total pay-outs and reserves for events that occurred in recent years.

To determine whether the calculation of the sufficiency test has been performed correctly, various control mechanisms and control tools are used. One of them is run-off analysis (also run-off test). The main idea is the use of special absolute or ratio coefficients for RBNS, IRBN and TRPP analysis. We will evaluate the adequacy of provisions for claims using the coefficients that result in the percentage of overstatement (if the result is positive) or understatement (if the result is negative). Run-off analysis can be performed either as run-off analysis of only RBNS, only IBNR or RBNS and IBNR together, i.e. run-off TRPP analysis. We will now discuss the particular cases.

[5] Odborná smernica SSA no. 1 v3: The adequacy of the technical reserves in life insurance http://www.aktuar.sk/userfiles/Odborna_smernica_SSA_c1_LAT_ZP_v3.pdf.

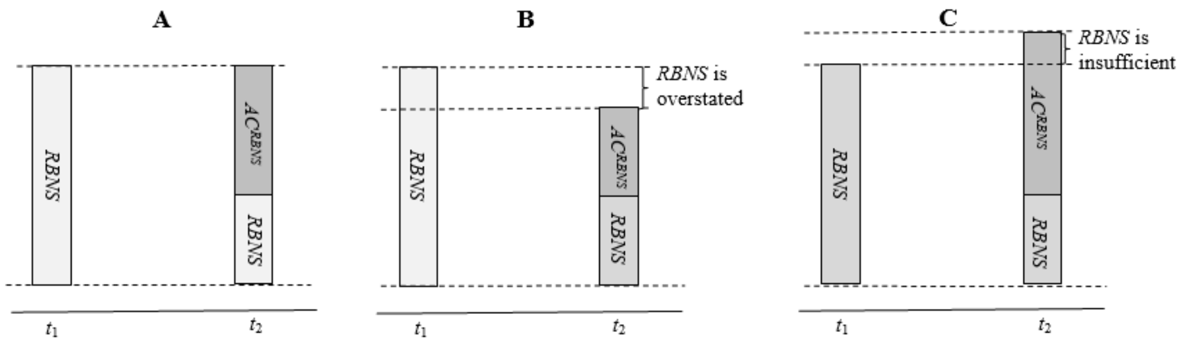
[6] Included in [2] and [3].

4.1 Run-off analysis for RBNS reserve

By RBNS it sets up gradually the liquidation of claims reported since the beginning of the reporting period. Checking the correctness of the RBNS calculation is very important as its estimate is based on the experience of the claims liquidator and it is good for the insurer to have an overview of whether this experience is reliable.

As we can see in Figure 1, the part of the RBNS at the beginning of the period, i.e. calculated at the date of determination of the technical provisions (TR), is converted into claims payment (in life insurance - payment of a sum assured), part of this reserve remains for the event which has been reported from the beginning of the period, but not liquidated till the end, i.e. till the date of the testing. By comparing the state at the beginning and the end of the reporting period, we obtain three possible results: either RBNS is sufficient (A), sufficient but overstated (B) or insufficient (C).

Figure 1: Development of RBNS change over the time period (source: self processing)



Formula for the calculation of the coefficient to check the RBNS sufficiency using the run-off test is

$$\frac{RBNS_{t_1} - RBNS_{t_2} - AC_{t_1-t_2}^{RBNS}}{RBNS_{t_1}} \qquad (1)$$

where $RBNS_{t_1}$ – is amount of *RBNS* to the date t_1 for claims incurred up to date t_1,

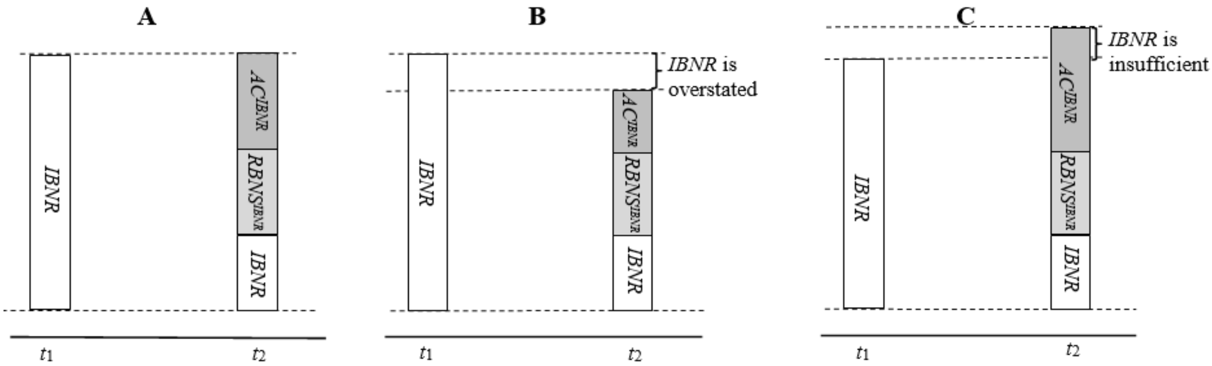
$RBNS_{t_2}$ – is amount of *RBNS* to the date t_2 for claims incurred up to date t_2,

$AC_{t_1-t_2}^{RBNS}$ – is the amount of claims paid between the dates t_1 and t_2, reported to the date t_1.

4.2 Run-off analysis for IBNR reserve

The IBNR control procedure with respect to the beginning of the reference period is similar to the control procedure with respect to the end of the reference period. This reserve can also be decomposed in several parts in time for decomposition. During the reporting period, some insured events are reported but not paid out and for these RBNS needs to be created. For some damages, insurance benefits, whether partial or total, are paid and part remains in the IBNR reserve, as can be seen in Figure 2. There can occur three cases: either IRBN is sufficient (A), sufficient but overestimated (B) or insufficient (C).

Figure 2: Evolution of IBNR change over the reporting period
(Source: self processing)



Run-off coefficient for IBNR is

$$\frac{IBNR_{t_1} - IBNR_{t_2} - AC_{t_1-t_2}^{IBNR} - RBNS_{t_2}^{IBNR}}{IBNR_{t_1}} \qquad (2)$$

where $IBNR_{t_1}$ − is amount of *IBNR* to the date t_1 for claims incurred up to date t_1,

$IBNR_{t_2}$ − is amount of *IBNR* to the date t_2 for claims incurred up to date t_2,

$IBNR_{t_2}$ − is the amount of claims paid between the dates t_1 and t_2, reported between the dates t_1 and t_2.

$RBNS_{t_2}^{IBNR}$ − the amount of *RBNS* to the date t_2 for the claims occurring to the date t_1 and reported between the dates t_1 and t_2.

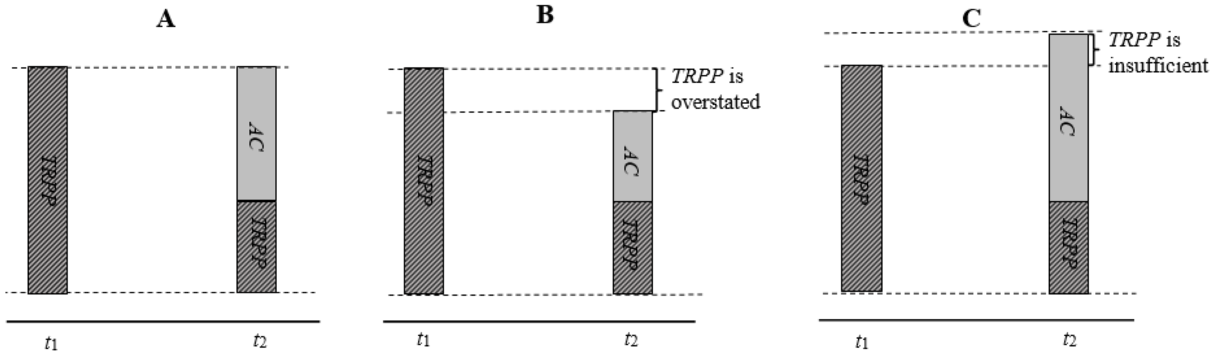
The informative value of the run-off analysis is influenced by the length of the monitored period - the longer is monitored period, the greater is credibility of special ratio coefficients.

The RBNS analysis is more relevant because IBNR is also based on the estimate of this reserve.

4.3 Run-off analysis for TRPP reserve

Finally, we will carry out the TRPP sufficiency check procedure, which is the sum of RBNS and IRBN reserves. As shown in Figure 3 and also as was seen in both previous examples, also now can occur three cases: A –TRPP is sufficient, B –TRPP is sufficient but overestimated, and C - TRPP is insufficient. It should also be noted that in this case we are no longer creating RBNS for IRBN.

Figure 3: Trend change in TRPP over the reporting period
(Source: self processing)



Formula for the calculation of run-off analysis coefficient to check TRPP's sufficiency is as follows

$$\frac{TRPP_{t_1} - TRPP_{t_2} - AC_{t_1-t_2}}{TRPP_{t_1}} \tag{3}$$

where $TRPP_{t_1}$ – is amount of *IBNR* to the date t_1 for claims incurred up to date t_1,

$TRPP_{t_2}$ – is amount of *IBNR* to the date t_2 for claims incurred up to date t_2,

$AC_{t_1-t_2}$ – is the amount of claims paid between the dates t_1 and t_2, reported to the date t_1 and also reported between the dates t_1 and t_2.

while $TRPP_{t_1} = RBNS_{t_1} + IRBN_{t_1}$

$$TRPP_{t_2} = RBNS_{t_2} + IRBN_{t_2}$$

(4)

$$AC_{t_1-t_2} = AC^{RBNS}_{t_1-t_2} + AC^{IBNR}_{t_1-t_2}$$

5. Practical demonstration of control of reserve sufficiency by means of run-off analysis

Now we will perform run-off analysis for all types of the above reserves at particular values. The values are hypothetical, but sufficient for practical demonstration. Suppose that a life insurance company realise a run-off analysis to the date 31/12/2019 (at the date of testing the adequacy of technical provisions) with respect to the data it has as of 31/12/2018. The insurance company has read the following data from its financial statements:

Table 1: Values of RBNS, IBNR and claims in observed dates in €
(source self processing)

RBNS k 31.12.2018 for the claim incurred to the date of the determination of technical reserves in €	2 549 936
RBNS k 31.12.2019 for the claims incurred to the date of determination of technical reserves in v €, of which - reported do 31.12.2018 - not reported between 31.12.2018 a 31.12.2019	2 453 291 2 281 151 172 140
IBNR k 31.12.2018 for the claims incurred to the date of determination of technical reserves in v €,	1 019 703
IBNR k 31.12.2019 for the claims incurred to the date of determination of technical reserves in v €,	707 890
Total claims paid between 31.12.2018 a 31.12.2019 v €, of which - reported by 31.12.2018 - Not reported between 31.12.2018 a 31.12.2019	325 032 165 611 159 421

In the calculations in particular formulas we will write only 2018 for the time t_1 (31/12/2018) and only 2018 for the time t_2 (31/12/2019).

Run-off for RBNS

First, we calculate the run-off coefficient for RBNS using formula (1) and thus

$$\frac{RBNS_{2018} - RBNS_{2019} - AC_{2018-2019}^{RBNS}}{RBNS_{2018}} =$$

$$= \frac{2\ 549\ 936 - 2\ 281\ 151 - 165\ 611}{2\ 549\ 936} = 0.0405\ \%$$

The calculated coefficient is positive, so the reserve is sufficient.

Run-off for IRBN

The run-off coefficient for RBNS is calculated by relationship (2), so we have that

$$\frac{IBNR_{2018} - IBNR_{2019} - AC_{2018-2019}^{IBNR} - RBNS_{2018}^{IBNR}}{IBNR_{2018}} =$$

$$= \frac{1\ 019\ 703 - 707\ 890 - 159\ 421 - 172\ 140}{1\ 019\ 703} = -\ 0.0194\ \%$$

The calculated coefficient is negative, so the IRBN margin is insufficient.

Run-off for TRPP

Before calculating the run-off coefficient for this type of provision, we determine the technical provisions for claims as of 31/12/2018 and 31/12/2019 as well as claims between the following two dates:

$$TRPP_{2018} = RBNS_{2018} + IRBN_{2018} = 2\ 549\ 936\ +\ 1\ 019\ 703 = 3\ 569\ 639\ €$$

$$TRPP_{2019} = RBNS_{2019} + IRBN_{2019} = 2\ 453\ 291 + 707\ 890\ =\ 3\ 161\ 181\ €$$

$$AC_{2018-2019} = AC^{RBNS}_{2018-2019} + AC^{IRBN}_{2018-2019} = 325\ 032\ €$$

Then the run-off analysis coefficient for TRPP is from formula (3)

$$\frac{TRPP_{2018} - TRPP_{2019} - AC_{2018-2019}}{TRPP_{2018}} =$$

$$= \frac{3\ 569\ 639 - 3\ 161\ 181 - 325\ 032}{3\ 569\ 639} = 0.0234\ \%$$

and from the result it is clear that the value of the TRPP reserve is sufficient and slightly overvalued.

6. Conclusion

By business activities of Life insurance Company it is necessary to act so as to be sure that the obligations arising from the insured risks assumed during the insurance process will be reimbursed. The ability of an insurance company to pay at a given time and required by its liabilities is defined as the solvency of the insurance company. By valuation of solvency, it is important in which mode or regime is valued the solvency. Accepted valuation regimes include e.g. going-concern, run-off analysis and Winding-up mode. Run-off analysis is a regime for which is no longer expected the continuation of insurance transactions. However, capital adequacy and technical provisions allow the insurer to pay insurance claims from previous transactions, which means that the solvency requirements must be higher. Given example about adequacy of technical provisions and solvency of life insurance company is a good illustration that run-off test is adequate demonstration of analysis and control of the insurance company's activities and management. And although the coefficient for IRBN is negative, as mentioned above, the coefficient for RBNS and, of course, the coefficient for TRPP have a higher significance.

References

[1] Kaderová, A., Krátka, Z, Páleš, M. (2018). *Volatility Modelling in Market Risk Analysis.* In *Journal of Applied Economic Sciences*. Spiru Haret University, Craiova.

[2] Sakálová, K. (2006). *Aktuárske analýzy.* Ekonóm Publishers, Bratislava.

[3] Sakálová, K. (2001). *Oceňovanie produktov životného poistenia*. Ekonóm Publisher, Bratislava.

[4] Odborná smernica SSA č. 1 v3: The adequacy of the technical reserves in life insurance. <http://www.aktuar.sk/userfiles/Odborna_smernica_SSA_c1_LAT_ZP_v3.pdf>.

[5] Legislation: Zákon č. 39/2015 Z. z. Zákon o poisťovníctve a o zmene a doplnení niektorých zákonov. < https://www.epi.sk/zz/2015-39>.

Acknowledgements:

This paper is supported by the Grant Agency of Slovak Republic − VEGA, grant no. 1/0120/18 - *Modern risk management tools in internal models of insurance companies in the context of the Solvency II Directive*.

Analysis of Audit Companies that Audit Public Interest Entities

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Abstract

The goal of the research work is to analyze the relationship between the bonity score of the audit companies and the number of audited public-interest entities. We proceed research by detailed analysis of Transparency Reports focusing on the structure and requirements of the report of all auditors or audit companies that carry out audits in public interest entities in the Slovak Republic, together with the analysis of their financial reports. The result of this study is the implementation of this knowledge into a demonstration in which a number of audited public-interest entities correlate with bonity scores. For the purpose of this research we used Altman Z-score and score IN-95, in both cases the correlation was positive. Unfortunately, we must also conclude that the quality of Transparency Reports of audit companies in Slovakia is very low and a lot of information in these reports is missing and does not fulfill the legal accuracy of the content of the Transparency Report.

Key words

Statutory audit, public-interest entity, Transparency Report, bonity scores.

JEL Classification: M42

1. Introduction

There are currently many audit companies that are required to prepare a Transparency Report. The various parts of the legislation contain a lot of obligations which the auditor or audit company is obliged to include in the Transparency Report, as well as their mentioning the mandatory parts that such a report should contain.

All statutory auditors or audit companies carrying out audits in public interest entities are required to publish Transparency Reports on their own websites within a maximum period of four months from the end of each financial year. (Kareš, 2015) These reports shall, as a minimum, include a list of all public-interest entities in which audit companies have carried out a statutory audit in the previous financial year.

The term "public interest entity" refers to entities whose management is of increased interest to the public. Listed companies are entities which have securities traded on a stock exchange and other entities that are also considered to be in the public interest in the European Union member state under individual laws. (Pakšiová, Janhuba, 2012)

Public-interest entities are defined in the Slovak Republic in the Act on Accounting, in the Statutory Audit Act, and also in Directive 2014/56 / EU of the European Parliament and of the Council, which replaced Directive 2006/43 / EC of the European Parliament and of the

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Council. Slovak legislation characterizes public interest entities as listed companies to which other entities have been assigned according to individual legal standards, which are also considered to be public interest entities in the Slovak Republic.

International Financial Reporting Standards and International Auditing Standards also mention listed companies. They are directly defined by International Auditing Standards, while International Financial Reporting Standards merely describe them, and do not directly define them. (Parajka, 2015)

In general, under Slovak law, we can characterize as an entity of public interest any entity whose securities have been issued and admitted to trading in any Member State of the European Union.

Under Act No. 431/2002 Coll. on Accounting (hereinafter referred to as the “Act on Accounting”) and Act no. 423/2015 Coll. on Statutory Audit and on amendment of Act No. 431/2002 Coll. on Accounting, as amended (hereinafter referred to as the “Statutory Audit Act”), we can see in the following table public interest entities, which are also characterized by these laws. However, each of these laws also has a list of public interest entities which vary from one law to another.

Table no. 1: Public interest entities according Accounting Act and also Statutory Audit Act

Listed companies
Banks and branches of foreign banks
Export Import bank of the Slovak Republic
Insurance and branches of foreign banks
Reinsurance and branches of foreign reinsurance
Health insurance
Asset management companies and branches of foreign asset management companies
Pension management companies
Supplementary pension management companies
Stock exchanges

Source: own processing according Accounting Act and Statutory Audit Act

As we can see in table no. 2 and 3, the Accounting Act and the Statutory Audit Act, do not consider the same entities as public interest entities. Nine entities are defined in both laws in the same way. However, for example, a payment institution, an investment company and a pension fund are subject to the public interest only under the Accounting Act. The Act on Statutory Audit defines, for example, railways in the Slovak Republic or cities, municipalities and neighborhoods that meet certain conditions that we can see in the table.

Table no. 2: Public interest entities according Accounting Act

Central securities depository
Brokers
Payment institutions
Electronic money institutions
Collective investment undertakings
Pension funds
Branches of foreign financial institutions
Accounting entities ty according §17a par.2

Source: own processing according Accounting Act

Table no. 3: Public interest entities according Statutory Audit Act

Railways of the Slovak Republic
Accounting entities preparing the consolidated financial statements of the central government
Higher territorial unit
An accounting entity that is a municipality, city, or city district under special regulations, from an accounting period preceded by at least two consecutive accounting periods in which it met the following conditions: the total amount of assets, where the amount of assets means the amount determined by the consolidated financial statements of the public administration entity >100 000 000 and the population >50 000

Source: own processing according Statutory Audit Act

When comparing Act No. 423/2015 Coll. to the statutory audit currently in force and Act No. 540/2007 Coll. on Auditors, Audit and Audit Oversight valid until 2015, we can see differences in the content of public interest entities. Until 2015, the following were considered to be public interest entities under this act: the National Bank of Slovakia, the Central Securities Depository, securities traders, entities that prepare consolidated financial statements under the Accounting Act and the branches of foreign securities brokers when they prepare financial statements in accordance with international financial reporting standards. On the contrary, until 2015 the following entities were not considered to be public interest entities: a higher territorial unanimity in the present Act if they meet certain criteria set out in this Act.

International Financial Reporting Standards and International Auditing Standards do not list public-interest entities, instead listing listed companies. According to International Auditing Standards, listed companies are those whose shares, bonds or shares are listed or traded in accordance with the rules of this recognized exchange or other similar organizations. (Kontsyvaya et al., 2019)

International Financial Reporting Standards in IFRS 33 - Earnings per share, although not explicitly described, describe listed companies in terms of the preparation of separate or consolidated financial statements as entities that have publicly traded ordinary shares (domestic or foreign stock exchange) or entities that submit their financial statements to organizations for the purpose of issuing ordinary shares (Akbas, Zeytinoglu, 2017). This standard is only for listed companies.

According to IFRS 10, (Antoniuk, Chizevska, Semenyshena, 2019) the obligation to present consolidated financial statements applies to all parent companies except those that meet all the conditions to this standard, including, for example, that an entity does not trade equity instruments, or public debt instruments and also that it does not present financial statements of organizations for the purpose of issuing ordinary shares.

Auditors carrying out audits in public interest entities may provide other non-audit services that comply with the conditions set out in the Statutory Audit Act. A statutory auditor who carries out a statutory audit in a public interest entity shall enter into an audit contract for a minimum of two years and a maximum of three years if the auditor has entered into a contract with that entity for the first time. Any subsequent contract may be concluded for a maximum of three years if the auditor is approved by a general meeting, or another body of the audited entity, who also recalls it.

If all renewed engagements at the audited entity, including the initial engagement, exceed ten years, then the engagement may be extended by a maximum of 10 years if a statutory auditor is tendered under the Statutory Audit Act, or by 14 years if at least two statutory auditors are appointed at the same time and have submitted an auditor's report pursuant to the

Statutory Audit Act, Section 27, Par. 4, which stipulates that if more than one statutory auditor is auditing an entity, they must agree on the results and prepare a joint auditor's report with an opinion. If the opinion of these auditors differs, each auditor shall state his or her own opinion in a separate paragraph of this report, together with a statement of their reasons.

The statutory auditor responsible for audits in public-interest entities may audit the same entity for a maximum of five consecutive years from the date of the appointment of that auditor. It may re-audit the same entity up to at least three years after the last statutory audit has been performed.

The statutory auditor is obliged to communicate to the Supervisory Authority, from the end of the accounting period, a maximum of four months, a list of public interest entities in which the statutory audit was performed. (Krišková, Užík, 2016)

Shareholders holding at least 5% of the entity's capital or 5% of the voting rights, the supervisory or management body of the audited entity and the supervisory authority may also submit a reasoned application to the audit performance.

2. Literature review

An Australian study analyzed (Fu Y., Carson E., Simnett R., 2015) the information disclosed by leading Australian audit companies in their first-time audit companies' transparency reports (Australia has mandated the preparation and release of transparency reports by audit companies in 2013). The authors found that minimum transparency report disclosure requirements are met by audit companies and they have different approaches to governance in the areas which may impact audit quality.

Similar research was done by Spanish authors (Zorio-Grima, Ana; Antonia Garcia-also Benau, Maria; Grau-Grau, Alfredo J., 2018), their analyses Transparency reports were published in Spain in 2010 and 2013. They declare that it is a pioneering research in this field. According to their results there is a growing level in the quality of these reports from 2010 to 2013, however, there is a decrease in voluntary information in 2013. Our study reveals that bigger audit companies and companies less dependent on fees from consultancy services are the ones with higher transparency levels in their Annual Transparency Reports.

Another study (Zorio-Grima, A. and Carmona, P., 2019) examines whether audit companies use transparency reports as a tool to standardize their brand image or whether the semantic and content analysis in these reports indicates a higher importance of country effects. Their research sample included 28 transparency reports published in English by the Big-4 audit companies in the UK, Ireland, Luxemburg, Hungary, Malta, USA and Australia. This research found that there is different language used in transparency reports across audit companies, jurisdictions and countries.

Transparency report disclosure was also the topic of research in Croatia (Cular, 2017). Croatian listed companies were analyzed as well as audit companies which audited Croatian listed companies in 2015. As a method a multiple regression model was used and results indicate that only 32% of audit companies were transparent.

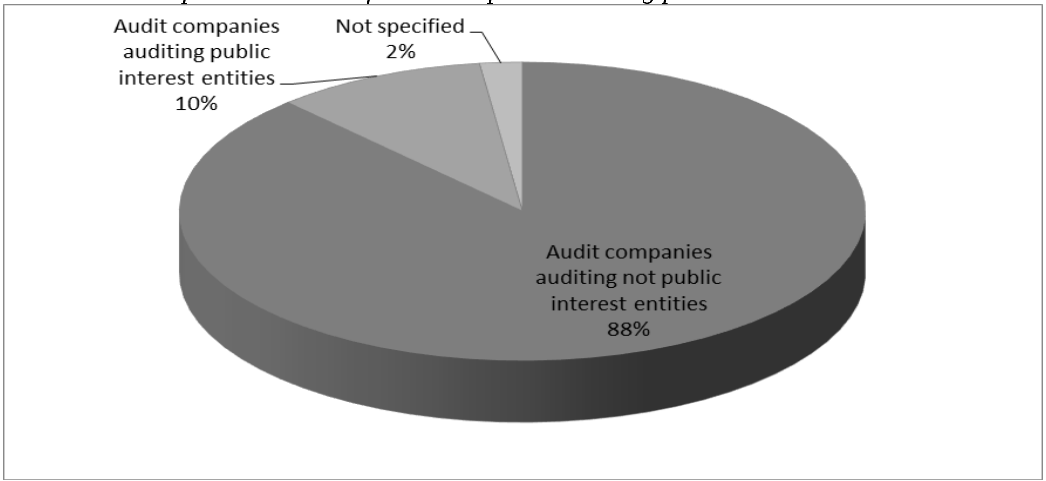
Another study (La Rosa, F., Caserio, C., Bernini, F., 2019) examines corporate governance disclosure in audit companies' transparency reports and whether more disclosure is associated with the audit position in the country. On the basis of content analysis, different measures of a corporate governance disclosure index were calculated for a sample of 122 auditing companies. The study provides an analysis of audit companies transparency based on the information disclosed in mandatory transparency reports.

3. Methodology and Data

In this paper, we examined the relationship between the number of public-interest entities audited and the creditworthiness of these audit companies by analyzing Transparency Reports. From the publicly available data on the website of the Audit Oversight Office, we obtained a list of all audit companies (total number of audit companies in the Slovak Republic is 236), from which we chose those which compiled the Transparency Report for 2018 (25 audit companies) and we analyzed these reports in detail.

The following chart shows how many audit companies in the Slovak Republic were audited in public interest entities in 2018. It also shows how many audit companies do not know whether they are auditing in public interest entities because this information was missing in the transparency report or the transparency report was not published at all.

Graph no. 1: Ratio of audit companies auditing public interest entities



Source: Own processing

Based on the database we have worked with, which includes 236 audit companies in the Slovak Republic, only 10% of audit companies were auditing public interest entities. In 2% of the audit companies we were unable to determine whether they had audited public interest entities in 2018 or not.

Table no. 2: List of analysed audit companies

1.	KLT Audit, spol. s r. o.	**13.**	A P X, k. s.
2.	D. E. A. Consult Trenčín, s. r. o.	**14.**	Deloitte Audit, s. r. o.
3.	AGV audit, spol. s r. o.	**15.**	INTERAUDIT Group, s. r. o.
4.	AUDITCON SLOVAKIA, s. r. o.	**16.**	RENTABIL BRATISLAVA, spol. s r. o.
5.	BDR, spol. s r. o.	**17.**	TPA AUDIT, s. r. o.
6.	INTERAUDIT Zvolen, spol. s r. o.	**18.**	Ernst & Young Slovakia, spol. s r. o.
7.	RVC Senica, s. r. o.	**19.**	AUDIT – EXPERT, s. r. o.
8.	BDO Audit, spol. s r. o.	**20.**	KPMG Slovensko, s. r. o.
9.	Mazars Slovensko, s. r. o.	**21.**	PricewaterhouseCoopers Slovensko, s. r. o.
10.	Boržík & partners, s. r. o.	**22.**	VGD SLOVAKIA, s. r. o.
11.	INTERAUDIT INTERNATIONAL, s. r. o.	**23.**	ACCEPT AUDIT & CONSULTING, s. r. o.
12.	Grant Thornton Audit, s. r. o.		

Source: Own processing

For the purposes of calculating the correlation, we used two models - Altman Z score and credit score IN05.

Table no. 3: Selected data from financial statements

	Assets	Profit / loss	Revenues	Altman Z-score	Score IN 05
BDR, spol. s r.o.	879091	293843	1811273	4.68	6.09
INTERAUDIT BENETIP s.r.o.	69401	1336	215500	4.02	1.19
INTERAUDIT Zvolen, spol. s r.o.	109304	1339	203657	5.35	1.63
Deloitte Audit s.r.o.	4933538	240618	13811437	3.45	5.73
KPMG Slovensko spol. s r.o.	6453524	3286115	21543170	6.26	6.64
Grant Thornton Audit, s. r. o.	239019	87239	771180	5.79	3.04
INTERAUDIT INTERNATIONAL, s.r.o.	369818	55927	349601	2.89	2.87
RENTABIL BRATISLAVA spol. s r.o.	90967	449	157969	1.77	0.64
ACCEPT AUDIT & CONSULTING, s.r.o.	195899	42985	677783	4.94	4.25
AGV audit spol. s r.o.	436937	50202	243072	4.72	2.14
D.E.A. Consult Trenčín, s.r.o.	55684	2156	292383	6.48	1.81
ACCONT AUDIT, s.r.o.	253216	23676	198168	1.48	3.08
PricewaterhouseCoopers Slovensko	24135761	5758367	30956918	5.32	7.81
Mazars Slovensko, s.r.o.	1134394	-125736	2153404	1.75	-0.04
Ernst & Young Slovakia, spol. s r.o.	3676180	1880574	9076302	5.68	3.71
A P X, k.s.	173964	8309	274406	2.62	0.95
KLT AUDIT, spol. s r.o.	211645	14098	344963	5.19	2.06
VGD SLOVAKIA s. r. o.	2251605	734400	5791215	4.25	9.06
RVC Senica s.r.o.	61874	36410	176289	6.39	4.18
AUDIT - EXPERT, s.r.o.	4682	-7486	34083	3.36	-4.42
TPA AUDIT, s.r.o.	826157	451383	868621	4.48	9.29
BDO Audit, spol. s r. o.	252052	16778	822870	3.55	1.27
AV Audit, s. r. o.	16308	34045	55454	12.3	12.23
Boržík & partners, s. r. o.	476319	138194	495963	3.80	2.39

Source: Own processing according www.registeruz.sk

Altman's model is the most used bonity score but it was developed by statistical research by American companies, it can be concluded that its testimony in Slovak conditions is not the same as in the American environment. Similarly, changes in the Slovak economy, from the existence of a large number of insolvent companies, through rapid economic growth to the financial crisis, certainly had an impact on the model's predicative ability. (Andrejovská, 2012) That's why we also included the IN05 score.

Z-score: (net working capital / total assets) x 0.717 + (retained earnings / total assets) x 0.847 + (EBIT / total assets) x 3.107 + (equity / liabilities) x 0.420 + (sales / total assets) x 0.998

where:

Z> 2,9 - the current and expected future financial situation will be good,

1,2 ≤ Z ≤ 2,9 - gray zone,

Z <1.2 - the financial situation is bad, high probability of bankruptcy .

Altman attaches the greatest weight to the return on assets. According to Altman, return on assets affects the financial health of the company most significantly

The Neumaier family in 2005 created the fourth model, which was tested on a group of Czech companies. They re-incorporated the EBIT / interest indicator into the model (Ondrušová, 2011) and, based on the results of the discriminatory analysis, attributed the following weights to the indicators:

0.13 x total capital / liabilities + 0.04 x EBIT / interest expense + 3.97 x EBIT / total capital + 0.21 x income / total capital + 0.09 x current assets / current liabilities

IN05> 1,6 = enterprise generates value

IN05 0.9 ≤ and ≤ 1.6 = gray zone

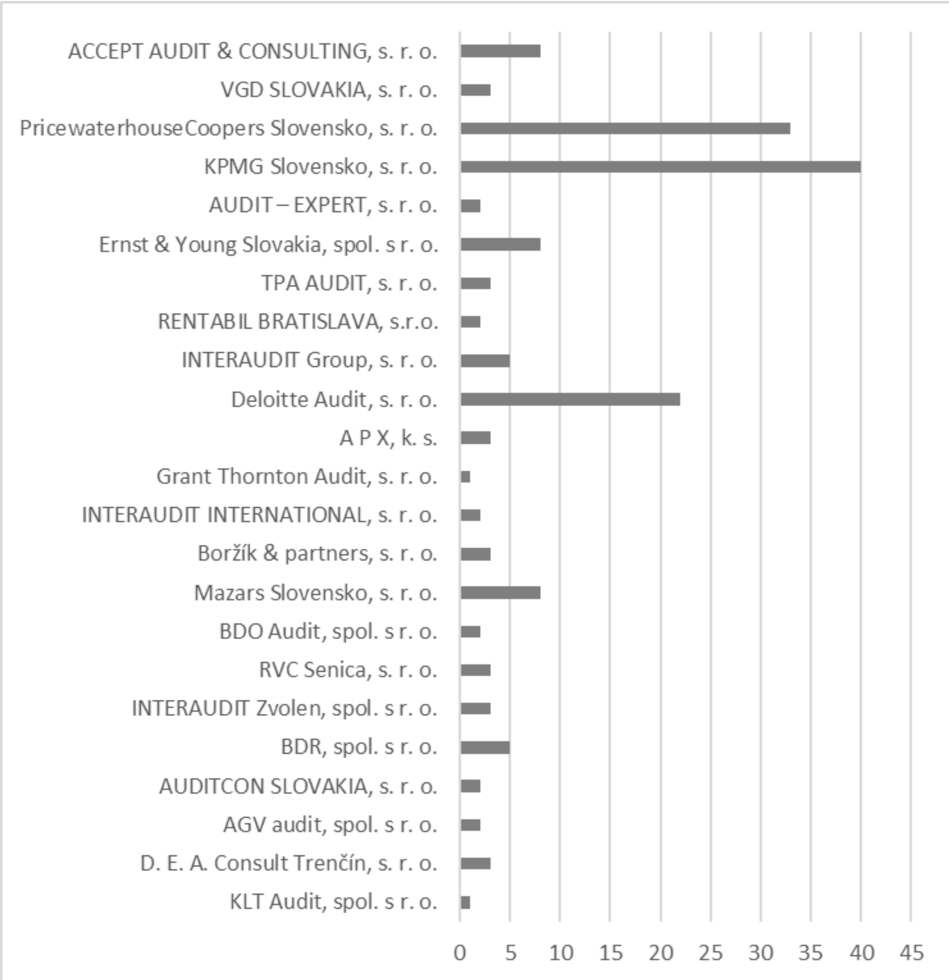
IN05 <0.9 = the company is going to bankrupt

Czech index IN05 copies results Altman Z-score. Together with Altman's model, it is one of the most accurate models that can be applied in our conditions.

4. Results

In total 25 audit companies had to prepare Transparency Reports, but only 23 did so and reported the number of audited public-interest entities. In the following chart, there are individual audit companies and the number of public interest entities audited by them. As we can see the lowest value is 1, while the highest value is 40. On average, there are approximately 6 public interest entities per audit company. Examining this data, we see that only the big four has the highest number of audited entities so KPMG Slovensko, then PricewaterhouseCoopers Slovakia, Deloitte Audit and Ernst & Young Slovakia had the lowest number of audited public interest entities within the Big Four.

Graph no. 3: Number of audited public interest entities



Source: Own processing

Then the correlation between the results of bonity models (Altman's Z-score and score IN05) were examined. A positive correlation was found in both models. In the case of Altman's Z-score was 0,525 higher in comparison to score IN05 with a value 0,409.

Table no. 4 Correlation of the number of public interest entities audited and bonity models

	Altman Z-score	Score IN05
Correlation	0,525	0,409

Source: Own processing

The results can be attributed to the fact that companies that audit public interest entities are more financially stable, or that they also charge higher amounts for their services if they charge public interest entities. Also, the payment discipline of public interest entities is probably higher. The results are limited in that it was not possible to determine the extent to which audit companies audit public interest entities and other entities, which may be the subject of further research.

Based on the analysis of the transparency reports we can conclude that some audit companies have published transparency reports in detail, while some, especially large audit companies that have extensive transparency reports, report information that we believe is not so important, they make the report confusing and difficult to read, because within so much information it is harder to find the most relevant. For example, information about the no. of employees and the percentage of women and men and many other similar statistics are considered unnecessary for this type of report and this information should be included in the annual report.

For some audit companies, on the basis of the transparency report, we considered that those audit companies that audited the public-interest entities probably do not put much emphasis on the accuracy of the data in the transparency report because the 2018 transparency report probably copied data from transparency reports from previous years.

We also found audit companies that did not audit the public interest entities in 2018, nevertheless prepared a Transparency Report. They made it beyond their obligations and could have different reasons for doing so, for example, they were unsure whether they were obliged to do so under the legislation and therefore made it for sure or audited the public interest entities in the previous year.

5. Conclusion

The goal of the research was to analyze relationships between bonity scores of the audit companies and the number of audited public-interest entities. We worked with a database of all the auditors and audit companies performing audits in the Slovak Republic. We have obtained this database from a publicly available source, on the website of the Audit Oversight Board. Based on this database, we have set up audit companies that carry out audits in public interest entities. We have found a positive correlation between the number of audited public entities and Altman Z-score. A similar correlation was also found with score IN05. We must state that the reports on the transparency of audit companies in the Slovak Republic are not prepared with due attention and greater pressure should be exerted by the control authorities to increase the quality of these reports.

Acknowledgments

This article is an output of the project APVV no. APVV-16-0602 “Enhancement of the relevance of the accounting data in the SR - from expenses to value”

References

[1] Akbas, H. E., Zeytinoglu, E. (2017). In the Light of Accounting Academicians' Perceptions, the Features of an Ideal Accounting System and International Financial Reporting Standards within this Perspective. *Eurasian Business & Economics Journal*, vol. 10, pp. 28-44.

[2] Andrejovská, A. (2012) Zhodnotenie ekonomickej situácie v obciach pomocou pomerových a absolútnych ukazovateľov. *Verejná správa a spoločnosť : [vedecký časopis].* - Košice : Fakulta verejnej správy Univerzity Pavla Jozefa Šafárika, 13th ed. vol. 1, pp. 102-108.

[3] Antoniuk, O., Chizevska, O., Semenyshena, N. (2019) Legal regulation and trends of audit services: what are the differences (evidence of Ukraine). *Independent journal of management & production.* vol. 10, issue 7, (Special Edition -May- PDATU, 2019).

[4] Barker, R., Kirk, J. and Munday, R.J., (1988). *Narrative analysis.* 3rd ed. Bloomington: Indiana University Press.

[5] Cular, M. (2017) Transparency report delay and disclosure by Croatian audit firms *Croatian operational research review.* Volume: 8 Issue: 1 Pages: 299-316.

[6] Fu Y., Carson E., Simnett R. (2015) Transparency report disclosure by Australian audit firms and opportunities for research, Managerial auditing journal. Volume: 30 Issue: 8-9 Pages: 870-910.

[7] Kareš, L. (2015) *Teória auditu*. Bratislava: Wolters Kluwer, 2015. Pages: 21 .ISBN 978-80-8168-149-3.

[8] Kontsyvaya, S., Alborov, R. Kontsevaya, S., Smutka, L., Makunina, I., Turebekova, B. (2019) Accounting and Analysis of Biological Asset Transformation Results in Agricultural Companies of the Russian Federation. *Conference Hradec Economic Days*, Volume 10(1), pp. 336-343.

[9] Krišková, P., Užík, J. (2016) Management of audit risk and its impact on the audit of financial statements. Managing and modelling of financial risks: proceedings: 8th international scientific conference: 5th - 6th september 2016, Ostrava, Czech Republic, pp. 460-466.

[10] La Rosa, F., Caserio, C., Bernini, F. (2019) Corporate governance of audit firms: Assessing the usefulness of transparency reports in a Europe-wide analysis. Corporate Governance, vol. 27, issue 1., Pages: 14-32.

[11] Lehenchuk, S. F., Valinkevych, N. V., Vyhivska, Irina M., Khomenko, H. Y. (2020) The Significant Principles Of Development Of Accounting Support For Innovative Enterprise Financing. *International Journal of Advanced Science and Technology*, *29*(8s), pp. 2282 – 2289.

[12] Ondrušová, L**.** (2011) Management decisions leading to ownership transactions. Strategic management : international journal of strategic management and decision support systems in strategic management, vol. 16, no. 3, pp. 18-22.

[13] Pakšiová, R., Janhuba, M. (2012) *Teória účtovníctva v kontexte svetového vývoja*. Bratislava: Iura Edition.

[14] Parajka, B. (2015) Are information needs of financial entities served by financial statements in the Slovak Republic. *Financial management of firms and financial institutions*, pp. 950-957.

[15] Weiss, T.D. and Coatie, J.J. (2010). *The World Health Organization, its history and impact.* London: Perseus.

[16] Zorio-Grima, A. Antonia Garcia-Benau, M., Grau-Grau, A. J. (2018) The transparency report issued by audit firms: Evidence from the Spanish market 2010-2013 By; et al. *Spanish journal of finance and accounting-revista espanola de financiation y contabilida.* Volume: 47 Issue: 2 Pages: 280-305.

[17] Zorio-Grima, A., Carmona, P. (2019) "Narratives of the Big-4 transparency reports: country effects or firm strategy?", *Managerial Auditing Journal*, Vol. 34 No. 8, Pages: 951-985.

Municipalities and fiscal rule: post-COVID-19 period

Eva Lajtkepová [1]

Abstract

Act Number 23/2017 Coll. stipulates the so-called fiscal rule vis-à-vis the regulation of indebtedness. Should a municipality fail to adhere to the rule and not reduce the debt, it faces the risk that the transfer of taxes from the state budget will be suspended, i.e. having its revenue significantly reduced. The period following economic stagnation (spring of 2020) will bring specific risks for the economic performance and indebtedness of municipalities. This article deals with the identification and analysis of these risks. The said analysis is conducted using qualitative methods; the level of risks is determined by qualified estimates. The most relevant risks on the part of municipalities are economic, political and legislative. A new risk is moral risk: disintegration of the current rules and fiscal (and ultimately decision-making) centralisation.

Key words

Fiscal rule, budget, budget responsibility, municipality, post-COVID-19 period

JEL Classification: H63, H72

1. Introduction

The status, rights and obligations of municipalities are governed by Act Number 128/2000 Coll., on municipalities (establishment of municipalities), while the fundamental framework of their economic performance is governed by Act Number 250/2000 Coll., on budgetary rules. Municipalities in the Czech Republic operate within the framework of their budget under the conditions stipulated by the applicable legislation, and they are fully responsible for their activities; the Ministry of Finance is not authorised to intervene directly in their economic performance and possible indebtedness.

The first real milestone in terms of regulating the indebtedness of municipalities (or more precisely, all regional self-governing units, not just municipalities) was the adoption of Act Number 23/2017 Coll., on budget responsibility rules, and the enforcement of the so-called fiscal rule. However, the period of time following the freezing of the economy in the spring of 2020 (which, for lack of a better term, shall be referred to here as the "post-COVID-19 period") will be very tough on municipalities and their economic performance; the strict application of fiscal rule may force tough decisions by assemblies on the future development of municipalities, as well as changes extending far beyond matters municipalities' economic performance.

2. The fiscal rule and the budget responsibility indicator

Act Number 23/2017 Coll. implements Council directive 2011/85/EU of 8 November 2011 on requirements for budgetary frameworks of the Member States, and refers to other EU

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legislation (in particular Regulation 1466/1997, Regulation 1467/1997 and Regulation 479/2009). It introduces new or stricter obligations for self-governing territorial units in order to regulate their over-indebtedness.

A novelty introduced by the act is the obligation on the part of municipalities to adhere to the so-called fiscal rule (Section 17 of Act Number 23/2017 Coll.). Pursuant to this rule, the budget responsibility indicator is calculated as follows:

$$BR = \frac{D}{R} \tag{1}$$

where:
BR – budget responsibility indicator (in %),
D – debt of the municipality,
R – average revenue of the municipality over the last four years.

Where the value of the budget responsibility indicator exceeds 60 % (fiscal rule), the municipality is obliged to reduce the debt at least by 5 % of the difference between the current amount of debt and the value corresponding to 60 % of the average revenue over the last four years. Should the municipality fail to reduce the debt by the mandatory minimum, the state shall suspend, temporarily, the transfer of tax revenue in the amount corresponding to said minimum. In other words, municipalities may exceed the value of the indicator, but they are motivated by financial sanctions to repay debts, not accumulate them.

In describing the budget responsibility indicator, its questionability becomes evident. The numerator is represented by the total sum of debt as the total of all liabilities (short-term and long-term loans, discounted short-term bonds and notes, long-term bonds issued, notes payable and long-term notes payable, short-term and long-term financial assistance repayable, short-term and long-term liabilities arising from security and other short-term loans), but they are not distinguished in any way. All liabilities included here have the same weight, be they short-term or long-term liabilities or liabilities towards banks as well as non-banking institutions. In reality, each category of liability constitutes a different level of risk, and the structure of debt is a key factor vis-à-vis the overall indebtedness of a municipality (in terms of time and subject-matter).

Sanctions imposed on municipalities in the case of a breach of the fiscal rule may be very painful: even though the state suspends the transfer of tax revenue only temporarily (i.e. the sanctioned municipality does not lose this amount permanently), if the municipality were to breach the fiscal rule several years in a row, the suspended sum could bring about swingeing budget cuts. Tax revenue transfers are a key source of income for all municipal budgets; even temporary partial loss could be very painful for many municipalities – especially where the municipality already has major debts and is obliged to repay the principal amount along with interest. Therefore the sanction *de facto* multiplies.

There are many risks municipalities have to face with respect to budgets and controlling indebtedness in the form of the fiscal rule: these risks can be referred to as “conventional”, but some new risks may transpire as well. During the period when the economy struggles to recover from the COVID-19 pandemic (spring of 2020), the risks will take a new form and changes may affect not only the economic performance of municipalities as such, but also decision-making processes vis-à-vis indebtedness as well as fiscal, political and legislative processes across the entire country.

3. Indebtedness in the related literature

Public indebtedness becomes an important issue during economic crises; we can therefore expect that much attention will be paid to it in the upcoming months (although experts' responses to the period after the spring of 2020 have been understandably limited). In the first place, authors have been examining the indebtedness of countries (Baldi & Staehr, 2016; Greiner, 2012; Tamborini, 2014), before quickly moving on to analyse the indebtedness of regions and municipalities. This is understandable: the indebtedness of regions and municipalities contributes to the public debt of the country and, at the same time, their economic performance directly affects the ordinary lives of all citizens.

Generally speaking, indebtedness (be it state, regional or municipal) is accepted within the framework of the public economics theory (Buchanan, 1998; Musgrave & Musgraveová, 1994); that being said, indebtedness (be it in the form of a loan or bonds) is only acceptable during recession or with respect to asset acquisition and investment (Holtz-Eakin, 1991; Cropf & Wendel, 1998).

Factors affecting the indebtedness of regions and municipalities are addressed by several authors, both internationally (Kiewiet & Szakaty, 1996; Rivers & Yates, 1997; Veiga & Veiga, 2007; Guillamón *et al.*, 2011; Guillamón *et al.*, 2013; Fisher & Wassmer, 2014; Balaguer-Coll *et al.*, 2016; Vera, 2018) and in the Czech context (Hájek & Hájková, 2009). Generally speaking, they agree that indebtedness can be affected by a variety of factors – especially demographic (population, in some countries with migration as a contributing factor), economic (unemployment, economic development level of the region, personal income of the local population), political, and others.

The significance of regulating the indebtedness of municipalities or regions by higher government authorities is emphasised by Rodden (2002) or Bröthaler *et al.* (2015). Monacelli *et al.* (2016) warn that while the enforcement of fiscal rules can control indebtedness at the local level, it may result in reduced investment in local infrastructure. Outside control and regulation are to some extent counterbalanced by examinations into the influence of municipalities' internal controls on the level of indebtedness (Gras *et al.*, 2014).

The impact of budgetary responsibility rules adopted in Europe (in response to the financial crisis) on lower levels of government is studied, among others, by Bröthaler *et al.* (2015) or Vera (2018). They agree that these rules may indeed have a positive effect on the level of indebtedness of municipalities or regions; however, these lower tiers of government always have reduced opportunities for loans and indebtedness than the state.

4. Aim, methodology, identification and analysis of risks

The primary aim of this article is to identify and classify the risks associated with implementation of the fiscal rule after the COVID-19 pandemic in the spring of 2020. The risks are identified in relation to indebtedness of the municipalities, but also to political and legislative space.

The risk analysis employs qualitative methods (Smejkal & Rais, 2003: 85). The level of risks is estimated using qualified empirical estimates: taking into account a certain level of subjectivity associated with this approach (comp. Tichý, 2006: 147–151).

There are many risks associated with calculating the budget responsibility indicator and the implementation of the fiscal rule, and municipalities are able to influence only some. Many risks which municipalities are about to face in the near future, while not new, will entail different aspects and different levels. Many risks overlap, from effects on the budgeting of municipalities to the general status of municipalities as fundamental units of self-governance. Other risks are appearing for the very first time.

In particular, the following risks are acknowledged (the given order does not reflect importance):

Economic risk: The fiscal authority of Czech municipalities is limited – they can only decide on real estate tax and local fees. Their revenue largely depends on transfers from the state budget, especially on shared tax revenue, as the nationwide revenue from these taxes is redistributed among the state, regions and municipalities. However, the collection of shared taxes (VAT, personal income tax, corporate tax) highly depends on the economic cycle, i.e. in the event of economic recession or even crisis (which can be expected after the events of the spring of 2020), the nationwide level of the collection of these taxes will see a major reduction (see Table 1). This means a significant loss of revenue for municipalities which is hard to compensate from other sources. This fact would have direct impact on compliance with the fiscal rule: the denominator of budget responsibility indicator would gradually fall and, with indebtedness remaining the same, the risk of breach of the mandatory limit would increase.

Table 1. Tax revenues (state budget, January – May) – bil. CZK

Tax	2018	2019	2020
VAT	209.24	113.23	105.38
Personal income tax	48.41	56.19	46.85
Corporate tax	29.21	31.07	27.61

Source: Ministry of Finance of the Czech Republic

Political risk: This risk stems from electoral results, and can manifest itself on the national level (we shall ignore this aspect as the influence would be indirect) or the municipal level. Not all elected assembly members will pursue solely public goals and the public interest. The political cycle is four years, but decisions affecting investments resulting in the indebtedness of municipalities are longer than this period. This also carries future risk: should the economic situation (including a failure to adhere to the fiscal rule) have a harsh impact, resulting among other things in reduced revenue and the worsening quality of local assets and services, some elections at the local level could see the success of candidates who will not pursue the public interest (comp. Wasil *et al*., 2018).

Managerial risk: In their decision-making processes, assemblies must adhere to legal regulations (e.g. Act Number 250/2000 Coll.), but they also base these decisions on fundamental managerial principles and procedures. That being said, not all assembly members are properly educated or have the requisite managerial experience. Therefore, although their decision-making might have good intentions (or not), it can frequently err – causing high indebtedness that does not correspond to potential revenues, the economic outlook or reality.

Structural risk: This risk is closely related to the managerial risk. Due to decisions of an assembly the debt of their municipality may be too high, or its structure may be less than ideal. More advantageous options for municipalities include various forms of repayable financial assistance granted by other budgetary sector institutions; on the other hand, the issue of municipal bonds is permissible by law, but very costly for smaller municipalities.

Legislative risk: A significant legislative risk for the upcoming period may concern amendments to legal regulations with an impact on budgetary revenue: especially Act Number 243/2000 Coll., on the budgetary allocation of tax revenue, resulting in a reduced share for municipalities from shared taxes. This risk is higher during serious economic crisis, when the government needs tax revenue to protect the economy (public revenues must be centralised), and municipalities may be neglected as a result.

Moral risk: Even though all economic, legal and social processes in society always have their moral and ethical aspect, this aspect plays a more important role in "tough times". This is a novel risk and is directly related to the aforementioned risks. Significant economic decline may cause attempts on the part of government to re-allocate economic resources (or centralise them), or attempts to shift power (or centralise it). References to the "critical situation" may herald attempts to curtail local self-governing units and suppress democratic decision-making processes at local levels, to the detriment of municipalities or regions.

Natural and health ("black swan") risks: Even though these risks may at first appear unrelated, they have a similarly unexpected onset and significant consequences: an immediate increase of expenditures. Municipalities own assets which they must take care of while exercising due diligence. Natural disasters (typically in recent years floods or storms) cause damage to public property whose repair or reconstruction requires unplanned public investment. The situation is similar when it comes to health risks (being a novel risk): the events occurring over the past few months show that municipalities must respond to the consequences of epidemic in the form of increased public expenditures. Since not all municipalities budget for such contingencies, this may result in indebtedness.

Currency risk: Currency-related (foreign exchange) risks are not typical for municipalities: where a municipality has a loan or has issued bonds, they are in Czech crowns.

Bank risk: Due to the fact that most long-term liabilities of municipalities consist in loans from banks, an increase in interest rates (increase in debt service payments) is a risk. Recent events and the lowering of interest rates by the Czech National Bank have significantly reduced this risk.

Individual risk levels, the opportunity to influence them, and their impact on debt and revenue are all listed in Table 2.

Table 2. Risk levels and municipalities' ability to influence them

Risk	Level	Influence by municipality	Impact on debt (D) or revenue (R)
Economic	9	no	R
Political	5	yes	D, R
Managerial	5	yes	D, R
Structural	5	yes	D
Legislative	4	no	R
Moral	7	no	
Natural and health	7	no	D
Currency	1	no	D
Bank	2	no	D

Note: The level is evaluated on the 1–10 scale: 1 being very low and 10 being extremely high.
Source: own work

5. Discussion

Table 2 shows that municipalities have limited options in their ability to influence most of these risks (see Act No. 128/2000 Coll, Act No. 250/2000 Coll and Act No. 243/2000 Coll.); and where they can, the risks are "medium". It should also be noted that risks are not isolated, and often act in combination with one another.

The most significant risk which can have fatal consequences for the economic performance and indebtedness of municipalities is the overall economic situation of the country. During an economic crisis the revenue of municipalities from shared taxes is reduced significantly (due to reduced national tax revenues – see Table 1); hence even if indebtedness were to remain the same or was even reduced, the reduction in revenue will be much faster, causing municipalities to fail to adhere to the fiscal rule. It is fair to describe this as a "debt trap" at the municipal level. Of course, if the recession were to be serious and prolonged, it is legitimate to ask whether compliance with the fiscal rule would be enforced in the first place. If the state can soften its own limits to indebtedness (spring 2020, amendment to the Act on the State Budget for 2020), municipalities and regions have the right to ask why they should still be tied by the fiscal rule. If municipalities are unable to influence much of their revenue and, at the same time, their expenditures are now even higher, even a municipality exercising due diligence and employing sound management is not able to ensure a balanced budget. It is therefore legitimate to discuss how to improve the revenue side of municipalities and regions, as their dependency on transfers of taxes from the state budget may prove to be a risk and disadvantage in tougher times (comp. Rodden, 2002).

There are only three risks which can be influenced at the municipal level: these risks stem from the structure of the assembly and its actions (i.e. political, managerial and structural risks). All these risks are classified as "medium" and can be reduced only by active participation on the part of voters and public control. Active participation in local elections, interest in candidates and parties, as well as public oversight (formally, e.g. by attending assembly sessions and the regular reading of minutes of meetings of the city council or assembly, as well as informally, e.g. via social networks) – that is the only way to ensure real transparency in the economic performance of municipalities and make it literally a matter "public concern". Similarly, this is the way to reduce the risk of electing assembly members unpersuaded by the public interest: only public oversight and active public involvement at the municipal level can prevent these outcomes (comp. Wasil *et al*., 2018).

Even though legislative risk is deemed to be "medium", it is substantive for a prolonged period of time, and may even increase. The ease with which the amendment to the Act on the State Budget for 2020 was adopted, which resulted in a reduced obligation of the state to adhere to the fiscal rule, gives municipalities little comfort that a similarly far-reaching amendment to the Act on the Budgetary Allocation of Tax Revenue would not occur. Should the government decide that protecting the economy is going to require more financial resources at the expense of the needs of municipalities or regions, it has enough votes in the Chamber of Deputies to pass such amendment. This scenario could be disastrous for many municipalities, and could result in a substantial worsening in the quality (and quantity) of local public services (comp. Bröthaler *et al*., 2015 or Monacelli *et al*., 2016).

Even though economic risks are of key importance vis-à-vis compliance with the fiscal rule, we cannot ignore moral risks in the post-COVID-19 period. The relaxation of rules so that the state is not obliged to adhere to strict regulations while other public entitles (municipalities and regions) are, would lead to moral hazard. Decentralisation of decision-making processes may be compromised and centralised decisions – justified by the "need to govern during a crisis" – would force municipalities/regions to act as enforcers of orders adopted on a centralised basis, and the role of municipalities as the basic unit of self-governance would be eliminated. The risk of the centralisation of power is evident.

The risks which we have decided to call "black swan" (natural and health) cannot be influenced by the affected municipalities; unfortunately, such risks can be very strong and they are becoming increasingly dangerous. Sudden budget expenditure to compensate damage after natural disasters is not uncommon in recent years, and typically results in increased debts. Just a few months ago, health risks would not have even be considered: they are a brand new risk. The COVID-19 pandemic has made this risk a primary one: considering budgetary measures alone, the purchase of PPE and disinfection etc. may increase the indebtedness of many municipalities simply by being an unexpected and unplanned expenditure.

Bank and foreign exchange risks are not very significant for municipalities for the time being.

6. Conclusion

We must first emphasize that the indebtedness of municipalities is not yet felt as a burning problem. Although half of the total number of municipalities is in debt, almost half of the debt has long been in the four largest Czech cities. However, 517 municipalities did not meet the criterion of budgetary responsibility (winter 2018/19), 10 municipalities managed with a "higher level of risk" (Ministry of Finance of the Czech Republic).

The aim of this article has been the identification and analysis of risks borne by municipalities in relation to implementing the fiscal rule during the post-COVID-19 period. It should be noted that even though many risks currently affect and are going to affect municipalities in terms of their economy (increased indebtedness, lost revenue, failure to adhere to the fiscal rule and the resulting sanctions), most risks have non-economic consequences. We can even expect purely economic risks to diminish over time, while other risks (political, legislative and moral) will increase.

In addition to the immediate effects of economic recession on municipal revenues (being the result of lower shared tax revenue), potential amendments to certain laws also appear significant, especially regarding the Act on the Budgetary Allocation of Taxes and a reduced share granted to municipalities from nationally collected taxes. The fact that municipalities cannot substitute this tax revenue with other sources of revenue would lead to the overall partial loss of revenue and, inevitably, to a reduction in local public services or the worsening quality and quantity of these services (comp. Bröthaler *et al.*, 2015 or Monacelli *et al.*, 2016).

If the economic recession were to be long and severe, it would be tough for municipalities to adhere to the fiscal rule. With the state having reduced its obligations vis-à-vis indebtedness relatively easily in the spring of 2020, it is legitimate to ask whether self-governing territorial units might be entitled to do the same.

A significant risk, one which goes beyond considering risks related solely to municipalities, includes not just the redistribution of economic resources, but the reallocation of power itself. Fiscal centralisation could thus be accompanied by the centralisation of power, leading to the weakening of the authority of municipalities as fundamental self-governing units. Even though this risk is long-term, municipalities and the general public must be very alert to any signs in this regard.

References

[1] Act Number 23/2017 Coll.
[2] Act Number 128/2000 Coll.
[3] Act Number 250/2000 Coll.
[4] Act Number 243/2000 Coll.
[5] Balaguer-Col, M. T., D. Prior and E. Tortosa-Ausina (2016). On the Determinants of Local Government Debt: Does One Size Fit All? *International Public Management Journal,* 19(4), pp. 513–542.
[6] Baldi, G. and K. Staehr (2016). The European debt Crisis and fiscal reactions in Europe 2000–2014. *International Economics and Economic Policy,* 13(2), pp. 297–317.

[7] Bröthaler, J., M. Getzner and G. Haber (2015). Suistainability of local government debt: a case study of Austrian municipalities. *Empirica*, 42(3), pp. 521–546.

[8] Buchanan, J. M. (1998). *Veřejné finance v demokratickém systému*. Brno: Computer Press. ISBN 80-7226-116-9.

[9] Cropf, R. A. and G. D. Wendel (1998). The Determinants of Municipal Debt Policy: a Pooled Time-series Analysis. *Environment and Plannning C: Government and Policy*, 16(T), pp. 211–224.

[10] Fisher, R. C. and R. W. Wassmer (2014). The Issuance of State and Local debt During the United States Great Recession. *National Tax Journal*, 67(1), pp. 113–150.

[11] Gras, E., J. Hernandez and M. Palacios (2014). An Explanation of Local Debt in Spain Based on Internal Control systém. *Lex Localis*, 12(4), pp. 775–792.

[12] Greiner, A. (2012). Public capital, sustainable debt and endogenous growth. *Research in Economics*, 66(3), pp. 230–238.

[13] Guillamón, M. D., F. Bastida and B. Benito (2011). Evalución de la deuda pública local en Espaňa. *Revista Española de Financiación y Contabilidad*, Apr.–June, pp. 251–285.

[14] Guillamón, M. D., F. Bastida and B. Benito (2013). The ellectoral budget cycle on municipal policy expenditure. *European Journal of Law and Economics*, 36(3), pp. 447–469.

[15] Hájek, P. and V. Hájková (2009). Debt analysis of Czech municipalities. *Scientific papers of the University of Pardubice. Series D,* 14, pp. 36–42.

[16] Holtz-Eakin, D. (1991). Bond Market Conditions and State-Local Capital Spending. *National Tax Journal*, 44(4), pp. 105–120.

[17] Kiewiet, D. R. and K. Szakaty (1996). Constitutional Limitations on Borrowing: An Analysis of State Bonded Indebtedness. *Journal of Law, Economics and Organization*, 12(1), pp. 62–97.

[18] Ministry of Finance of the Czech Republic (https://www.mfcr.cz).

[19] Monacelli, D., M. G. Pazienza and Ch. Rapallini (2016). Municipality Budget Rules And Debt: Is the Italian Regulation Effective? *Public Budgeting&Finance*. 36(3) pp. 114–140.

[20] Musgrave, R. A. and P. B. Musgraveová (1994). *Veřejné finance v teorii a praxi*, Praha: Management Press. ISBN 80-85603-76-4.

[21] Rivers, M. J. and B. M. Yates (1997). City Size and Geographic Segmentation in the Municipal Bond Market. *The Quaterly Review of Economics and Finance*, 37(3), pp. 633–645.

[22] Rodden, J. (2002). The Dilemma of Fiscal Federalism: Grants and Fiscal Performance around the World. *American Journal of Political Science*, 46(3), pp. 670–687.

[23] Smejkal V. and K. Rais (2003). *Řízení rizik*. Praha: Grada. ISBN 80-247-0198-7.

[24] Tamborini, R. (2014). Interest-Rate Spread and Public Debt Dynamies in Two-Country Monetary Union Portfolio Model. *Open Economies Review*, 25(2), pp. 243–261.

[25] Tichý, M. (2006). *Ovládání rizika. Analýza a management*. Praha: C. H. Beck. ISBN 80-70179-415-5.

[26] Veiga, L. G. and F. J. Veiga (2007). Political business cycles at the municipal level. *Public Choice,* 131(1-2), pp. 45–64.

[27] Vera, J. O. (2018). Explanatory factors and limitations of Spanish local debt. *Academia Revista Latinoamericana de Administración*, 31(2), pp. 360–377.

[28] Wasil, J., M. Sidor and K. Kuć-Czajkowska (2018). Consequences of the Electoral Systém in Polish Municipalities – Pathologies and Abuses. *Polish political science,* 47(3), pp. 477–490.

Assessment of Factors Influencing Final Corporate Income Tax of Manufacturing Industry in the Czech Republic

Karolina Lisztwanová, Iveta Ratmanová [1]

Abstract

Tax policy is an important part of fiscal policy. Tax policy is able to create sources for financing government expenditures but on the other hand is able to support desirable activities and to change behaviour of individual subjects. This role of tax policy can be observed in case of corporate income tax as one kind of direct taxation. The paper is concentrated on assessing the impacts of individual items determining final corporates tax liability. Main attention is devoted to selected sector - manufacturing industry. For determining main factors affecting the total tax liability of corporates the pyramidal decomposition of top indicator is used. Respecting the fact that pyramidal decomposition determines multiplication relationship among individual items, the analysis of variances is adjusted to this fact. As a source information data provided by Finanční správa is used and observed period is limited by years 2005–2018.

Key words

Corporate income tax, total tax liability, pyramidal decomposition, functional method, manufacturing industry

JEL Classification: H20, K34, H25

1. Introduction

The tax is the income of public budgets, where in the form of a mandatory and usually regularly recurring payment is deducted on a non-refundable basis from the nominal income of the entity according to the tax law. Generally speaking, taxes are mandatory, irreversible, and inequivalent. First, compulsory means that taxpayers with tax obligations must abide by the state`s tax laws. Main aim of taxation is creating sources to finance government expenditures, to redistribute incomes and wealth of individual subjects and to support certain activities. By this way the role of taxes can be identified in influencing of economic growth, in reducing inequality between rich and poor people, in reduction of poverty etc.

Tax system can be described as the sum of all taxes that exist in economy including another revenues of public budgets (e.g. duties, social security contribution) and fiscal nature charges (administrative, judicial, local fees, etc.). The tax system is influenced by economic, political, cultural, globalization factors and technological progress. A good tax system should meet five basic conditions: fairness, adequacy, simplicity, transparency, and administrative ease according to Kubátová (2015) or Musgrave, Musgrave (1994). Tax system concludes direct and indirect taxes.

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Income tax is one of the direct taxes. The income tax may be levied on income of individuals or corporates. Regarding corporates income tax is imposed by a tax jurisdiction on the income or capital of corporations or analogous legal entities. The tax object is the earnings (profits) of companies. Such taxes are usually assessed on the total incomes of corporations from all sources and not simply profits generated by production.

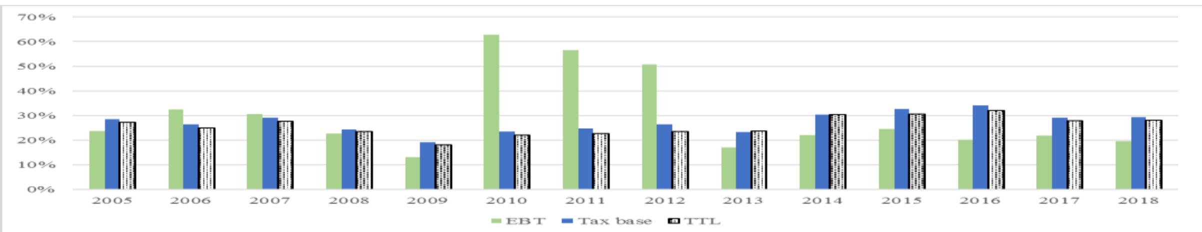
Tax liability of individual corporates is affected by tax policy. The assessment of real impacts of individual aspects on taxation has been already made according to Lisztwanová, Ratmanová (2017) for all sectors or Lisztwanová, Ratmanová (2018) for finance and insurance sector. The goal of the paper is to assess the impact of tax-deductible costs, changes of tax rate, items reducing tax base and tax reliefs on the final tax liability of taxpayers of manufacturing industry. The pyramidal decomposition and analysis of variances are used for assessing influence of individual items on top indicator in the selected time period between years 2005–2018 with usage data of Finanční správa.

2. Selected Information about Manufacturing Industry

The manufacturing industry is an important segment of the economy, which is a significant driver of the development of technology, knowledge and job opportunities. It has a long tradition in the Czech Republic and in its development, it has demonstrated the ability to maintain its position in a competitive environment, mainly due to the entry of foreign capital. High degree of integration and connection on foreign trade, however, at the same time makes this industry sensitive to changes in external conditions. According to Panorama zpracovatelského průmyslu 2018 this industry is the most important section of the economy, creating more than 25 % of gross value added (GVA) at current prices, 38.3 % of production at current prices and employing 26.6 % persons in 2018. The Czech Republic, with more than a quarter of manufacturing industry`s share in gross value added, is at the forefront of European countries. Only Ireland has a higher share (Panorama zpracovatelského průmyslu 2018). This kind of industry is specified, moreover by the fact that this industry transforms raw materials, subcontracting or components into products. Therefore, created revenues come from selling of own products and services. Revenues from the sale of goods, which are resold, generate a smaller part of total revenues.

The position of the manufacturing sector among other sectors of economy is clear from Figure 1. Generally, the ability to generate profit is obvious and specific trend reflecting changes in observed period as well. Certain willingness to persist and to overcome difficulties in economy is visible in data between the years 2010 and 2012 when the Czech economy had to pass over the second wave of recession. When it comes to tax base relatively stable level of the ratio of the manufacturing industry can be confirmed. This ratio moved from 18.98 % as the lowest value to 34.07 % as the highest value. Comparing changes in total tax liability (TTL) close relationship between this one and tax base is obvious. Generally, government tax incomes which come from the manufacturing industry represent approximately 18 % - 32 % of total corporate`s income tax liability.

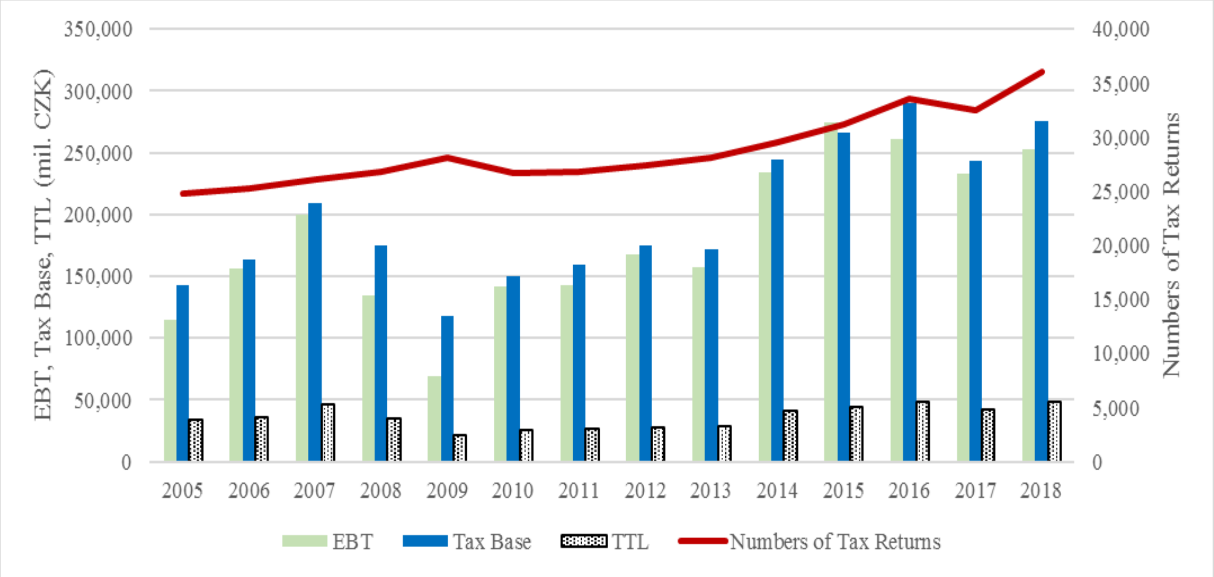
Figure 1: Ratio of EBT, tax base and total tax liability of the selected sector to the all sectors



Source: Authors` processing according data of Finanční správa

The figure 2 provides detailed information about absolute values of earnings before tax, tax base and total tax liability. In addition, attention is devoted to pointing out changes in numbers of tax revenues. Respecting the fact that the number of tax returns may reflect quantity of tax subjects the similar trend of numbers of tax returns and rest of indicators included in the figure can be observed. The value of earnings before tax shows impact of recession and declarative effect of second wave of recession in the year 2013. The year 2017 shows the decline of earnings before tax in spite of the fact that growth of GDP (4.5 %) can be observed. For the matter of that explaining can be found in leaving of exchange rate interventions which might have significantly affected economic results of some entities.

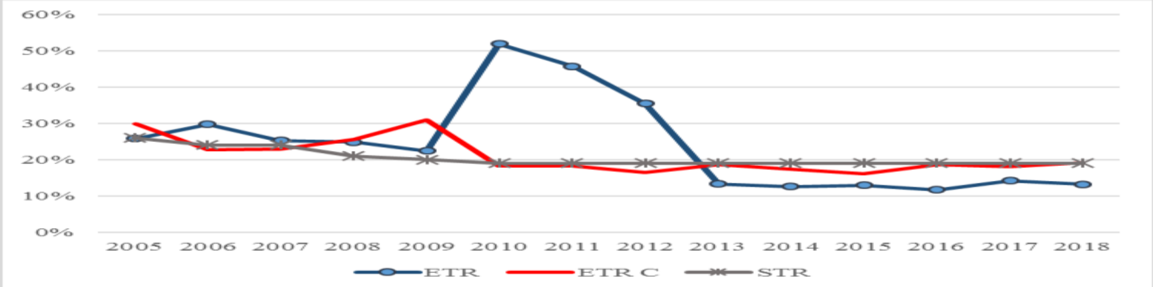
Figure 2: Changes of EBT, tax base, total tax liability and number of tax returns of the selected sector



Source: Authors` processing according data of Finanční správa

Data of the Figure 3 provides comparison among development of the effective tax rate of the manufacturing industry and the effective tax rate of all the sectors. The calculation of the effective tax rate is influenced by value or total tax and by value of earnings before tax according to Lisztwanová, Ratmanová (2017). Statutory tax rate is stable since the year 2010. The effective tax rate of all sectors shows impact of crisis. Nevertheless, assessing observed data it is important to realize the fact that our data came from data of entities of all sectors. Because of it, summarized data works not only with accounting profit but moreover with accounting loss. However, trend observed in the case of the manufacturing industry sector doesn`t declare so much distinct changes as it is obvious in case of all sectors. As conclusion it can be assumed that declining of profit had been lower in corporates which belong the observed manufacturing industry.

Figure 3: Changes of STR, ETR of the all sectors and of ETR of the selected sector



Source: Authors` processing according data of Finanční správa

3. Methodology

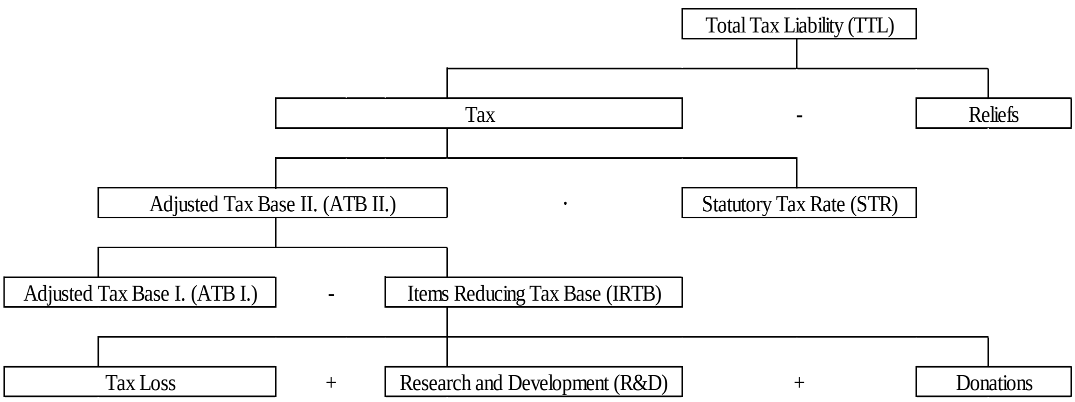
Income tax is as a direct tax concentrated on taxation of taxpayer`s income. Income can be created not only by individuals but moreover by companies. Corporate income taxation is considered as good addition taxation to personal income tax. Sometimes there is a risk that the part of total incomes should be taxed with personal income tax. Nevertheless, is not taxed. And this part can be taxed just with corporate income tax. Relying solely on personal tax revenue may cause delaying the taxpayer`s liability. The shareholders would like to reinvest of corporate profits rather than get the dividends. If a company is owned by foreign shareholders, earnings and consequently potential government`s revenues from personal income taxation may be transferred abroad. Next fact which support the idea of implementation of corporate taxation, is that taxation is way for paying for public services. Moreover, corporate income taxation supports desirable activities for example research and development, environmental activities, regions, industries, small firms etc. Regarding macroeconomic point of view of corporate taxation this one works like an automatic stabilizer. In "good times", for example, stable economic development, social stability, etc. creates a higher government`s tax revenues.

On the other hand, there are objections to the application of corporate taxation. The corporate profit is not a special kind of income. So, there is not a reason to use it. All corporations` profits will become personal incomes, so, there is a risk of economy double taxation and by it and corporate taxation weakens a fair effect of personal income. Moreover, it is not possible to identify respecting of the payment ability principle. The principle of payment ability means that the taxes should be levied according to the taxpayer`s ability to pay. In case of corporate tax if tax liability exists, tax must be paid.

The calculation of final tax liability is affected by determining of tax base. While the tax base as such can be reduced by deductions from the tax base. The adjusted tax base is the starting point for the tax calculation itself. The last aspect which may affect final tax liability is tax credit as the item decreasing calculated tax and determine final tax liability. This general procedure can be identified in tax liability calculations all over the world. There are observed efforts of fiscal policy by main instruments of corporate tax calculations to support desirable activities and such a way stabilize or stimulate economy.

Respecting tax legislation of the Czech Republic items reducing tax base comprise tax loss of previous periods, science and research expenses, expenditure on vocational education and gratuitous transaction on specific purpose. Investment incentives and employment of disabled people indicate tax credits. Detail determination of final tax liability is mentioned in Lisztwanová, Ratmanová (2018).

Figure 4: Pyramidal decomposition of total tax liability



Source: Authors` processing according The Act no. 586/1992 Coll., on Income Taxes

Respecting details of figure 4, it is obvious that top indicator - total tax liability - can be expressed as a decomposition of individual indicators, which play significant role in changes of this top indicator. With regard to details of decomposition there are multiplicative and additive relationship among indicators. By this way there is no problem to identify indicator with the biggest and the lowest impact on selected top indicator (Lisztwanová and Ratmanová, 2018). According to Dluhošová et al. (2010), Zmeškal et al. (2013) for mathematical expression of influence of individual indicators on changes of top indicator in case of multiplicative relationship it has been used the functional method.

4. Assessment of Influence of Individual Indicators

For understanding the development of pointed out indicators, table 1 has been created. The data of the year 2005 is considered as basic data. With regard to our observation fluctuation of total tax liability is obvious and since the year 2011 stable value of statutory tax rate. When it comes to the tax loss impact of tax crisis is observed with certain understandable delaying. In the case of research and development expenditures the years 2010 and 2018 indicated the highest value during observed period. This fact can be interpreted as an effort to invest in this area by the individual companies. Fluctuation is a way how to describe changes in donations activities. If trend must be observed, it can be mentioned that donations are not regularly part of company`s decisions. Concerning the possibility to use the tax credits as a way for reduction of tax liability trend is decreasing in last six years of our observation.

Table 1: Year-to-year changes of selected items within period 2005–2018 in % of the selected sector

	2005	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**
TTL	100.0	104.0	129.4	74.9	62.5	119.7	101.8	105.2	105.6	139.8	108.8	110.2	86.4	114.4
STR	100.0	92.3	100.0	87.5	95.2	95.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
ATB I.	100.0	114.7	124.3	82.7	71.4	132.7	103.2	105.2	87.1	160.0	107.1	106.1	83.8	113.9
Tax Loss	100.0	110.1	95.4	68.7	123.9	168.1	86.9	69.8	100.1	143.4	80.9	66.5	79.4	109.0
R&D	100.0	117.3	102.1	104.5	94.6	153.5	127.7	108.5	116.2	98.4	111.2	91.3	76.2	158.6
Donations	100.0	138.6	83.4	90.2	69.8	113.1	122.3	90.8	99.8	159.4	90.5	108.2	80.4	140.5
Reliefs	100.0	134.5	114.0	51.1	92.9	141.5	134.0	143.0	58.7	171.6	112.5	100.3	66.0	98.1

Source: Authors` calculation according data of Finanční správa

Table 2 and table 3 provide broader view of what happened in the manufacturing industry. Respecting the decomposition of the total tax liability there are details of impact of individual indicators on changes of the top indicator. Data mentioned in the table 2 shows magnitude of influence and data of table 3 as a percentage value shows the power of the influence. As far as total tax liability is concerned, trend is clearly growing. Decrease can be observed only in the year-on-year change between years 2007 and 2008, between 2008 and 2009 and finally between years 2016 and 2017. Adjusted tax base which can be understand as accounting profit corrected by tax legislation can be mostly assessed as indicator with the strongest impact on the change of the top indicator. Moreover, the following fact is observed in year-on-year changes. If the adjusted tax base grows, the total tax liability grows as well. Respecting absolute values of indicators tax loss is the indicator with the highest impact followed by research and development and donations. When it comes to tax credit it directly decreases final tax liability. As to the statutory tax rate importance of this indicator completely disappeared because of stable value of it in recent nine years.

Table 2: Power of influences of individual indicators of the selected sector, absolute changes in mil. CZK

	05-06	06-07	07-08	08-09	09-10	10-11	11-12	12-13	13-14	14-15	15-16	16-17	17-18
Δ TTL	1 361	10 426	-11 512	-12 888	4 235	453	1 356	1 535	11 573	3 581	4 520	-6 645	6 047
Reliefs	923	502	-1 997	-152	819	961	1 579	-2 212	2 214	662	19	-2 036	-73
STR	-3 043	0	-5 731	-1 455	-1 336	0	0	0	0	0	0	0	0
ATB I.	5 889	10 723	-8 925	-11 096	8 603	1 092	1 840	-471	15 003	3 640	3 345	-9 494	6 827
Tax Loss	-398	191	1 167	-558	-1 876	593	1 185	-2	-1 187	751	1 064	435	-151
R&D	-114	-16	-32	37	-327	-253	-99	-205	24	-162	140	350	-656
Donations	-50	29	13	34	-10	-18	9	0	-53	14	-11	28	-46

Source: Authors` calculation according data of Finanční správa

Details of the table 3 declare the power of influence. As long as the change of the top indicator is assessed as the one compact unit, data of the table 3 displays not only the shares of these indicators in this unit, but moreover positive or negative effect of them. That is why the table 3 creates a clearer idea about the particular impact of individual indicators.

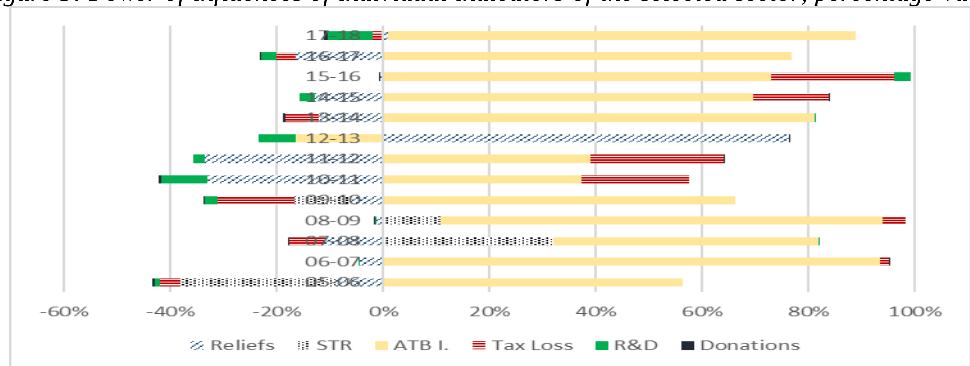
Table 3: Power of influences of individual indicators of the selected sector, percentage value

	05-06	06-07	07-08	08-09	09-10	10-11	11-12	12-13	13-14	14-15	15-16	16-17	17-18
Δ TTL	100	100	100	100	100	100	100	100	100	100	100	100	100
Reliefs	-67.80	-4.81	-17.34	-1.18	-19.33	-212.09	-116.42	144.14	-19.13	-18.48	-0.42	-30.64	1.21
STR	-223.61	0.00	49.79	11.29	-31.55	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ATB I.	432.76	102.85	77.53	86.10	203.13	241.08	135.65	-30.66	129.64	101.64	74.01	142.86	112.90
Tax Loss	-29.27	1.83	-10.14	4.33	-44.30	130.96	87.40	-0.15	-10.26	20.98	23.55	-6.55	-2.50
R&D	-8.40	-0.15	0.28	-0.28	-7.72	-55.95	-7.30	-13.34	0.21	-4.52	3.10	-5.26	-10.85
Donations	-3.68	0.28	-0.11	-0.26	-0.23	-4.00	0.68	0.01	-0.46	0.38	-0.23	-0.41	-0.75

Source: Authors` calculation according data of Finanční správa

Respecting data of this table it is a clear deep influence of the adjusted tax base I. on the top indicator and loose of influence in the case of the statutory tax rate. Regarding items reducing tax base, observation doesn`t differ from observation mentioned in table 3 and the strongest impact of the tax loss can be confirmed in the group of items reducing tax base. Also, it is necessary to mention that in the case of tax relief its influence was significant. This indicator contributed to decrease of the change of the total tax liability. Following figure 5 visually help to understand changes which have been observed during selected period.

Figure 5: Power of influences of individual indicators of the selected sector, percentage value



Source: Authors` calculation according data of Finanční správa

5. Conclusion

Corporate income tax is important part of tax systems of all countries from all over the world. Its role is not as significant as the role of consumption taxes, nevertheless it is not right to assume that this tax will be cancelled in the future. That is why it is important to know its development and to know better aspects which may influence it. This paper concentrates on decomposition of the total corporate tax liability on individual indicators respecting the Czech tax legislation and with usage of data of the manufacturing industry. For assessing the impact of them analysis of variances has been used.

As it was mentioned in previous text, power and direction of indicators influences are clear from assessment of percentage value of the indicators. Following table 4 provides information about the same facts but respecting greater clarity. The final assessment is showed as order of individual indicators.

Table 4: Order of influences of individual indicators of the selected sector

	05-06	**06-07**	**07-08**	**08-09**	**09-10**	**10-11**	**11-12**	**12-13**	**13-14**	**14-15**	**15-16**	**16-17**	**17-18**
Reliefs	3	2	3	4	4	2	2	1	2	3	4	2	4
STR	2	6	2	2	3	6	6	6	6	6	6	6	6
ATB I.	1	1	1	1	1	1	1	2	1	1	1	1	1
Tax Loss	4	3	4	3	2	3	3	4	3	2	2	3	3
R&D	5	5	5	5	5	4	4	3	5	4	3	4	2
Donations	6	4	6	6	6	5	5	5	4	5	5	5	5

Source: Authors` calculation according data of Finanční správa

In conclusion, it can be stated that adjusted tax base I. is the most important indicator influencing changes of the total tax liability of the selected sector. The tax credit the most influenced top indicator but its impact was lower compared to ATB I. As far as the tax loss is concerned it follows the tax relief. When it comes to research and development the influence of this indicator was not as strong as the tax loss. The statutory tax rate and donations were indicators with the lowest impact on the changes of the total tax liability in the manufacturing industry during the observed period.

Acknowledgments

This paper was supported by the SGS Project VŠB – TU Ostrava SP2020/124 "Finanční modelování a rozhodování podniků a finančních institucí za rizika, flexibility a interakce”.

References

[1] *The Act no. 586/1992 Coll.*, on Income Taxes.

[2] Dluhošová, D. (2004). Přístupy k analýze finanční výkonnosti firem a odvětví na bázi metody EVA – Economic Value Added. *Finance a úvěr – Czech Journal of Economics and Finance*, 54, vol. 11–12, pp. 541–559.

[3] Dluhošová, D., Z. Zmeškal, D. Richtarová and J. Valecký. (2010). *Finanční řízení a rozhodování podniku.* 3th ed. Praha: Ekopress.

[4] Finanční správa (2019). *Analýzy a statistiky: Daňová statistika k roku 2005–2018*. Retrieved from: http://www.financnisprava.cz/cs/dane/analyzy-a-statistiky/danova-statistika.

[5] Kubátová, K. (2015). *Daňová teorie a politika*, 6th ed. Praha: Wolters Kluwer.

[6] Musgrave, R. A. and P. B. Musgrave. (1994). *Veřejné finance v teorii a praxi*. Praha: MANAGEMENT PRESS.

[7] Lisztwanová, K. and I. Ratmanová (2017). Assessment of Impact of Items Reducing Tax Base and Tax on Total Amount of Corporate Income Tax in the Czech Republic in Selected Periods. In: *European Financial Systems 2017. Proceedings of the 14th International Scientific Conference.* Brno: Masaryk University, pp. 498–505.

[8] Lisztwanová, K. and I. Ratmanová (2018). Assessment of Factors Influencing Final Corporate Income Tax of Financial Institutions in the Czech Republic. In: *Managing and Modelling of Financial Risks. 9th International Scientific Conference. PROCEEDINGS (Part II.)* Ostrava: VŠB - Technical University of Ostrava, pp. 459-467.

[9] Zalai K., L. Kalafutová and J. Šnircová. (2007). *Finančno-ekonomická analýza podniku.* Bratislava: SPRINT vfra.

[10] Zmeškal, Z., Dluhošová, D. and T. Tichý. (2013). *FINANČNÍ MODELY. Koncepty, metody, aplikace.* 3rd ed. Praha: Ekopress.

[11] https://www.mpo.cz/assets/cz/prumysl/zpracovatelsky-prumysl/panorama-zpracovatelskeho-prumyslu/2019/10/panorama_cz_web.pdf

[12] https://www.czso.cz/csu/czso/cri/tvorba-a-uziti-hdp-4-ctvrtleti-2017

Potential product development andproduction gaps in the Czech Republic, 1997-2019

Jiří Mihola, Jakub Nábělek, Luka Slemnik[1]

Abstract

The paper deals with the development of potential product and production gap in the Czech Republic. Using an econometric model and own calculations, this paper analyses data on sources of gross value added both in individual sectors and in the entire national economy. The data obtained from information available from 1997 to the present serve to define the economic development in the Czech Republic in the past in all sectors. The main reasons of growth or decline in each sector are then explained and whether all sectors copy the trend of total GVA.

Keywords

Hodrick-Prescott filter, gross value added, stationarity, potential product, production gap

JEL Classification: G0

1. Introduction

Data on sources of gross value added (GVA) in individual industries from the Czech Statistical Office were used to determine the development of potential product and production gap in the Czech Republic. The potential product values themselves were then estimated using the Gretl econometric program. The aim of this paper is to evaluate the development of potential product and production gap in the Czech Republic using econometric tools. By analysing the development, it is possible to trace when there was a boom and when there was a recession in individual sectors. Each sector contributes to a certain proportion of the total GVA, i.e. also to the total economic cycle of the Czech Republic.

The introductory part examines the nature of difficulties related to estimating the potential product. Its problematic nature is proved not only by several calculation methods, but also by many authors, who often do not even agree on the basic parameters of their calculation. This part begins with an overview of the conclusions of international, domestic theoretical and empirical research. Next, the essence of two basic methods of calculating the potential product is explained, after which the paper focuses on the one that will be used in the empirical part to analyse the development. The method of calculation using the HP filter is explained and finally the data for individual industries and their time series are defined.

The empirical part first examines the original data and its time series and tests whether the individual time series are stationary. For each time series of individual industries, its peaks, bottoms are highlighted, as well as whether there are any differences from the development of other industries. The second chapter deals with the HP filter application, from which we obtained data on the potential product and the production gap. Subsequently, their development over time was analysed for each sector and in the whole national economy. The conclusion

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provides a summary and clarification of the results obtained from the empirical part whether the aim of the paper was met.

2. Theoretical foundations of the paper

2.1 Potential product

Macroeconomic approaches to the estimation of potential product are based on finding the real GDP development trend (M. Hájek and V. Bezděk, 2000). A potential product is defined as the real optimal level of output that can be produced given the existing production technology and production factors. The value of a potential product cannot be measured, but only estimated (M. Hloušek and J. Polanský, 2007). K. McMorrow and W. Roeger (2001) pay great attention to measuring potential output. Because the potential product is a directly unobservable variable, the authors discuss the essence of the concept used before the measurement itself. They then use statistical methods or econometric analyses to estimate the potential product.

In this paper, the potential product will be estimated using the Hodrick-Prescott filter (HP filter). This method leads to obtaining the potential product value by smoothing out GDP (R. J. Hodrick and E. C. Prescott, 1980). The second common method is to identify the potential product and the production gap using the production function. This method is most often estimated using the two-factor Cobb-Douglas production function (E. Jašová, 2011). Both of these methods have been recommended and applied to developed countries by the OECD (C. Giorno et. al, 1995). Other methods include Baxter-King bandpass filter, the Kalman filter, the multivariate Hodrick-Prescott filter, or the unobservable components models. The identified potential product is then used to calculate the production gap. The production gap is defined as the difference between the actual and potential product to the potential product. Production gap (in %) = (Y - Y*)/ Y* (J. Hráčková, 2015)

2.2 Hodrick-Prescott filter

Estimating a potential product using an HP filter is a relatively simple econometric operation that does not require input data. The filter used by us is a logarithmic version which is defined by this algorithm (M. Hájek and V. Bezděk, 2000):

$$\mathrm{Min}\left\{\sum_{t=1}^{T}(\ln Y_t - \ln Y_t^*)^2 + \lambda \sum_{t=2}^{T-1}\left[(\ln Y_{t+1}^* - \ln Y_t^*) - (\ln Y_t^* - \ln Y_{t-1}^*)\right]\right\} \quad (1)$$

in this relation Y denotes the actual product, Y* is the potential product, λ is the parameter determining the smoothness of the trend smoothing. If $\lambda = 0$, the potential product is equal to real GDP, for $\lambda \rightarrow \infty$ the trend will be a straight line. This parameter expresses the minimisation of the differences of moving averages between the actual development and the calculated trend component, λ reduces the variability of the trend component, which means that the filter output will have a smoother course (I. Křivý, 2012).

The ideal value used for the decomposition of macroeconomic time series is usually 1600 for quarters, 100 for annual cycles and 700 for half-yearly cycles (Hájek and Bezděk, 2000 also used this smoothing for the conditions of the Czech Republic). In contrast, the authors V. Bezděk, A. Dybczak and A. Krejdl (2003) used the value 480 to smooth the quarterly data due to structural breaks affecting the previous and subsequent observations. According to a previously unpublished study, the ideal value for product analysis in the Czech Republic is 110. This value is closest to the ideal values of the ratio of the squares of the variance of the cyclical component and the variance of the growth rate of the trend component (D. Tráge, 2011). In our paper, the estimation of a potential product is always performed using the HP filter. When using it, it is not necessary to use other factors (e.g. consumer prices and import prices) that are not statistically captured (E. Jašová, 2016). According to A. C. Harvey and A. Jaeger, 1993, the HP filter can generate business cycle dynamics even if none are present in the original data.

The advantages of the HP filter according to M. Hájek and V. Bezděk (2000) are:

- Low input data demand factor (the only necessary variable is the actual values of GDP at constant prices)
- Easy application and transparency and simplicity of interpretation

The disadvantages of the HP filter according to M. Hájek and V. Bezděk (2000) are:

- Problems of the smoothing parameter λ, where there is no criteria according to which it would be possible to reliably determine it, or to determine the most suitable value
- Possibility of estimates deviation at the beginning or end of the series, if the beginning and end of the time series do not capture a similar place in the economic cycle
- Unlike other methods, the HP filter does not capture structural changes in the economy

For the purpose of this paper, we believe the disadvantages of HP filter do not have an influence on given data and our analysis, so we prefer HP filter thanks to its simplicity.

2.3 Data sources used

The stationarity of the initial time series was verified before calculating the estimates of the variables. The stationarity of published volume indices means that its probability distribution is constant and independent over time, and at the same time relevant within the econometric model. Dynamic models of economic time series are assumed to be constructed from the observation of economic variables that meet the stationarity requirement (R. Hušek, 2007). Information and data from the Czech Statistical Office (CZSO) were used. Subsequently, the data were divided into several sectors, which are the sources of the total gross value added of the Czech Republic. The development of gross value added will be important for the calculation and analysis of potential production and production gap. These are the following 10 sectors (CZSO, 2019):Agriculture, forestry and fisheries

- Industry, mining and quarrying
- Building industry
- Trade, transport, accommodation and hospitality
- Information and communication
- Financial and insurance activities
- Real estate activities
- Professional, scientific, technical and administrative activities
- Public administration and defence, education, human health and social work activities
- Other service activities

3. Analytical part

3.1 Original data and stationarity tests

As already mentioned in the theoretical part, data from the CZSO were used to prepare the paper, which was then divided into individual categories. All-time series begin with Q1 1997 and end with Q2 2019. Each time series expresses a volume index, i.e. an index of the development of the total volume in a given category. This is a year-on-year index, also referred to by the abbreviation SPLY = 100, i.e. the same period of the previous year is equal to 100. Everything above 100 means growth, everything below 100 means a decrease compared to the same period of the previous year (Statistika&My, 2015).

The development of time series of all 10 sectors is diverse, it cannot be said that in times of economic boom all volume indices were above 100 and vice versa that during recessions they were all below 100. However, it is true that most time series indices behaved like this. The first calculation itself was the conversion of volume indices to percentages, as growth (positive number) or decrease (negative number) is better expressed in percentages.

Gross value added (GVA), which to some extent copies the development trend of gross domestic product, declines in the first 6 quarters of the time series, after which it shows growth until the economic crisis in 2009. The peak was in Q1 2006 (8.4%), while the bottom was in Q3 2009 (-6.4%). The crisis is followed by a slight increase and then a slight decrease in 2012 and 2013. In the last 6 years, GVA values have been in positive numbers (from 1.4 to 5.0%).

All of these time series were tested for stationarity. This test was applied using the extended Dickey-Fuller test. In order for the time series to be stationary, i.e. for the time series to not increase or decrease continuously, the p-value must always be less than or equal to 0.10 and this was the case for all industries, except for the time series of total gross value added, but when applying a delay, this time series was also stationary in the end (see Table 1).

	Agriculture, forestry and fisheries	Industry, mining and quarrying	Building industry	Trade, transport, accommodation and hospitality	Information and communication	Financial and insurance activities	Real estate activities	Professional, scientific, technical and administrative activities	Public administration and defence, education, human health and social work	Other service activities	Total potential product of gross value added	**GVA delay**
p-value	0,001228	0,02799	0,001119	0,001375	0,005798	0,002567	0,0007848	0,006365	0,0001444	0,07204	0,1609	0,0198
test statistics	-4,17838	-3,12851	-4,20621	-4,14455	-3,69075	-3,95299	-4,31075	-3,65965	-4,7871	-2,73593	-2,34363	-3,20382
estimated value	-0,327103	0,201327	-0,331827	-0,320545	-0,249442	-0,304311	-0,315621	-0,255556	-0,395787	-0,162968	-0,112981	-0,15099
stationary?	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	NO	YES

Table 1: Stationarity test
Data source: https://www.czso.cz/csu/czso/hdp_cr; Gretl; own calculation in Excel

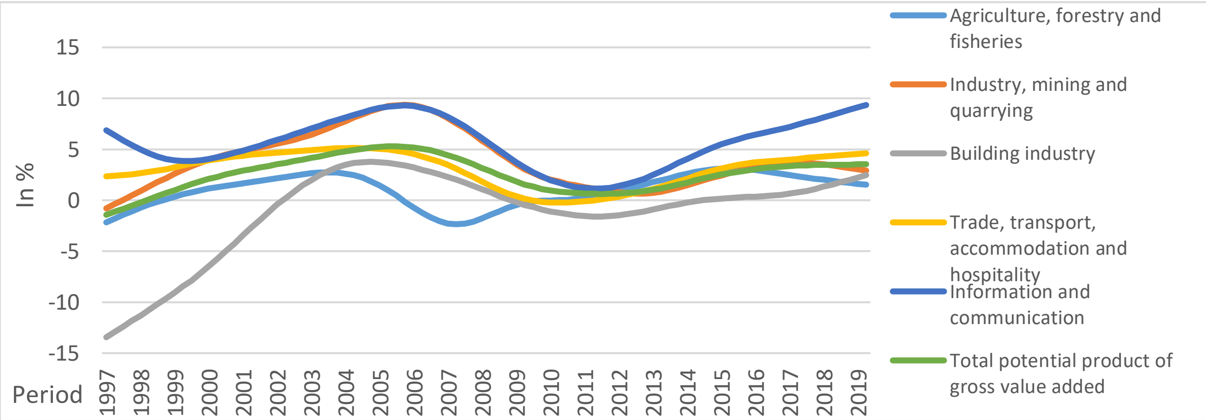
3.2 HP filter

In the next part, the HP filter with $\lambda = 1600$ was applied, because we will use time series with quarterly data. Using the HP filter, the quarterly smoothed potential product will be determined from the gross value added in individual industries. The time series of potential product for **agriculture, forestry and fishing** starts with negative numbers in 1997 and increases with each subsequent quarter until it peaks in 2003. After reaching the peak, the values fall and reach the bottom in 2007 (-2.4%), which was caused by Hurricane Kyrill, which caused enormous damage to forests. Another peak is observed in 2015 (3.1%); since then it has been declining slightly. This time series does not copy the trend of the overall economy or other sectors. The bottom in 2007 in Chart 1 stands out from the trend of other industries.

The initial development was similar for **industry, mining and quarrying**, which gradually rose from -0.8% to 9.4% in the last quarter of 2005. The sector's performance then declines, most markedly during the economic crisis in 2009, but the fall continues until a mild recession in 2012, but never reaches negative values (the lowest is 0.6%). The second peak occurred in Q2 2017, and it has been declining slightly in recent periods. At the beginning of the reference period, the **building industry** was in the worst numbers of all sectors (-13.4% at the beginning of 1997), but its subsequent increase was all the more pronounced, and this trend stopped in the last quarter of 2004 at 3.8%. The trend was then gradually declining until 2011 and then it was growing again until the present. In the building industry, it can be observed that its development is slowing down, which can be seen, for example, in 2012-2014, when all sectors were already growing after the recession, but the only building industry was still in negative numbers.

Another monitored sector is **trade, transport, accommodation and hospitality**. This, as one of the few, starts on the positive side in 1997 and remains positive until 2009. After weak negative numbers in 2010, the sector is growing steadily and still has not reached its peak at the end of the observed series. There are several reasons why the industry is still growing with only a few exceptions. The main ones are ever-increasing globalisation and tourism. The only sector that has shown an increase throughout the reference period is **information and communication activities**. It started at 6.9% in 1997, then with slight oscillations the industry bottomed out in 2011, but the value was still positive (1.1%). Since then, it has been rising again and the latest available figure is 9.3%. This sector is still growing because of the intensive development based on continuous innovation. People working in this sector are among the best paid on the labour market. **The total potential product of gross value added** in 1997 starts at -1.4% and gradually grows until Q3 2005 to 5.3%. It then declines to a bottom of 0.6% during Q3 2011. Since then, it has been growing steadily again; in the last few quarters, a slowdown in the growth trend can be seen and the value has stabilised at 3.5% (see Chart 1).

Chart 1: Potential product of the first 5 industries and total potential GVA product

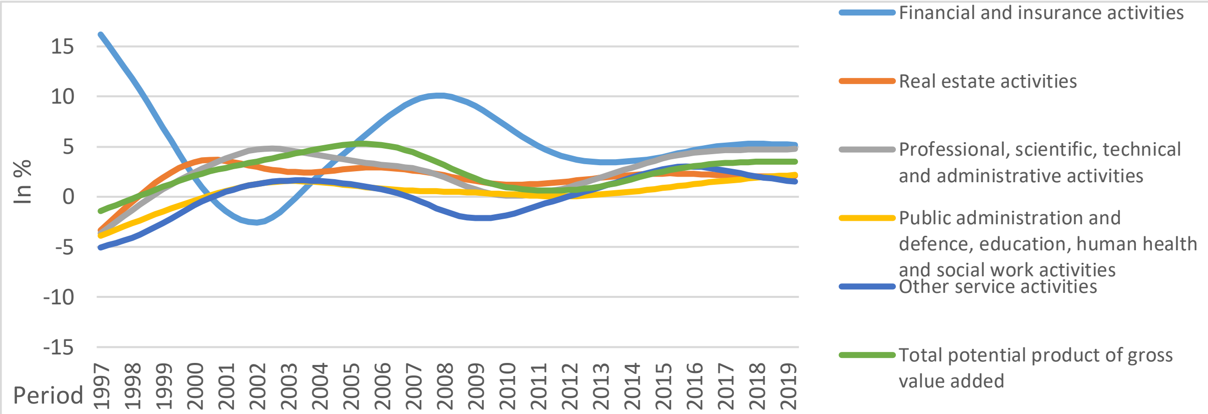


Data source: own calculation based on data from the CZSO: https://www.czso.cz/csu/czso/hdp_cr

Relatively turbulent developments are recorded in **financial and insurance activities**. The time series of this sector starts remarkably high (16.2%), and its development decreases by 18.8 pp by 2002. Then it grows to double digits again. It declines during the crisis, but not so sharply, and subsequent growth is not so high (see Chart 2). The initial significant development of this sector is caused by the consequences of events happening before 1997, when the change of regime to a capitalist one led to the entry of many new entities into the market and to the subsequent demand for their products by clients. The development of **real estate activities** starts at -3.3%, but its trend was growing and peaked in Q3 2000. The development then slightly decreased and stabilised at values from 1.2% to 2.9% and did not show any significant fluctuations for the rest of the reference period. The real estate market is limited by the slow construction of new flats so the period of boom and recession does not have a significant impact.

The development of the **professional, scientific, technical and administrative activities** started, similarly to the real estate sector, with an increase, but did not reach its peak of 4.8% until 2008. However, both the economic crisis of 2009 (the bottom with a value of 0.1%) and the subsequent rise had a more pronounced effect here. Economic development does not have a significant impact on this sector either, but it is a sector that is dependent on state contributions, and in a period of recession, the state must reduce its funding in this area as well. **Public administration and defence, education, human health and social work activities** also begin with negative values and a subsequent growing development. The peak was reached in the first half of 2003 (1.6%), after which it decreases very slowly until 2011. In recent years, the growing development of this sector has been more pronounced. It is a sector that includes basic human needs and is most tied to the state budget, so the state cannot afford its dynamic development. The last sector is **other service activities**, which starts with a growth from -5.1% to 1.6% in 2003. It is gradually reaching negative values and the decline will not stabilise until Q2 2009, which is the lowest value recorded during the economic crisis of all sectors. After the crisis, there is a growth to 3% at the turn of 2015 and 2016 and it has been declining since then.

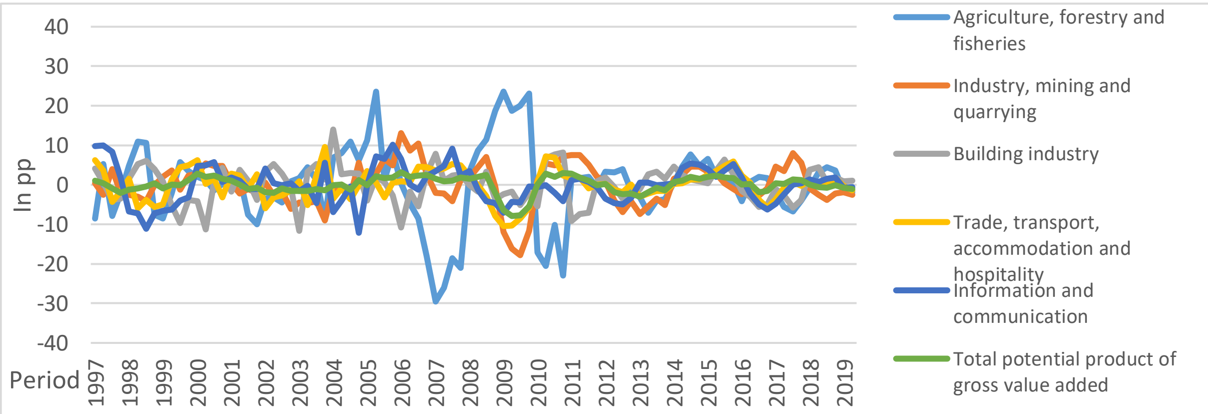
Chart 2: Potential product of the other 5 industries



Data source: own calculation based on data from the CZSO: https://www.czso.cz/csu/czso/hdp_cr

Using the HP filter, the development of the phases of the potential product is also determined, i.e. in which period there was a positive or negative "gap" or production gap. The production gap is expressed in percentage points (pp). If the value for a given quarter is positive, it indicates a positive production gap or boom, and conversely; a negative value indicates a period of negative production gap and a period of recession. In general, the values of the production gap in all-time series are fluctuating and therefore difficult to interpret. Nevertheless, a certain trend can be traced from them, whether it is the trend of individual sectors or the overall trend in the whole economy.

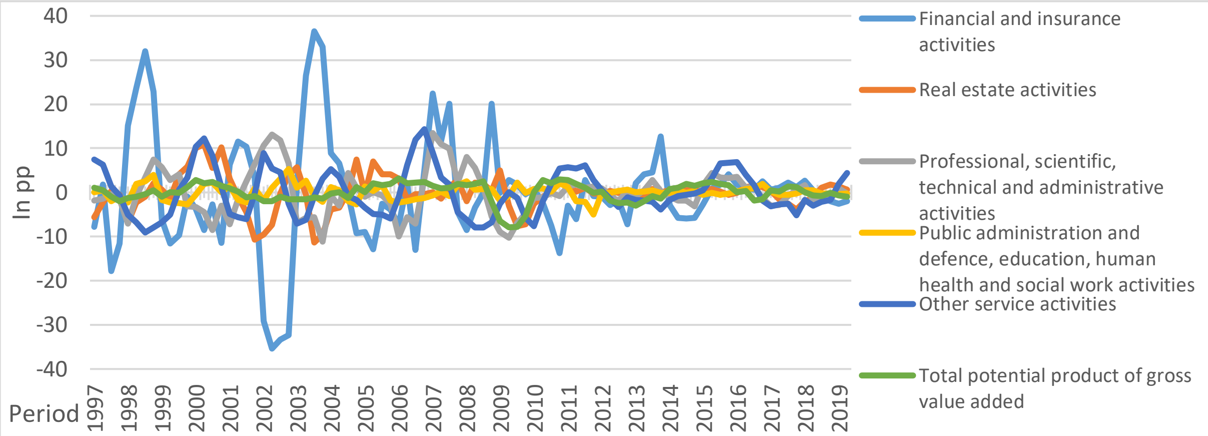
Chart 3: Production gap of the first 5 industries and total potential GVA product



Data source: own calculation based on data from the CZSO: https://www.czso.cz/csu/czso/hdp_cr

The **agriculture, forestry and fisheries** sector is more volatile and, as already mentioned, also does not follow the general trend of other sectors. It recorded a significant boom in 2005 and a significant recession in 2007 (bottom in Q1 with a value of almost -30 pp). This is followed by a longer-lasting boom in 2009 and again by another recession throughout 2010 (see Chart 3). Another sector with significant fluctuations is **Financial and insurance activities**, which recorded a strong boom in 1998, then a recession in 2002 (bottom in Q2 with a value of -35.4 pp) and then a significant boom in 2003 (peak in Q3 with a value of 36.5 pp). Other larger booms occurred in 2007 and 2013 (see Chart 4).

Chart 4: Production gap of the other 5 industries



Data source: own calculation based on data from the CZSO: https://www.czso.cz/csu/czso/hdp_cr

If we combine the data of all sectors and try to find some similarities in their development, we can see that the most sectors experienced a boom in 2000, 2005 and 2015, on the contrary, the recession was most common in 1999, 2009 and 2013. In the last two monitored quarters (first half of 2019), building industry, real estate activities and other activities service are in a boom phase. Other sectors are in recession according to the development of the production gap.

4. Conclusion

The aim of the paper was to use the HP filter application to obtain a tool for mutual comparison of the development of the industry, especially in terms of the trend expressed by the development of the potential product. In our case, it turned out that most industries copy the trend of total GVA. We consider it valuable to have found out which sectors are sensitive to the financial crisis in their development, or whether they are more seasonal in nature or they react to other phenomena of a more general nature.

The **agricultural, forestry and fishing** sector bottomed out not during the economic crisis, but during a natural disaster. The **building industry** was not very successful at first, but then its development stabilised. **Industry, mining and quarrying** sector has not declined since 1998, not even during the economic crisis. **Trade, transport, accommodation and hospitality** sector faced only a small decline during the crisis. The record holder in constant growth is the **information and communication activities** sector. The initially high growth of the **financial and insurance activities** sector was due to the events before 1997 and the entry of new market participants. The period of general boom and recession does not have a significant effect on the development of **real estate activities**. The **professional, scientific, technical and administrative activities** sector is similar, although it is dependent on state funding. The **public administration and defence, education, human health and social work activities** sector includes basic human needs, so the state cannot afford a possible significant decline. **The total potential product from gross value added** copies the trend of GDP development. The development of the production gap – is also important. The results of the production gap were very volatile and difficult to interpret. The result was the finding that after averaging the developments in individual sectors, the highest economic growth took place in 2000, 2005 and 2015 and the recession in 1999, 2009 and 2013. All these findings mentioned above can also be used to predict the future development of the production gap, although with the pandemic taking place right now, the economic development in years ahead is very uncertain.

References

[1] BEZDĚK, V.; DYBCZAK, A.; KREJDL, A. (2003). *Czech Fiskal Policy: Introductory Analysis*. Czech National Bank, Working Paper. 2003, no. 7.

[2] Česká národní banka (2020). Hlavní ukazatele [online]. [cit. 7.2.2020]. Available at: https://www.cnb.cz/cnb/STAT.ARADY_PKG.hlavni_ukazatele?p_iframe=0&p_lang=CS

[3] Český statistický úřad (2020). *Hrubý domácí produkt – Časové řady ukazatelů čtvrtletních účtů* [online]. [cit. 7.2.2020]. Available at: https://www.czso.cz/csu/czso/hdp_cr

[4] Český statistický úřad (2020). *Nejnovější ekonomické údaje* [online]. [cit. 7.2.2020]. Available at: https://www.czso.cz/csu/czso/aktualniinformace

[5] Fileplanet. *Gretl*. Available at: https://gretl.fileplanet.com/

[6] GIORNO, C., et al. (1995). *Potential Output, Output Gaps and Structural Budget Balances*, OECD Economic Studies, No. 24, 1995/I, OECD, Paris.

[7] HÁJEK, M; BEZDĚK V. (2000). *Odhad potenciálního produktu a produkční mezery v ČR*. WP Česká národní banka

[8] HARVEY, A. C.; JAEGER, A. (1993). Detrending, Stylized Facts and the Business Cycle. Journal of Applied Econometrics, Vol. 8, No. 3. (Jul. - Sep., 1993), pp. 231-247

[9] HLOUŠEK, M.; POLANSKÝ J. (2007). *Produkční přístup k odhadu potenciálního produktu – aplikace pro ČR*. Národohospodářský obzor. Brno: Ekonomicko-správní fakulta MU, 2007, č. 4.

[10] HODRICK, R.; PRESCOTT, E. 1980. Post-war Business Cycles: An Empirical Investigation. Working Paper, Carnegie-Mellon University.

[11] HRÁČKOVÁ, J. (2015). *Mezera výstupu a historie jejího užití*. s.29-30 [online]. Masarykova univerzita. [cit. 7.12.2019]. Available at: https://is.muni.cz/th/r1se3/Diplomova_prace.pdf

[12] HUŠEK, R. (2007). *Ekonometrická analýza*. Praha: Oeconomica. ISBN 978-80-245-1300-3.

[13] JAŠOVÁ, E. (2011). *Vybrané otázky z oblasti NAIRU, potenciálního produktu a produkční mezery na publikovaných datech a predikci vývoje v ČR do konce roku 2010* [online]. Business & IT [cit. 7.2.2020]. Available at: http://bit.fsv.cvut.cz/issues/01-11/full_01-11_02.pdf

[14] JAŠOVÁ, E. (2016). *Koncept a metody odhadu NAIRU a hospodářského cyklu na trhu práce v zemích Visegrádské skupiny*. s. 106. Národohospodářská fakulta VŠE v Praze

[15] KŘIVÝ, I. (2012). *Analýza časových řad* [online]. [cit. 7.12.2019]. Available at: http://www1.osu.cz/~bujok/files/ancas.pdf

[16] KOŠŤÁKOVÁ, T. (2015). *Meziroční a meziměsíční srovnání* [online]. Měsíčník českého statistického úřadu [cit. 7.2.2020]. Available at: https://www.statistikaamy.cz/2015/06/mezirocni-a-mezimesicni-srovnani/

[17] MORROW MC, K.; ROEGER, W. (2001). *Potential Output: Measurement Methods, "New" Economy Influences and Scenarios for 2001-2010 - A Comparison of the EU15 and the US*. Economic and Financial Affairs (ECFIN) of the European Commission, No. 150, 4-21.

[18] TRÁGE, D. (2011). *Časově-frekvenční analýza hrubého domácího produktu ČR*. s.6 [online]. Vysoké učení technické v Brně [cit. 7.12.2019]. Available at: https://www.vutbr.cz/www_base/zav_prace_soubor_verejne.php?file_id=36049

Application of Stochastic Dominance for Stock Selection and Comparison of Correlation Measures in Portfolio Optimization Approach

David Neděla [1]

Abstract

Over the last decades a significant number of scientists focuses on the portfolio optimization problem topic. The portfolio optimization model provides a tool for finding proportion of investment into particular assets. At the beginning of investment, it is important to choose the right investment instruments, especially particular investment assets together with the portfolio model. However, various parameters may be modified in portfolio model as measurement of dependency between assets. For this reason, the selection of stocks based on stochastic dominance approach and comparison between the several correlation measures in portfolio optimizations models are solved in this paper. The stock markets used for empirical analysis were the US, UK, Chinese, German, and the Japanese markets, with stock data drawn at, boom and crisis time periods to allow comprehensive comparison across markets and time periods.

Key words

Portfolio optimization; stochastic dominance; mean-variance model; correlation coefficient

JEL Classification: C10; G11

1. Introduction

Over the last decades, a significant number of papers focused on portfolio optimization problems have been published. One of the representatives in portfolio optimization was Harry Markowitz (1952), who proposed the well-known mean-variance approach that minimizes the variance of a portfolio whilst achieving the expected return. The portfolio optimization model or strategy provides a tool for finding a proportion of investment into particular assets. It is appropriate to align an investor's risk attitude with potential portfolio return. The investor chooses the indicator which mainly takes into account priorities for his decision-making such as payback period, level of risk, mean return, among others. One of these decision criteria can be considered stochastic dominance (SD). It can be defined as a statistical method that can be used in portfolio optimization. Stochastic dominance can be first, second or even higher order. More used type is second-order stochastic dominance.

Give the above information, the first contribution of this paper is an application of stochastic dominance approach for selection of eligible assets and the second contribution is empirical analysis on the influence of different types of correlation measures in the portfolio theory (model) in different world markets and comparison with benchmarks.

The motivation of this analysis is to provide a recommendation of investment markets and a comparison of different general dependency measures in the portfolio model, which is useful for individual investors. Whereas the stochastic dominance is a modern approach used in finance, investors can follow this approach in the stock selection process.

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The whole paper is divided into 5 sections. The introduction is described in section 1. In section 2, a brief literature review of the related research is made. The descriptions of the mathematical formulation and portfolio model are introduced in section 3. Application of the described approaches in the empirical analysis is provided in section 4, and in section 5, the whole paper is summarized and concluded.

2. Literature Review

In his path-breaking work on a portfolio optimization problem, Markowitz (1952) considers how investors can maximize an expected return for a given risk level, or equivalently, minimize the risk for a given expected return. In original Markowitz's formulation and later specifications, e.g. including the CAPM of Sharpe (1964), the measure of risk is considered for a variance of returns in an investor's overall portfolio. By this meaning, the mean return vector and covariance matrix of individual asset returns provide us with everything necessary to carry out a portfolio selection. One of the following modifications was provided by Konno and Yamazaki (1991) who proposed the use of approximation to variance instead of using variance as a measure of risk. They concluded that the linear model for portfolio optimization is computationally efficient, and the results are not different from the Mean-Variance model. A comparison of the different dependency indicators, e.g. correlation coefficients included in the portfolio model has already been provided by Ortobelli et al. (2015) and Kouaissah et al. (2018).

Two different approaches in portfolio selection under the condition of uncertainty which respect the utility theory can be distinguished. The first one is the reward – risk analysis and the second is the stochastic dominance approach (SD), see Kouaissah et al. (2018). SD allows to compare different random variables e.g. asset returns or different risk indicators, when it is deeply related to the theory of utility functions, see Bawa (1978). The SD takes into account the whole statistical distribution instead of its parts such as moments, see Wolfestetter (1996). The advantage of the first and second order of SD is having a clear economic interpretation. The application of SD in the portfolio optimization theory was dealt with e.g. Kopa (2019).

The necessarily known indicator in portfolio theory and in other financial optimization problems is the dependency structure of financial variables. In this paper, different correlation measures, both linear and non-linear, are used. Firstly, the Pearson correlation coefficient is mostly used for optimizing the diffusion of portfolio returns. It is well known that the Pearson correlation coefficient works well with the normally distributed data (Gaussian distribution), but this assumption was rejected by many authors e.g. Mandelbrot (1963), Fama (1965) or Neděla (2020). Rank (non-linear) correlation measures also satisfy the requirement for dependency indicator, see Ortobelli et al. (2015). The second possible approach to measure the dependency between random variables is the Spearman coefficient of correlation (Spearman′s rho) where is no requirement of data normality meaning a non-parametric statistic. It assesses how well the relationship between two random variables can be described formulating a monotonic function, see Tsay (2010). The Spearman′s rho between two variables is equal to the Pearson correlation between the rank values of those two random variables. Thirdly, the Kendall correlation coefficient (Kendall′s tau) is also permissible to use. It is a non-parametric test as well based on the rank of two random variables. After comparison, the Kendall′s Tau usually acquire smaller values than Spearman′s rho, see Tsay (2010).

3. Theoretical Background and Portfolio Optimization Models

Before constructing a portfolio model, the choice of the investment instruments and particularly the performance measure has an important character, and for investors, it is difficult to argue for one measure against another. Similarly, the dependence structure of asset returns plays a crucial role in portfolio theory.

Let $p_{i,t}$ denote the price of i-th stock at time t, observed for $T+1$ time periods, i.e., $t \in \{0,1,\ldots,T\}$. Then the continuous form of i-th asset return at time t is given by $R_{i,t} = \ln\left(\frac{p_{i,t}}{p_{i,t-1}}\right)$. The mean i-th asset return is calculated as $E(R_i) = \frac{1}{T}\sum_{i=1}^{T} R_i$ and the variance of i-th asset return is defined as $\sigma_i^2 = \sum_{i=1}^{T}\left[R_{i,t} - E(R_i)\right]^2$ from which the standard deviation formula $\sigma_i = \sqrt{\sigma_i^2}$ can be easily derived.

3.1 Stochastic Dominance

The most popular order of stochastic dominance relations is the first-order (FSD), the second-order (SSD) and the third-order (TSD). In this paper, only the first two variants are considered. Let A be a random variable and $F_A(x)$ its distribution function.

3.1.1 First-order Stochastic Dominance

The FSD can be defined as followed: random variable A first order stochastically dominates the random variable B, written $A \succ_{FSD} B$, if for any x applies

$$\Pr(A > x) \geq \Pr(B > x) \tag{1}$$

where $\forall x \in R$ and there is at least one x for which a strong inequality applies.

Another option is to use investor's preferences which can be represented by a utility function. Firstly, let define F_A and F_B are two cumulative distribution functions of the return. Thus, the FSD for distribution function can be defined as:

$$F_A(x) \leq F_B(x) \tag{2}$$

where there are at least one x for which a strict inequality applies. As FSD relates to non-decreasing utility function $U \in U_1$, it can be defined as

$$E_A[U(x)] \geq E_B[U(x)] \tag{3}$$

for all $U \in U_1$ with a strong inequality for at least one $U_0 \in U_1$. The preference ranking is reversed for decreasing utility function.

3.1.2 Second-order Stochastic Dominance

The integrated distribution function can define as

$$F_A^{(2)}(x) \leq \int_{-\infty}^{x} F_A(z)dz \tag{4}$$

Given the previous equation, the random variable A second order stochastically dominates the random variable B, written $A \succ_{SSD} B$, if for any x applies

$$\int_{-\infty}^{x} F_A(z)dz \leq \int_{-\infty}^{x} F_B(z)dz \tag{5}$$

where there is at least one x for which a strong inequality applies. This relation is sometimes called a weak SSD (Kopa, MME 2019). For the increasing utility function, the equation is identical as equation (3), however the equation can be adjusted to form

$$\int_{\alpha}^{x} [F_B(z) - F_A(z)]dz \geq 0 \qquad \forall x \in [a,b] \tag{6}$$

3.2 Correlation Measures

The first and the commonly used indicator is the Pearson coefficient of correlation which is based on linear dependency, see Tsay (2010). The formulation of the linear Pearson correlation coefficient between two random variables X and Y can be defined as:

$$\rho_{x,y} = \frac{Cov(X,Y)}{\sigma(X)\cdot\sigma(Y)} = \frac{\sum_{t=1}^{T}[x_t - E(x)][y_t - E(y)]}{\sqrt{\sum_{t=1}^{T}[x_t - E(x)]^2 \cdot \sum_{t=1}^{T}[y_t - E(y)]^2}} \tag{7}$$

where $E(x)$ and $E(y)$ are the sample mean of X and Y, respectively, and it is assumed that the variances exist.

The Spearman coefficient of correlation (Spearman′s rho) is defined as:

$$\rho_{S;x,y} = \frac{Cov(rX,rY)}{\sigma(rX)\cdot\sigma(rY)} \tag{8}$$

where rX is rank of variable X, rY is rank of variable Y, $\sigma(rX)$ and $\sigma(rY)$ are the standard deviations of the rank variables. If all n ranks are distinct integers, it can be used the formula

$$\rho_{S;x,y} = 1 - \frac{6\sum d_i^2}{n(n^2-1)} \tag{9}$$

where d_i is the difference between the ranks of corresponding variables.

As the third in order, the Kendall correlation coefficient (Kendall′s tau) is defined. Let`s consider two random variable vectors where each vector size is n then we know that the total number of pairings is $n(n-1)/2$. The formula used to the calculation is

$$\rho_{\tau;x,y} = \frac{n_c - n_d}{\frac{1}{2}n(n-1)} \tag{10}$$

where n_c is the number of concordant pairs, n_d is the number of discordant pairs.

3.3 Portfolio Model and Selected Performance Measures

Whereas the comparison is based on the different types of correlation measures ρ, the portfolio model needs to be adapted accordingly.

Before using this model, it is necessary to calculate variables that are inserted into the model. As stated by Ortobelli et al. (2015), if $\boldsymbol{x} = [x_1, x_2, \dots, x_n]$ is vector of asset weights, If $\boldsymbol{r} = [r_1, r_2, \dots, r_n]$ is vector of gross returns, then the portfolio dispersion is formulated as

$$d_{\rho,\sigma}(\boldsymbol{x}'\boldsymbol{r}) = \sqrt{\boldsymbol{x}'\boldsymbol{Q}_{\rho,\sigma}\boldsymbol{x}} \tag{11}$$

where $\boldsymbol{Q}_{\rho,\sigma} = \left[\sigma_{r_i}\sigma_{r_j}\rho_{i,j}\right]$, for correlation coefficient $\rho_{i,j}$ is still used Pearson correlation coefficient, but the deviation measures $\sigma_x = CVaR_{0.05}[X - E(X)]$.

A modified portfolio optimization problem can be defined as the following formulation:

$$\begin{gathered}\arg\max \frac{E(\boldsymbol{x}'\boldsymbol{r})}{d_{\rho,\sigma}(\boldsymbol{x}'\boldsymbol{r})}\left[\rho_1\left(\boldsymbol{x}'\boldsymbol{r}, \max_i \boldsymbol{r}_i\right) - \rho_2\left(\boldsymbol{x}'\boldsymbol{r}, \min_i \boldsymbol{r}_i\right)\right] \\ \sum_{i=1}^{n} x_i = 1 \\ x_i \geq 0, \qquad \text{for } i = 1,2,\dots,\mathrm{n}\end{gathered} \tag{12}$$

where three different factors of the type $\left[\rho_1\left(\boldsymbol{x}'\boldsymbol{r}, \max_i \boldsymbol{r}_i\right) - \rho_2\left(\boldsymbol{x}'\boldsymbol{r}, \min_i \boldsymbol{r}_i\right)\right]$ are considered expressing various relations between portfolio return and the upper and lower bounds, more characterized in subsection 3.2. The correlation coefficient ρ_i is of the same type, which causes that $(\rho_1 - \rho_2) > 0$.

Generally, the comparison criteria are based on performance indicators. Commonly used indicators are final wealth, mean return, mean standard deviation, Sharpe ratio, Rachev ratio, and Value-at-Risk, see among others Sharpe (1994), Biglova et al. (2004), Kresta (2016).

Another permissible risk measure focusing on the potential extreme losses can be express through the modified Sharpe ratio (MSR), see Ardia et al. (2015). In this indicator is included the skewness and kurtosis of the return distribution. The formula of MSR is:

$$MSR = \frac{E(R_p) - r_f}{MVaR_\alpha}, \tag{13}$$

where the $MVaR_\alpha = E(R_p) - \zeta \cdot \sigma_p$ and ζ is the Cornish-Fisher asymptotic expansion for the quantile of a non-gaussian distribution defined as following formula:

$$\zeta = \left\{Q_\alpha + \frac{1}{6}\left[{Q_\alpha}^2 - 1\right]S_p + \frac{1}{24}\left[{Q_\alpha}^3 - 3Q_\alpha\right]K_p - \frac{1}{36}\left[2{Q_\alpha}^3 - 5Q_\alpha\right]S_p^2\right\}, \tag{14}$$

where Q_α is the quantile of the return distribution, S_p is skewness of portfolio return, K_p is kurtosis of portfolio return.

4. Empirical Analysis

The content of this section is a description of used data, but primarily focusing on empirical analysis where the approaches and methods mentioned in the previous section are applied.

4.1 Data Description

For the empirical analysis, daily adjusted close prices of stocks included in indices DAX, FTSE 100, Hang Seng, NASDAQ-100, and NIKKEI 225, traded on the German, UK, Chinese, US, and Japanese stock markets are used[2]. These particular indices also represent the benchmarks used for the final comparison. The length of the time period starts from January 1, 2014, to December 31, 2019, i.e. 6-year horizon. For performance indicator calculations, the risk-free rates expressed by 10 years bond yield of each market are necessary[3] (7 years bond yield for the Chinese market) to date December 31, 2019. Several stocks time series are not included in the analysis due to the incomplete data.

4.2 Application of SD for Stock Selection

At the beginning of the empirical analysis, the application of FSD and SSD approaches for the selection of the most suitable stocks included in particular indices are provided. The main reason for applying these methods is to reduce the number of investment instruments according to the dominance of the time series of returns. Filtering out inappropriate stocks will also reduce the time of calculating weights in the optimal portfolio.

The total number of stocks included in the indices, the number of stocks eligible under the FSD, as well as the number of stocks eligible under the SSD are in the Table 1.

Table 1: Stock selection by stochastic dominance approach

	Total	FSD	SSD	SSD/FSD
DAX	29	29	5	0.1724
FTSE 100	89	89	9	0.1011
Hang Seng	47	47	10	0.2127
NASDAQ-100	98	98	13	0.1327
NIKKEI 225	204	204	18	0.0882

[2] Available from the Yahoo Finance website: https://finance.yahoo.com/
[3] Available from the Trading Economics website: https://tradingeconomics.com/

It is apparent from the values obtained in Table 1 that the application of the FSD approach does not affect the selection of stocks, due to the same amount of stocks as in the whole index. However, in the second case, when the SSD method is used, a sharp reduction in the number of considered stocks is seen, such as from 204 to 18 in the case of the Japanese stock index, which represents about 9%, or a reduction to 10% in the case of UK stocks.

Before compiling the portfolio, it is advisable to compare the performance measures founded as the mean value of the particular stock's indicators contained in the particular indices. The results can be found in the Table 2.

Table 2: Parameters computed from daily returns of particular stocks included in market indices and preselected stocks by SD

	Total / FSD						SSD					
	E[R]	σ	*SR*	*MSR*	*RR*	$VaR_{0.05}$	*E[R]*	σ	*SR*	*MSR*	*RR*	$VaR_{0.05}$
DAX	$3\text{x}10^{-4}$	0.0158	0.0219	0.0097	0.9885	0.0241	$7\text{x}10^{-4}$	0.0128	0.0548	0.0291	0.9906	0.0202
FTSE 100	$5\text{x}10^{-5}$	0.0362	0.0189	0.0099	0.9595	0.0241	$7\text{x}10^{-4}$	0.0122	0.0552	0.0295	1.0402	0.0181
Hang Seng	$3\text{x}10^{-4}$	0.0169	0.0103	0.0057	1.0672	0.0258	$7\text{x}10^{-4}$	0.0159	0.0331	0.0184	1.0470	0.0240
NASDAQ-100	$6\text{x}10^{-4}$	0.0182	0.0332	0.0170	0.9652	0.0273	$1\text{x}10^{-3}$	0.0146	0.0629	0.0329	0.9918	0.0216
NIKKEI 225	$2\text{x}10^{-4}$	0.0193	0.0111	0.0061	0.9909	0.0294	$7\text{x}10^{-4}$	0.0169	0.0386	0.0214	1.0345	0.0260

Already at the first comparison of the basic time series indicators ($E[R]$ and σ), it is visible that these values have improved using the SSD approach in all markets, which is desirable for diversification of financial assets. The reduction in the risk level is evident from the decrease in the $VaR_{0.05}$ value. Looking at the values of the *SR* and *MSR*, the selection of assets also helps to improve these indicators, due to higher values.

Overall, this choice can be summarized as beneficial and then, it is acceptable that the stocks selected by SSD should be considered for further analysis in portfolio theory.

4.3 Application of Portfolio Model with Particular Correlation Coefficient

In the following part of the analysis, selected stocks from the previous subsection are used for creating portfolios by the modified portfolio model characterized by formula (12). To include the risk measures into the model, the relationship from the equation (11) is used, where mentioned correlation coefficients from subsection 3.2 are varied.

It is assumed that at the beginning of each month the portfolio is re-optimized, i.e. the monthly back-testing procedure with the one-year rolling window is applied. For the selected six-year time periods, the daily returns of the first year are used for optimizing the created portfolio, where the investment itself begins on the first trading day of the following year. The investment will last until the end of 6 years. In relation to the investor preferences, it is assumed aversion to the risk. In all portfolio investments, the amount of initial wealth W_0 is equal to 1.

The determined values of particular portfolio performance measures with respect to different correlation coefficients are recorded in Table 3. The differences in the mean returns of the portfolio $E[R_p]$ are very small, due to that these values are in the low order. However, in the long-time horizon, the effect of different returns can be seen, causing the differences in the final W. In most cases, the Kendall correlation in the portfolio model seems to be the most suitable, with regard to the values of the W, $E[R_p]$ or σ_p, similarly as in Ortobelli et al. (2015). For US stock market, the Pearson coefficient is optimal when comparing return, but when comparing return with risk, the conclusion is different. Nevertheless, the differences between Spearman and Kendall correlation in portfolio optimization are very slight. This conclusion is primarily based on a comparison of W, *SR*, *MSR*, and $VaR_{0.05}$. Interestingly, when comparing RR, the summary is the opposite of that based on other indicators.

If a specific market for investment were to be recommended, the US market would be rationally considered before the UK market. The investment is valued about 3.5 times in the US market, and over 2 times in the UK market. However, it can be observed that the profitability of the US market in the case of the Pearson coefficient significantly exceeds other cases.

Table 3: Results of application particular correlation coefficients in portfolio model

	W	*E[R_p]*	σ_p	*SR*	*MSR*	*RR*	$VaR_{0.05}$
Pearson correlation coefficient in portfolio model							
DAX	2.0028	0.0006	0.0130	0.0494	0.0273	1.0267	0.0204
FTSE 100	2.0710	0.0006	0.0119	0.0516	0.02631	0.9967	0.0170
Hang Seng	1.6810	0.0006	0.0182	0.0256	0.0134	0.9905	0.0253
NASDAQ-100	6.5641	0.0017	0.0196	0.0820	0.0436	1.0569	0.0274
NIKKEI 225	3.4749	0.0011	0.0167	0.0682	0.0373	1.0377	0.0252
Spearman correlation coefficient in portfolio model							
DAX	2.3499	0.0007	0.0098	0.0745	0.0383	0.9623	0.0151
FTSE 100	2.4683	0.0008	0.0091	0.0794	0.0393	1.0389	0.0141
Hang Seng	2.3698	0.0008	0.0122	0.0536	0.0288	0.9748	0.0193
NASDAQ-100	3.6726	0.0011	0.0110	0.0925	0.0492	1.0102	0.0169
NIKKEI 225	1.9889	0.0006	0.0130	0.0491	0.0257	0.9443	0.0202
Kendall correlation coefficient in portfolio model							
DAX	2.3740	0.0007	0.0098	0.0755	0.0387	0.9623	0.0151
FTSE 100	2.5085	0.0008	0.0088	0.0837	0.0458	1.0674	0.0139
Hang Seng	2.4046	0.0008	0.0115	0.0569	0.0304	0.9742	0.0190
NASDAQ-100	3.4223	0.0010	0.0112	0.0862	0.0445	0.9892	0.0168
NIKKEI 225	2.1188	0.0007	0.0124	0.0549	0.0287	0.9220	0.0190

Given that the stocks contained in the indices are compared, it is permissible and correct to compare the results of the model with the indices that can be defined as a benchmark.

Table 4: Benchmark's performance indicators

	W	*E[R_p]*	σ_p	*SR*	*MSR*	*RR*	$VaR_{0.05}$
DAX	1.2350	0.0002	0.0110	0.0213	0.0113	0.9082	0.0183
FTSE 100	1.1487	0.0001	0.0087	0.0083	0.0073	0.9740	0.0137
Hang Seng	1.1162	0.0002	0.0111	0.0026	0.0014	0.9034	0.0184
NASDAQ-100	1.8889	0.0006	0.0102	0.0473	0.0244	0.8396	0.0167
NIKKEI 225	1.2512	0.0003	0.0120	0.0211	0.0109	0.8661	0.0197

The comparison with the benchmark (Table 4) is surely influenced most significantly by the absence of transaction costs in the portfolio models calculations. It can be observed that the $E[R_p]$ of benchmark investment is lower which leads to approximately half the value of the W with slightly higher values of risk indicators i.e. σ_p or $VaR_{0.05}$ and low values of SR as well as MSR and RR. For this reason, the investor would be advised to use the SD approach for asset selection instead and then use the portfolio model to optimize the portfolio in this situation.

5. Conclusion

The first objective of this paper is an application of SD approach for selection of eligible assets and the second contribution is empirical analysis on the influence of different types of correlation measures in the modified portfolio model in different world markets and comparison with the benchmarks.

After the application of FSD, the number of stocks did not change from the amount in the whole index. However, in the case, when the SSD method was used, a sharp reduction in the number of considered stocks were seen. It could be observed that all performance and risk indicators had improved in all markets after using the SSD approach, that is desirable for diversification of financial assets.

In most cases, the Kendall correlation in the portfolio model was the most suitable, but in US market, using the Pearson correlation was appropriate, but the differences were very slight. Based on the results and rational thinking of the investor, the US market should be recommended before the UK market. While the results are compared with benchmarks, the expected return of portfolio investment is higher which leads to approximately half the value of the final wealth with slightly lower values of risk indicators.

Acknowledgements
This work was supported by the VSB-TU Ostrava under the SGS project SP2020/11.

References

[1] ARDIA, D., BOUDT, K. (2015). Testing equality of modified Sharpe ratios. *Finance Research Letters*, 13, 97-104.

[2] BAWA, V. S. (1978). Safety-First, Stochastic Dominance, and Optimal Portfolio Choice. *The Journal of Financial and Quantitative Analysis,* 13(2), 255–271.

[3] BIGLOVA, A., ORTOBELLI, S., RACHEV, S.T., STOYANOV, S. (2004). Different Approaches to Risk Estimation in Portfolio Theory. *The Journal of Portfolio Management,* 31(1), 103-112.

[4] DENTCHEVA, D., RUSZCZYŃSKY, A. (2006). Portfolio optimization with stochastic dominance constraints. *Journal of Banking & Finance*, 30(2), 433-451.

[5] FAMA, E. (1965). The behavior of stock market prices. *Journal of Business,* 38, 34–105.

[6] KONNO, H., YAMAZAKI, H. (1991). Mean-Absolute deviation portfolio optimization model and its applications to Tokyo stock-market. *Management Science*, 37(5), 591–531.

[7] KOPA, M. (2019). Second order stochastic dominance constraints in decision dependent randomness portfolio optimization problems. In *Houda* and *Remeš* (eds.). *The 37th conference Mathematical Methods in Economics 2019*. University of South Bohemia in České Budějovice.

[8] KOUAISSAH, N., ORTOBELLI, S., TICHÝ., T. (2018). *Portfolio Theory and Conditional Expectations: Selected Models and Applications.* SAEI, vol. 59, VŠB-TU Ostrava. ISBN 978-80-248-4343-8.

[9] KRESTA, A. (2016). *Kvantitativní metody investování s aplikacemi v prostředí Matlab*. SAEI, vol. 43, VŠB-TU Ostrava. ISBN 978-80-248-3898-4.

[10] MARKOWITZ, H. (1952). Portfolio Selection. *The Journal of Finance,* 7(1), 77–91.

[11] NEDĚLA, D. (2020). Data Analysis and Testing with Respect of Portfolio Selection Problem. In *Branžovský*, J. and J. *Pavelek* (eds.). *Proceedings of the 22nd International Conference MEKON 2020.* VSB – TU Ostrava.

[12] MANDELBROT, B. (1963). New methods in statistical economics. *Journal of Political Economy,* 71(5), 421-440.

[13] ORTOBELLI, S., TICHÝ, T. (2015). On the impact of semidefinite positive correlation measures in portfolio theory. *Annals of Operations Research*, 235(1), 625-652.

[14] SHARPE, W.F. (1994). The Sharpe ratio. *Journal of Portfolio Management*, 21, 49-58.

[15] TSAY, R.S. (2010). *Analysis of Financial Time Series. 3rd ed.* Cambridge, Mass.: Wiley, 2010. ISBN 978-0-470-41435-4.

[16] URYASEV, S. (2000). Conditional value-at-risk: optimization algorithms and applications. *Financial Engineering News,* 14, February 2000.

[17] WOLFESTETTER, E. (1996). *Stochastic Dominance: Theory and Applications*. SFB 373 Discussion Papers from Humboldt University of Berlin, No 1996, 40.

Comparison of Multinomial Rating Models Based on Used Methods and Datasets

Martina Novotná [1]

Abstract

Some various approaches and techniques can be used to develop multinomial rating models. In this study, we applied discriminant and logistic regression analysis. In the case of discriminant analysis, we used linear, quadratic and logistic approach. As far as the second approach, we applied the methods of multinomial and ordinal logistic regression analysis. In this study, two ways were used and compared to create an experimental and hold-out sample. Based on the overall results, it can be concluded that higher classification accuracy is associated with a lower number of rating categories as dependent variables and a higher number of financial, or independent variables. Furthermore, it also appeared that when we estimated models based on the previous years, and then we used these models to predict a rating for the following year, we achieve a better classification ability.

Keywords

Credit rating, discriminant analysis, logistic regression analysis, model, prediction

JEL Classification: C52, C53, G24

1. Introduction

The main principle of rating models is to give the necessary information about the company's operating characteristics; to identify alert factors and assign a corresponding rating. With the aid of this assessment, comparison among companies can be made, and the detection of problematic companies can be performed. Rating models are quantitative models that provide rating typically based on publically available information, mostly on financial statements. By such means, it is possible to assess unquoted companies, compare them according to the credit risk, and identify those with the highest probability of default.

Some rating agencies focus almost exclusively on quantitative data, which they incorporate into a mathematical model. Thus, a well-estimated model partially fulfils the role of rating agencies. Together with financial statement information, rating agencies consider other analytic areas such as management's reputation, reliability, experience, and past performance. While rating agencies emphasise that both financial and non-financial factors matter in the prediction of default and bond rating, the academic literature has focused primarily on the ability of economic indicators to predict ratings. However, recent studies have shown that other factors, such as the state and perspective of the overall economy or the industry characteristics, may improve the predictive ability of rating models.

The main objective of this paper is to estimate and compare rating models developed by the methods of discriminant and logistic regression analysis. The purpose of this study is to assess the importance of selected financial variables on rating prediction. Next, we also focus on the

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effect of the sample selection and the used method on the classification ability and adequacy of estimated models.

2. Overview of Approaches to Rating Modelling

Rating systems can be developed using different approaches and methods, such as expert-based approaches, statistical-based models, heuristic and numerical approaches. While expert-based techniques are usually based on the mix of both judgmental and model-based analyses, statistical-based models refer to quantitative financial models. Although these models are based on simplifying assumptions about the variable to be predicted, they present a useful way for accessing the creditworthiness of a borrower or prediction the probability of default. The purpose of traditional statistical approaches is to distinguish between 'good' and 'bad' borrowers in terms of their creditworthiness in an automated way. Financial institutions widely use these techniques for retail or SME loans, and scoring models typically represent them. The main principles of scoring models can also be applied to the problem of rating to distinguish between 'more' and 'less' credit risk in terms of rating categories. Although some authors argue that credit scoring is not highly sophisticated approach (i.e. De Servigny and Renault, 2004), it is undoubtedly widely used approach for assessing the credit risk that can be applied on any borrower.

Statistical-based models, sometimes referred to as the traditional way for credit risk modelling, are widely applied by rating agencies and financial institutions to access the credit risk of bonds and corporate loans. Since they are based on a judgmental approach supported by specific critical financial numbers, these models can be considered as fundamental-based models. In addition to these traditional models, alternative and more quantitative methods were developed to model and measure credit risk, for example, the Merton model. As Crouhy et al. (2014) emphasise, these models complement, and to a degree compete with traditional approaches to measuring credit risk and offer an independent check on judgment ratings. Firm-specific or fundamental-based credit risk models are primarily based on the company's financial reports. The main principle of this approach is to find key statistics that can be used for the prediction of the likelihood that cash flow generated from the firm's assets will be sufficient to cover its debt obligations (Moody's Analytics, 2012). This approach relies on historical values that are recorded and carried over from one period to the next so that historical values may differ from current or expected values, and the company's prospects may not be accurate in some cases. On the other hand, the advantage of these models lies in the fact that since financial reporting is subject to accounting rules, it increases their objectivity and comparability of financial ratios. Fundamentally oriented models are statistical-based models that provide a useful way for accessing the creditworthiness of a borrower or prediction the probability of default. In other words, the primary purpose of these models is to provide an automated rating or scoring system that can be used for assessing the credit risk.

The first models were developed using the linear discriminant analysis that is a method suitable for the problem of separation of groups. Currently, the models are mostly derived using the methods of logistic regression analysis, classification trees, random forests or neural networks. Regardless of the used method, the primary purpose is to develop an accurate model with a strong discriminating predictive ability from the input, multiple variables. As Royston et al. (2009) suggest, although there is no consensus on the ideal method for developing a model, adequate sample size and proper data play a crucial role. When reporting the final models, authors should pay attention to the details of data handling, modelling methods and especially to an exact coding and reporting of used predictors. The predictive performance or accuracy is also an essential part of model development. In general, the insufficient prediction is usually the result of an inappropriate methodology or weaknesses in the data; however, even with a high-quality model, there may be a too much-unexplained variation to generate accurate predictions. Thus, external validity is typically required to assess the quality of a final predictive model.

3. Description of Methodology

In this chapter, the essential characteristics of the methods used in this study are described. The aim is to provide a basic description of the methods used, while the work of the authors listed below can be used for a deeper understanding.

3.1 Discriminant Analysis

Discriminant analysis is a standard statistical method used for separation of groups. The description of discriminant analysis and techniques can be found for example in Rencher (2002), Manly (2004), Huberty and Olejnik (2006), Tabachnik and Fidell (2007) or Hair et al. (2014). The analysis can be used for two primary objectives: the description of group separation and the prediction or allocation of observations to groups. For example, Huberty and Olejnik (2006) distinguish between descriptive discriminant analysis (DDA) and predictive discriminant analysis (PDA). The purpose of DDA is usually the study of comparison among a certain number of groups for each of which we have several outcome variable scores. However, suppose the single set of response variables are used as predictors and there is a single grouping variable. In that case, the primary purpose is to analyse how well group membership of analysis units may be predicted, using PDA. Correspondingly, Rencher (2002) differentiates between discriminant and classification functions. Discriminant functions are those used to separate groups, while classification functions can be used to assign individual units to one or more groups. In the case of group separation, linear functions of variables are used to describe the differences between two or more groups, and the main objective is to identify the relative contribution of p variables to separation. The latter problem is focused on the prediction or allocation of observations to groups, which is a common goal of linear discriminant analysis (LDA). A prediction rule then consists of a set of linear combinations of predictors, where the number of combinations reflects the number of groups. Discriminant functions are linear combinations of variables that best separate groups, for example, the k groups of multivariate observations.

In addition to LDA, which is one of the well-known methods, we use quadratic and logistic methods of discriminant analysis in this study. Quadratic discriminant analysis (QDA) is a variant of LDA that allows for non-linear separation of data. Logistic discriminant analysis (LogDA) in which the posterior probabilities are estimated by multi-nominal logistic regression (MLR). The ability of classification procedure for group membership prediction is usually based on the probability of misclassification, or the error rate. The complement to the error rate is called the correct classification rate (Rencher, 2002). The classification procedure can be carried out using the same data that has been used to compute the classification functions; then the method is called resubstitution. The results are typically shown in a classification table.

2.2 Logistic Regression Analysis

In finance, logistic regression is mostly used in its bivariate context; however, it can be easily modified for outcome variable with more than two possible values. When exploring relationships among rating and firms' indicators, multinomial logistic regression should be applied, since there are more than two categories of the dependent variable. In this case, the number of categories comes from the number of rating groups. There is a vast literature on logistic regression methods, for example, Hosmer, Lemeshow and Sturdivant (2013), Menard (2010), Harrel (2010), or Tabachnik and Fidell (2007).

Usually, if we intend to describe the relationship between an outcome (dependent) variable and a set of independent (predictor or explanatory) variables, logistic regression is a suitable method. Hosmer et al. (2013) distinguish among several types of logistic regression models according to the number of variables used in the model. The simplest binary model contains only one independent variable and a dependent variable with two possible outcome values. If we consider more than one independent variable in the model with two possible outcomes, then the model is called the multiple, or multivariable logistic regression model. This model can be further modified for the outcome variable with more than two levels of responses, and it is called multinomial, polychotomous, or polytomous logistic regression model. Moreover, in the case that the outcome is an ordinal scale, we can use ordinal logistic regression.

The dichotomous logistic model of a single independent variable can be further generalised for more than one independent variable. If we assume a collection of p independent variables, then the model is called the multiple logistic regression model. The model with more than two dependent categories is called multinomial, or polychotomous. Generally, the multinomial logistic regression can be used if the outcome variables are not nominal but ordinal scale. However, it is suggested to preferably use the ordinal logistic regression, which respects the natural ranking of the categorical outcome. Menard (2010) describes ordinal variables as either crude measurement of a variable that could be measured on an interval or ratio scale or measurement of an abstract characteristic for which there is a no natural metric or unit of measurement. Hosmer et al. (2013) argue that even in these cases, the multinomial logistic regression could be used. Still, it must be realised that not taking into account the natural ordering of outcome variable may lead to the problem that estimated models might not address the questions asked of the analysis. Thus, the ordinal logistic regression seems to be an appropriate method for assessing bond rating that allows taking into account the rank ordering of rating categories. As Menard (2010) says, different models, make different assumptions about whether the dependent variable is intrinsically ordinal or reflects an underlying continuous interval or ratio variable. Various models can be proposed for the analysis of ordinal dependent variables, such as the cumulative logit model, the continuation ratio logit model, the adjacent categories logit model or the stereotype model. Since the cumulative logit model is the most widely used logistic regression model, it is also used in this study.

4. Comparison of Rating Models

Firstly, the used dataset is described, and then the attention is paid to the comparison of estimated models.

4.1 Description of Used Datasets

This study is focused on the analysis of corporate credit rating from eight countries from Central and Eastern Europe (CEE): Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Slovakia, and Slovenia. Although the data come from the period 2002-2008, we can use them for the analysis of rating and the main factors of rating prediction. The dataset includes 1249 very large and large industrial companies from mining, manufacturing and construction sector. We record companies characteristics over the observed period, depending on the year when the company was established or firstly rated. Thus, we have several observations available for each company according to individual years, which gives a total of 6,646 observations with an assigned rating for all years. For model estimation, the dataset is divided into an experimental and hold-out sample, where the experimental sample is used for estimation and the hold-out sample for validation. The partial aim of this study is to compare the models developed based on two approaches of data splitting, random and non-random.

Table 1 summarises the first, random way of dataset split. All observations for all companies and years are randomly divided into the experimental sample (83%) and hold-out, or validation sample (17%). It is a commonly used way of dividing a base data sample used in many studies.

Table 1: Experimental and hold-out sample I (random)

Rating	Code	Experimental sample		Hold-out sample	
		Number of observations	Percentage	Number of observations	Percentage
B	1	277	5.51	91	5.62
BB	2	893	17.76	290	17.92
BBB	3	2353	46.80	719	44.44
A	4	1212	24.11	414	25.59
AA	5	292	5.83	104	6.43
Total		5028	100	1618	100

Table 2 shows the second way of the splitting of the original sample. In this case, the experimental sample (84.3%) contains observations from the years 2002-2007 and the hold-out sample (15.7%) data from the year 2008. Although the hold-out sample does not have observations in the outer categories B and AA, it will be used for validation as it reflects the actual rating distribution of the firms in this particular year. Compared to the first method, this division is given by the year of observation, and for this study, the data set will be marked as non-random.

Table 2: Experimental and hold-out sample II (non-random)

Rating	Code	Experimental sample		Hold-out sample	
		Number of observations	Percentage	Number of observations	Percentage
B	1	368	6.57	0	0
BB	2	1183	21.12	644	61.63
BBB	3	2428	43.35	312	29.86
A	4	1314	23.46	89	8.52
AA	5	308	5.5	0	0
Total		5601	100	1045	100

4.2 Estimation and Comparison of Models

We use discriminant analysis and logistic regression analysis for the estimation of rating models. To compare the adequacy and prediction ability of estimated models, we develop several models with different characteristics (Table 3).

Table 3: Summary of rating models

	Discriminant analysis	Logistic regression analysis
Method	Linear Quadratic Logistic	Multinomial Ordinal

Dataset	Random Non-random	Random Non-random
Number of categories (rating groups)	3 5	3 5
Number of predictors (variables)	7 10	7 10

Regardless of the used method, we develop models using both experimental samples marked as random and non-random, as described in the above text. Since the number of outer rating categories (AA, B) is very low compared to others, we also estimate 3-category rating models for the middle classes only (A, BBB, BB).

The potential association between each independent variable and rating was firstly verified through the univariable analysis ($p<0.2$). In this study, we consider ten financial variables as predictors with a potential effect on rating: logarithm of total assets (lnta), return on assets (roa), return on equity (roe), equity to total assets (eqta), logarithm of interest coverage (lnintcov), logarithm of liquidity ratio (lnliqr), logarithm of cash flow (lncf), logarithm of current ratio (lncurr), logarithm of long-term debt to total assets (lnltdta) and the percentage of ebitda to total debt (ebitdar). In order not to include variables in the model that would interact with each other, based on the correlation analysis, three variables were removed (roa lncf, lnliqr) and seven of the original ten variables were left. The models are estimated using both sets of predictors for comparison (see Table 3).

a) Discriminant models

The main characteristics of the estimated models are summarised in Annex 1. According to the criterion of the overall error rate, we consider the model estimated through the logistic discriminant analysis based on 3 rating groups, ten variables and random sample as the one with the minimum value. The same model achieves the maximum classification ability on the estimation sample; however, if we are interested in the classification on the hold-out sample, then we choose the logistic model estimated based on non-random sample. Thus, the model developed by the non-random sample is more appropriate for rating prediction for the following year.

b) Logistic models

The main characteristics of logistic models are summarised in Annex 2. According to the criterion of Pseudo R^2, we find the MLR model based on 3 rating groups, ten variables and non-random sample as the one with the maximum value. The maximum classification ability on the hold-out sample is achieved by the MLR model based on three groups, ten variables and non-random sample. Therefore, we can say that the model developed by the non-random sample is more appropriate for rating prediction, similarly to the discriminant analysis.

5. Conclusion

The main objective of this study was to use selected methods to estimate rating models and to verify the influence of used financial variables on the rating. The partial goal was to compare the models estimated within the individual approaches and also based on different input assumptions concerning the number of input (independent) variables, the number of output (dependent) variables and the use of the experimental data set. By combining various assumptions, there were estimated 24 discriminant and 16 logistic models in total. There are different ways to compare the estimated models. In this study, the main comparative criterion

was the classification ability of the models using a hold-out sample. Besides, in the case of discriminant analysis, the error rate value was used; and in the case of logistic models, the pseudo R^2 was used as additional criteria for comparison.
Based on the overall results, it can be said that the best classification ability is achieved by 3-group models containing all input variables, estimated based on a non-random sample. These models were derived using LogDA, MLR and LDA and their classification ability are very similar. On the other hand, OLR 5-group models with seven variables have the worst classification ability, while the choice of sample does not affect this predictive ability. Among all models, the difference between the best and the worst correct classification is almost 15 percentage points. Based on the overall results, it can be concluded that higher classification accuracy is associated with a lower number of rating categories as dependent variables and a higher number of financial, independent variables. Furthermore, it also appeared that when we estimated models based on the previous years, and then we used these models to predict a rating for the following year, we reached better classification ability.
Based on the results, it can be argued that not only the number of input variables and the choice of the method used but also the process of original sample splitting, which we use in this study, has an impact on the classification ability of final models.

Acknowledgement

This paper was supported by the Operational Programme for Competitiveness under Project CZ.1.07/2.3.00/20.0296 and Project SGS 2020/124.

References

[1] CROUHY, M., GALAI, D., MARK, R. (2014). *The Essentials of Risk Management*. New York: McGraw-Hill Education.
[2] De SERVIGNY, A., RENAULT, O. (2004). *Measuring and Managing Credit Risk*. New York: McGraw-Hill.
[3] HAIR, J. F., BLACK, W. C., BABIN, B. J., ANDERSON, R. E. (2014). *Multivariate Data Analysis*. Harlow: Pearson.
[4] HARRELL, F. E. (2010). *Regression Modeling Strategies: with Applications to Linear Models, Logistic Regression, and Survival Analysis.* New York: Springer.
[5] HOSMER, D. W., LEMESHOW, S., STURDIVANT, R. X. (2013). *Applied Logistic Regression*. Hoboken: Wiley.
[6] HUBERTY, C. J., OLEJNIK, S. (2006). *Applied MANOVA and Discriminant Analysis*. Hoboken: John Wiley & Sons.
[7] MANLY, B. F. J. (2005). *Multivariate Statistical Methods: A Primer*. Boca Raton: Chapman & Hall/CRC.
[8] MENARD, S. W. (2010). *Logistic Regression: From Introductory to Advanced Concepts and Applications*. Los Angeles: Sage Publications
[9] Moody's Analytics (2012). *Public Firm Expected Default Frequency (EDF™) Credit Measures: Methodology, Performance, and Model Extensions*. [online] Available at: https://www.moodys.com/researchdocumentcontentpage.aspx?docid=PBC_143179 [Accessed 12 October, 2017]
[10] RENCHER, A. C. (2002). *Methods of Multivariate Analysis*. Hoboken: John Wiley & Sons.
[11] TABACHNICK, B. G., FIDELL, L. S. (2007). *Using Multivariate Statistics*. Boston: Pearson Allyn & Bacon.

Annexes

Annex 1: Discriminant models

Method	Number of cat.	Number of var.	Experimental sample (ES)	Number of obs. (ES)	Errorrate	Class. (ES)	Class. (hold)
LDA	3	10	non-random	3222	0.1142	0.8858	0.8986
LDA	3	7	non-random	3232	0.1782	0.8218	0.8507
LDA	3	10	random	3004	0.1122	0.8878	0.8882
LDA	3	7	random	3013	0.1696	0.8304	0.8198
LDA	5	10	non-random	3518	0.1489	0.8511	0.8775
LDA	5	7	non-random	3533	0.2273	0.7727	0.8043
LDA	5	10	random	3274	0.1476	0.8525	0.8600
LDA	5	7	random	3286	0.2158	0.7842	0.7661
LogDA	3	10	non-random	3222	0.094	0.9060	0.9096
LogDA	3	7	non-random	3232	0.1358	0.8642	0.8603
LogDA	3	10	random	3004	0.0895	0.9105	0.8977
LogDA	3	7	random	3013	0.1331	0.8669	0.8556
LogDA	5	10	non-random	3518	0.1154	0.8846	0.8801
LogDA	5	7	non-random	3533	0.1661	0.8339	0.8232
LogDA	5	10	random	3274	0.1139	0.8861	0.8851
LogDA	5	7	random	3286	0.168	0.8320	0.8287

QDA	3	10	non-random	3222	0.1996	0.8004	0.8781
QDA	3	7	non-random	3232	0.2048	0.7952	0.8685
QDA	3	10	random	3004	0.1937	0.8063	0.8006
QDA	3	7	random	3013	0.1952	0.8048	0.7914
QDA	5	10	non-random	3518	0.2166	0.7834	0.8636
QDA	5	7	non-random	3518	0.2166	0.7834	0.8636
QDA	5	10	random	3274	0.2126	0.7874	0.7857
QDA	5	7	random	3286	0.2258	0.7742	0.7700

Annex 2: Logistic models

Method	Number of cat.	Number of var.	Experimental sample (ES)	Number of obs. in ES	Pseudo R^2	Class. (ES)	Class. (hold)
OLR	3	7	random	3013	0.5627	0.8890	0.8681
OLR	3	7	non-random	3244	0.7031	0.8178	0.8082
OLR	3	10	non-random	3222	0.7244	0.8818	0.8882
OLR	3	10	random	3004	0.7300	0.8986	0.8811
OLR	5	7	non-random	3533	0.5325	0.7816	0.7642
OLR	5	7	random	3286	0.5300	0.7687	0.7613
OLR	5	10	random	3274	0.6988	0.8494	0.8600
OLR	5	10	non-random	3518	0.7021	0.8649	0.8468
MLR	3	7	non-random	3232	0.6576	0.8603	0.8642
MLR	3	7	random	3013	0.6586	0.8669	0.8556
MLR	3	10	non-random	3222	0.7706	0.9096	0.9060
MLR	3	10	random	3004	0.7730	0.9105	0.8977
MLR	5	7	non-random	3533	0.6582	0.8232	0.8339
MLR	5	7	random	3286	0.6565	0.8320	0.8287
MLR	5	10	random	3274	0.7537	0.8861	0.8851
MLR	5	10	non-random	3518	0.7583	0.8801	0.8850

Alternative derivation of integral method for quantifying effects of component ratios within pyramidal decomposition

Barbora Ptáčková, Jiří Valecký[1]

Abstract

The paper deals with the integral method for quantifying effects of change of component ratio on the change of the base ratio within pyramidal decompositions. We derived the integral method and, using artificial data in the numerical example, we demonstrate the limitations of this method. Specifically, we show that under certain conditions the method evaluates effects that are not coherent with the observed change of component ratio. In addition, we also identified that under the same conditions, the effects of all component ratios vary as well as their order of importance. Therefore, we provide an alternative derivation of the integral method that handles all these limitations and significantly facilitates the interpretation of the results.

Key words

Du Pont decomposition, financial analysis, pyramidal decomposition, analysis of deviance, integral method.

JEL Classification: C02, G3, G32

1. Introduction

The pyramidal decompositions are an integral part of the financial analysis, in particular when the profitability for shareholders is analysed, e.g. Du Pont analysis of Return on Equity (ROE) where the ROE is decomposed on several other ratios referred to as the component ratios. While the change of any component ratio yields an obvious change of the decomposed ratio, the observed change is given by the change of all component ratios simultaneously and the problem which ratio represents the main reason of such change arise.

The concept of decomposition of Return on Equity can be found with a number of authors. For example, Laitinen (1999) or Liesz (2002) considers decomposition of ROE as one of an important part for understanding the firm's profitability for shareholders. In 1918, F. Donaldson Brown recognized a mathematical relationship that existed between profitability measures and efficiency measure. This was the original Du Pont model. Gitman (1998) remarks that the attention shifted from ROA to ROE in the 1970s and this led to the first modification of the original Du Pont model. As Brigham and Houston (2001) noted, this modified Du Pont model became a standard for all financial management textbooks. The last modification was done by Hawawini and Viallet (1999), which yielded decomposition of ROE on five component ratios.

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However, the concept of Du Pont decomposition can be generalized and applied also on other financial indicators, such as Economic Value Added (EVA) which represents one of the modern financial performance indicators, see Ehrbar (1998), Maříková, Mařík (2005), Dluhošová (2010) or Laitinen (1999) for more details. The EVA is nowadays mostly preferred over traditional financial indicators, such as ROE or ROA, to evaluate the firm's financial performance due to the incapability of traditional financial indicators to provide a proper picture on the firm's financial performance for shareholders, Abdoli, Garkaz, Golami, Pourkazemi, (2011). Some specific application of EVA can be found in Richtarová (2016), Dluhošová, Ptáčková, Richtarová (2017).

Further, during the decomposition procedure, the various relationships must be considered, i.e. the additive, multiplicative or exponential relation. While the first two are very common, the third occurs rarely in economic practice.

To quantify the effect of component ratios on the base ratio, the analysis of deviation is very helpful, see Dluhošová (2010) or Zmeškal, Dluhošová, Tichý (2004). In contrast to an additive relationship where the effects are evaluated by the only method, the effects in case of multiplicative relationship can be determined by a method of gradual changes, a decomposition method with a residue, a logarithmic method, functional method or integral method, see Zmeškal, Dluhošová, Tichý (2004). While the effects of component ratios on base ratio have been calculated mostly according to the logarithmic or functional method, Dluhošová (2010) or Richtarová (2007), the integral method appeared to be preferred recently, see Dluhošová, Zmeškal (2014) or Gurný, Richtarová, Čulík (2017). However, this method is not referred to as uniformly. While some authors call it the integrand method, for instance, Dluhošová, Zmeškal (2014); the others, such as Gurný, Richtarová, Čulík (2017) as well as the authors of this paper prefer the term integral method.

Anyway, this method might become favourite for its simplicity, especially in contrast to the functional method, although the relative change of the base ratio is actually "roughly" approximated, yielding specific imperfections. We highlight in this paper that the function for the effect of given component ratio is not continuous, and we show that the evaluated effects are not always coherent, i.e. they do not correspond to the financial implications. For instance, increase (decrease) in the profit margin should yield a positive (negative) effect on the change of the ROE. Therefore, the goal of this paper is to propose an alternative integral method that solves these problems.

The remainder of the paper is organized as follows. The general pyramidal decomposition, as well as analysis of deviance, are described in Section 2. Section 3 focuses on the two derivation techniques of the integral method and the numerical example that shows the imperfection of original derivation is presented in Section 4. Section 5 provides the conclusions of this study.

2. Pyramidal decomposition and analysis of deviance

Thus, in financial practice, the base ratio x mathematically depends on the component ratios a_i, so $x = f(a_1, a_2, \ldots a_n)$. Then the change of base indicator can be expressed as

$$\Delta x = \sum_{j=1}^{n} \Delta x_j, \tag{1}$$

where Δx is the change of base ratio and Δx_j is the impact of the jth component ratio on the change of the base ratio. Note that it is possible to analyse absolute deviation, $\Delta x = x_1 - x_0$, as well as a relative deviation, $\Delta x / x_0$.

The function $f(\ )$ depends on the relationship among the component ratios. In general, we distinguish mostly the additive relationship and multiplicative relationship where the base ratio is given by $x = \sum_{j=1}^{n} a_j$ and $x = \prod_{j=1}^{n} a_j$ respectively. The exponential relation $x = a_1^{\prod_{j=2}^{n} a_j}$ may also occur, but it is rarely in financial practice.

While there is only one method for quantifying the effect of component ratios when the additive relation is considered, there exist several options in the multiplicative relation, i.e. method of the gradual change, a method of the decomposition with the surplus, logarithmic method, functional method and integral method that is addressed in this paper.

3. Derivation of the integral method

The basic idea of quantifying Δx_j involves the Taylor series, where the change of base ratio, Δx or $\Delta f(a_1, a_2, \ldots a_n)$ respectively, is approximated as follows

$$\begin{aligned}\Delta f(a_1, a_2, \ldots a_n) \approx \sum_{j=1}^{n} \frac{\partial f(\cdot)}{\partial a_j} \cdot \Delta a_j + \frac{1}{2!} \sum_{j=1}^{n} \sum_{k=1}^{n} \frac{\partial^2 f(\cdot)}{\partial a_j \cdot \partial a_k} \cdot \Delta a_j \cdot \Delta a_k + \ldots \\ + \frac{1}{3!} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} \frac{\partial^3 f(\cdot)}{\partial a_j \cdot \partial a_k \cdot \partial a_l} \cdot \Delta a_j \cdot \Delta a_k \cdot \Delta a_l + \cdots\end{aligned} \tag{2}$$

where only the first order is used for derivation of integral method. First, we reviewed the initial idea of derivation and then we propose another way how to quantify the effects of component ratios.

3.1 Original derivation

The initial idea for derivation is as follow

$$\Delta x = \frac{\Delta f\left(a_1, a_2, \ldots, a_n\right)}{\Delta f\left(a_1, a_2, \ldots, a_n\right)} \Delta x., \tag{3}$$

where the change of the function is approximated by Taylor series of the first order, thus

$$\Delta f\left(a_1, a_2, \ldots, a_n\right) \approx \sum_{j=1}^{n} \frac{\partial f(\cdot)}{\partial a_j} \cdot \Delta a_j . \tag{4}$$

Dividing by x_0, we get

$$\begin{aligned}\frac{\Delta f\left(a_1, a_2, \ldots, a_3\right)}{x_0} &= \frac{1}{x_0} \sum_{j=1}^{n} \frac{\partial f(\cdot)}{\partial a_j} \cdot \Delta a_j \\ &= \frac{\Delta a_1}{a_{1,0}} + \frac{\Delta a_2}{a_{2,0}} + \ldots + \frac{\Delta a_n}{a_{n,0}} = R_1 + R_2 + \ldots + R_n\end{aligned}.$$

Substituting that into

$$\Delta x = \frac{\Delta f\left(a_1, a_2, \ldots, a_n\right)}{x_0} \frac{x_0}{\Delta f\left(a_1, a_2, \ldots, a_n\right)} \Delta x., \tag{5}$$

we find that

$$\Delta x = \left(R_1 + R_2 + \ldots + R_n\right) \cdot \frac{1}{R'_x} \cdot \Delta x., \tag{6}$$

where $R'_x = \sum_{j=1}^{n} R_j$.

Particular effects assigned to factors are then

$$\Delta x_j = \frac{R_j}{R'_x} \cdot \Delta x, \text{ for } j = 1, 2, \ldots, n. \tag{7}$$

3.2 Alternative derivation

The alternative derivation of the integral method is also based on the first order of Taylor series approximation, but it is based on the different idea. Thus, the change of base ratio is firstly approximated by the Taylor series according to (4)

$$\Delta x \approx \Delta f\,(a_1, a_2, \ldots a_n)$$

and then it is expanded by $\frac{\Delta x}{\Delta x}$ and $\frac{x_0}{x_0}$, thus

$$\begin{aligned}
\Delta f\,(a_1, a_2, \ldots a_n) &= \Delta f\,(a_1, a_2, \ldots a_n) \frac{\Delta x}{\Delta x} \frac{x_0}{x_0} \\
&= \frac{\Delta f\,(a_1, a_2, \ldots a_n)}{x_0} \frac{x_0}{\Delta x} \Delta x \\
&= \left(\frac{\Delta a_1}{a_{1,0}} + \frac{\Delta a_2}{a_{2,0}} + \ldots + \frac{\Delta a_n}{a_{n,0}} \right) \frac{x_0}{\Delta x} \Delta x \\
&= \left(R_1 + R_2 + \ldots + R_n\right) \frac{x_0}{\Delta x} \Delta x
\end{aligned} \tag{8}$$

Note that $R_x = \frac{\Delta x}{x_0}$ represents the relative change of the base ratio, then

$$\Delta f\,(a_1, a_2, \ldots a_n) = \left(R_1 + R_2 + \ldots + R_n\right) \cdot \frac{1}{R_x} \cdot \Delta x \tag{9}$$

and the particular effects can be expressed as follows

$$\Delta x_j \approx \Delta f\,(a_j) = \frac{R_j}{R_x} \cdot \Delta x, \text{ for } j = 1, 2, \ldots, n. \tag{10}$$

4. Comparison of both derivations

In this section, using Du Pont decomposition involving three component ratios, we compare both derivations of the integral method and we highlight that the quantified effects of component ratios are not in coherence with financial theory, whereas the alternative derivation of the integral method provides coherent results.

Let ROE be decomposed as follows

$$ROE = \frac{EAT}{E} = \frac{EAT}{Sales} \cdot \frac{Sales}{A} \cdot \frac{A}{E}, \tag{11}$$

where *EAT*, *A* and *E* are net profit (Earning after Taxes), assets and equity respectively. Note that the first fraction corresponds to the Return on Sales (net profit margin), the second represents asset turnover and the last is financial leverage ratio. [2]

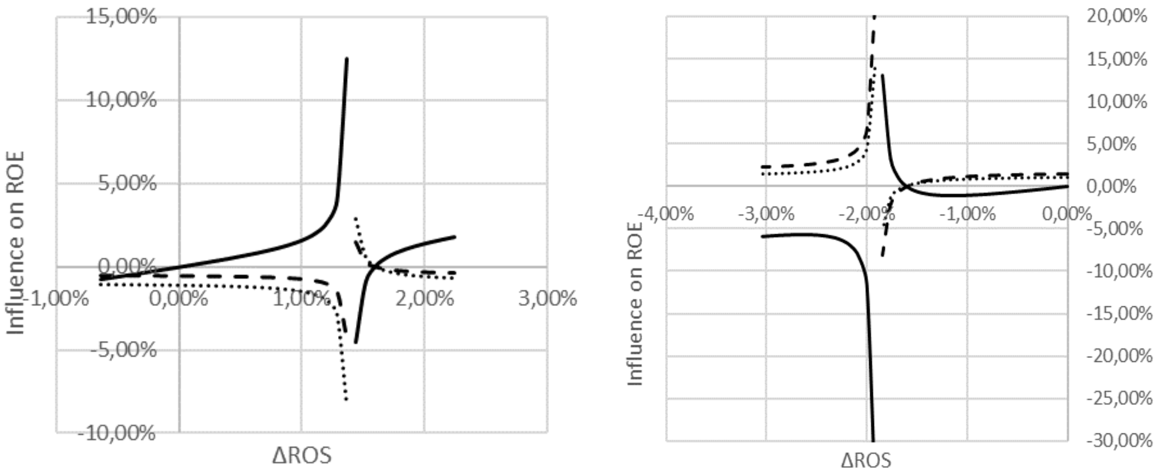
For the numerical example, we considered the unchanged sales across both periods, while the assets, as well as equity, have changed inter-periodically. The specific values of the relevant variables are shown in the next table.

Table 1 Pre-defined value of relevant variables

Value of financial variable	First period	Second period	
		Scenario 1	Scenario 2
Sales	1250	1250	1250
Assets	1600	1700	1400
Equity	1000	1200	800

Clearly, while we considered a decrease in asset turnover as well as financial leverage in the first scenario, the second scenario assumed an increase of both ratios. Finally, the initial value of EAT was set on 100 and varied in the second period for both scenarios to yield different value of ROS as well as different effect of ROS. The particular effect of component ratios evaluated by (7) as well as (10) are shown in next figure.

Figure 1: Effects of all component ratios on ROE for both scenarios (original derivation). Solid lines, the effect of ROS; dashed lines, the effect of asset turnover, dotted line, the effect of the financial leverage ratio.



The Figure 1 shows clearly that the effect of ROS is not in coherence with financial theory. Note that the positive change of ROS should always have a positive effect on ROE because higher net profit margin yields higher ROE in (11), while the negative change of ROS should affect the ROE negatively. However, the figure shows that positive change by 1.5 % has a negative effect on ROE and the negative change 1.75 % affects ROE positively.

In addition, the effects are infinitely large or small for certain change of ROS. It worsens not only the interpretation that each additional change of ROS yields non-constant additional effect on ROE, but also the effect of other component ratios varies exponentially depending on the change of ROS. Remind that although asset turnover and financial leverage changed inter-

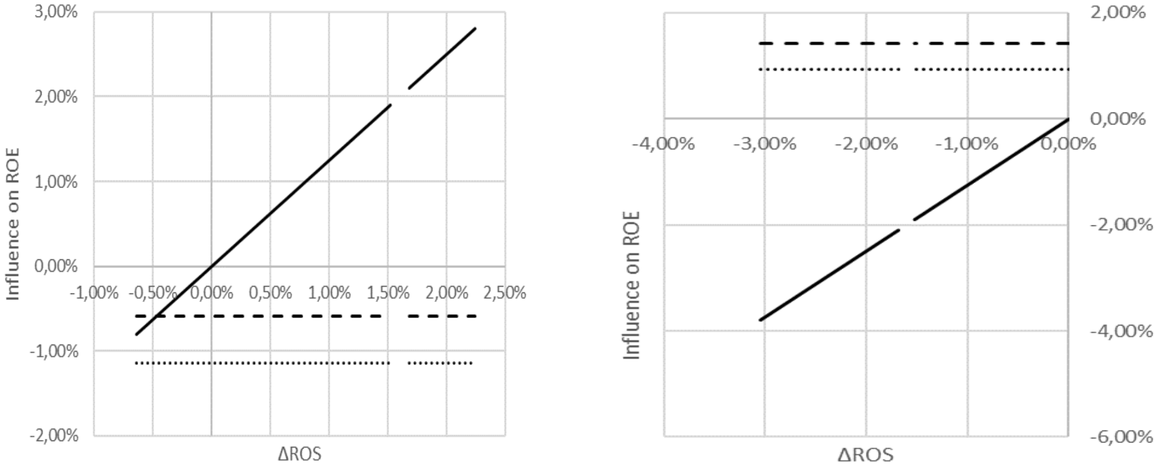
[2] Net profit margin describes the company‘s ability to generate profitable sales, the asset turnover measures the company’s overall ability to generate revenues with a given level of assets and the financial leverage ratio represents the solvency and measures the amount of assets supported for each one money unit of equity.

periodically in both scenarios, they are independent on the varying EAT. Although their varying effect is acceptable because of multiplicative relation, the exponential change of these effects is inadequate.

The figure also shows that the various change of ROS affects the order of importance. Having the change of ROS at 2.00 %, the most important ratios affecting the change of ROE is ROS itself, then asset turnover and financial leverage follow, while for the change of ROS at 1.5 %, the effect of the financial leverage ratio is higher than the effect of asset turnover.

By contrast, the alternative derivation handles these imperfections perfectly, but it suffers other limitation. Next Figure 2 shows the evaluated effects of ROS for both scenarios.

Figure 2: Effects of all component ratios on ROE for both scenarios (alternative derivation). Solid lines, the effect of ROS; dashed lines, the effect of asset turnover, dotted line, the effect of the financial leverage ratio.



Clearly, the effect of ROS is in accordance with its change and the effect of other component ratio is independent on the change of ROS. Moreover, the additional increase in ROS yields additional constant increase in effect on ROE which significantly facilitates the interpretation and the order of importance is also invariant to the change of ROS.

Note that the sum of all approximated effects does not equal to the true change of ROE. However, if the analyst focuses on the order of importance and interpretation rather than the exact value of effects on the change of ROE, the alternative derivation represents a reliable tool, while the original derivation might be used with cautions, i.e. the evaluated effects are always necessary to confront about the change of component ratios.

5. Conclusion

The paper dealt with the integral method that is used for quantifying effects of component ratios on the base ratio within pyramidal decomposition. In this paper, there are two derivation techniques that each out of them provides different results as well as different conclusions and, using a numerical example, we analysed both derivation technique in further details.

Thus, we demonstrated that the evaluated effects of component ratios by the original derivation are not always in coherence with financial implications. Under certain conditions, the evaluated effects were negative (or positive) although the component ratio increased (or decreased) which was against the expected results.

In addition, we showed that under the same certain conditions, each additional change of component ratio yields a non-constant additional effect on the base ratio, which significantly worsened the interpretation. Although the sum of all effects equalled still to the total analysed change of the base ratio, the effect of all component ratios varied exponentially despite they are not dependent on the net profit. Moreover, the interpretation was made more difficult because the order of importance varied. However, the original derivation always provides mathematically precise results.

By contrast, the alternative derivation evaluates effects that are in accordance with the change of component ratio and the effects of other component ratios are mutually independent. Moreover, the additional increase in component ratio yields an additional constant increase in the evaluated effect, which significantly facilitates the interpretation. Note also that the order of importance is invariant. However, the sum of all effects does not equal to the real change of base ratio, but only to the approximated change given by the first order of Taylor expansion.

Thus, we may conclude that if the analyst prefers the mathematical precision, he might use the original derivation. However, he must either check the evaluated effects if they are coherent with the change of component ratios. On the other hand, the alternative derivation represents a more reliable tool and facilitates the interpretation of the results. Its mathematical approximation is actually not as crucial because an analyst usually focuses on the importance rather than the exact values of evaluated effects.

Acknowledgements

This paper was supported within Operational Programme Education for Competitiveness – Project No. CZ.1.07/2.3.00/20.0296 and under the SGS project SP2019/132.

References

[1] Abdoli, M., Garkaz, M., Golami, Y., Pourkazemi, A. (2011). The comparative study of ranking company's efficiency based on data envelopment analysis (DEA) and traditional methods (DuPont's method). *International Conference on Economics and Finance Research*, p. 505-508, IACSIT Press.

[2] Akalu, M. (2002). Measuring and ranking value drivers: A Shareholder Value Perspective, *Tinbergen Institute Discussion Paper Series*, pp.1- 17.

[3] Brigham, E., Houston, J., F. (2001). *Fundamentals of Financial Management*. Concise Third Edition, Harcourt Publishers. ISBN 9781305480742.

[4] Dluhošová, D. (2004). Přístupy k analýze finanční výkonnosti firem a odvětví na bázi metody EVA – Economic Value Added. *Finance a úvěr - Czech Journal of Economics and Finance*, p. 541-559, vol. 54. ISSN 0015-1920.

[5] Dluhošová, D. (2010). *Finanční řízení a rozhodování podniku*. Praha: EKOPRESS. ISBN 978-80-86926-68-2.

[6] Dluhošová, D., Ptáčková, B., Richtarová, D. (2017). Application of selected financial performance methods to chosen industry in the Czech Republic. *European Financial Systems 2017: proceedings of the 14th international scientific conference*. Brno: Masaryk University, 2017. p. 67-75. ISBN 978-80-210-8609-8

[7] Dluhošová, D., Zmeškal, Z. (2014). Generalised approach to deviation analysis of non-linear company discounted value measure. *METAL 2014. 23. ročník mezinárodní konference metalurgie a materiálů*. Brno: Tanger.

[8] Ehrbar, A. (1998). *EVA: the real key to creating wealth*. New York: Wiley. ISBN 0-471-29860-3.

[9] Gitman, L., J. (1998). *Principles of Financial Management*. 8th Edition, Addison Wesley Publishers. ISBN 9781930789029.

[10] Gurný, P., Richtarová, D., Čulík M. (2017). Liquidity analysis and prediction in the processing industry by applying VG process: The case of the Czech Republic. *Ekonomická revue*, p 17-28, 2017, vol. 20(1). ISSN 1212-3951. (DOI: 10.7327/cerei.2017.03.02.)

[11] Hawawini, G., Viallet, C. (1999). *Finance for executives: managing for value creation*. Cincinnati: South Western College Pub. ISBN 0538853956.

[12] Laitinen, K. E. (1999). Du pont decision support system (DSS) for expenditure budgeting. *International Journal of Applied Quality Management*, p. 75-99, vol. 2. ISSN 1096-4738. (https://doi.org/10.1016/S1096-4738(99)80005-0.)

[13] Lau, K.CH. (2016). How corporate derivatives use impact firm performance? *Pacific-Basin Finance Journal.* p 102-114, vol. 40. ISSN 0927-538X. (https://doi.org/10.1016/j.pacfin.2016.10.001.)

[14] Liesz, T. (2002). Really modified Du Pont analysis: Five ways to improve return on equity. *Proceedings of the SBIDA Conference*.

[15] Maříková, P., Mařík, M. (2005). *Moderní metody hodnocení výkonnosti a oceňování podniku.* Praha: Ekopress. ISBN 80-86119-61-0.

[16] Richtarová, D. (2007). Analýza odchylek npv na bázi ukazatele eva a její využití při postauditu invesic. *Finanční řízení podniků a finančních institucí*. Ostrava: Ekonomická fakulta VŠB-TU Ostrava, p. 208-217.

[17] Richtarová, D. (2016). Aplikace analýzy rozptylu při hodnocení výkonnosti zpracovatelského průmyslu v České republice. *Managing and Modelling of Financial Risks.* Ostrava: VŠB - Technical University of Ostrava, p. 856-861, 2016. ISBN 978-80-248-3994-3.

[18] Seger, J., Hindls, R. (1995). *Statistické metody v tržním hodpodářství.* Praha: Victoria Publishing. ISBN: 80-7187-058-7.

[19] Vítková, E., Semenova, T. (2015). The Impact of Key Parameters Change on Economic Development of the Company. *Procedia Computer Science*, p. 744-749, vol. 64. ISSN 1877-0509. (https://doi.org/10.1016/j.procs.2015.08.605.)

[20] Zalai, K. (2007). *Finančno-ekonomická analýza podniku*. Bratislava: Sprint. ISBN: 978-80-89085-74-1.

[21] Zmeškal, Z., Dluhošová, D., Tichý, T. (2004). *Financial models*. Praha: Ekopress. ISBN: 80-248-0754-8.

Appendix

Table 1 Particular effect of component ratios evaluated by both derivation of integral method (scenario 1)

$EAT_0 = 100$, $S_0 = 1250$, $S_1 = 1250$, $A_0 = 1600$, $A_1 = 1700$, $E_0 = 1000$, $E_1 = 1200$

EAT_1	ROE_0	Change of ratio				Effect of component ratio			Alternative integral method		
		ΔROE	ΔEAT/S	ΔS/A	ΔA/E	EAT/S	S/A	A/E	EAT/S	S/A	A/E
113	9,42%	-0,58%	1,04%	-4,60%	-18,33%	1,75%	-0,79%	-1,54%	1,30%	-0,59%	-1,15%
114	9,50%	-0,50%	1,12%	-4,60%	-18,33%	2,10%	-0,88%	-1,71%	1,40%	-0,59%	-1,15%
115	9,58%	-0,42%	1,20%	-4,60%	-18,33%	2,67%	-1,05%	-2,04%	1,50%	-0,59%	-1,15%
116	9,67%	-0,33%	1,28%	-4,60%	-18,33%	3,98%	-1,46%	-2,85%	1,60%	-0,59%	-1,15%
117	9,75%	-0,25%	1,36%	-4,60%	-18,33%	12,47%	-4,32%	-8,41%	1,70%	-0,59%	-1,15%
118	9,83%	-0,17%	1,44%	-4,60%	-18,33%	-4,55%	1,49%	2,90%	1,80%	-0,59%	-1,15%
119	9,92%	-0,08%	1,52%	-4,60%	-18,33%	-0,95%	0,30%	0,58%	1,90%	-0,59%	-1,15%
120	10,00%	0,00%	1,60%	-4,60%	-18,33%	n/a*	n/a*	n/a*	n/a*	n/a*	n/a*
121	10,08%	0,08%	1,68%	-4,60%	-18,33%	0,48%	-0,13%	-0,26%	2,10%	-0,59%	-1,15%
122	10,17%	0,17%	1,76%	-4,60%	-18,33%	0,79%	-0,21%	-0,41%	2,20%	-0,59%	-1,15%
123	10,25%	0,25%	1,84%	-4,60%	-18,33%	1,02%	-0,26%	-0,51%	2,30%	-0,59%	-1,15%
124	10,33%	0,33%	1,92%	-4,60%	-18,33%	1,20%	-0,29%	-0,57%	2,40%	-0,59%	-1,15%
125	10,42%	0,42%	2,00%	-4,60%	-18,33%	1,36%	-0,32%	-0,62%	2,50%	-0,59%	-1,15%

*not available due to zero relative change of the base ratio

Table 2 Particular effect of component ratios evaluated by both derivation of integral method (scenario 2)

$EAT_0 = 100$, $S_0 = 1250$, $S_1 = 1250$, $A_0 = 1600$, $A_1 = 1400$ $E_0 = 1000$, $E_1 = 800$

EAT_1	ROE_0	Change of ratio				Effect of particular ratio					
						Integral method			Alternative integral method		
		ΔROE	ΔEAT/S	ΔS/A	ΔA/E	EAT/S	S/A	A/E	EAT/S	S/A	A/E
84	10,50%	0,50%	-1,28%	11,16%	15,00%	-1,04%	0,93%	0,61%	-1,60%	1,43%	0,94%
83	10,38%	0,37%	-1,36%	11,16%	15,00%	-0,96%	0,80%	0,53%	-1,70%	1,43%	0,94%
82	10,25%	0,25%	-1,44%	11,16%	15,00%	-0,79%	0,63%	0,41%	-1,80%	1,43%	0,94%
81	10,13%	0,13%	-1,52%	11,16%	15,00%	-0,51%	0,38%	0,25%	-1,90%	1,43%	0,94%
80	10,00%	0,00%	-1,60%	11,16%	15,00%	n/a*	n/a*	n/a*	n/a*	n/a*	n/a*
79	9,88%	-0,13%	-1,68%	11,16%	15,00%	0,99%	-0,67%	-0,44%	-2,10%	1,43%	0,94%
78	9,75%	-0,25%	-1,76%	11,16%	15,00%	3,31%	-2,15%	-1,41%	-2,20%	1,43%	0,94%
77	9,63%	-0,38%	-1,84%	11,16%	15,00%	13,05%	-8,11%	-5,32%	-2,30%	1,43%	0,94%
76	9,50%	-0,50%	-1,92%	11,16%	15,00%	-35,37%	21,05%	13,82%	-2,40%	1,43%	0,94%
75	9,38%	-0,63%	-2,00%	11,16%	15,00%	-11,67%	6,67%	4,37%	-2,50%	1,43%	0,94%
74	9,25%	-0,75%	-2,08%	11,16%	15,00%	-8,34%	4,58%	3,01%	-2,60%	1,43%	0,94%
73	9,13%	-0,88%	-2,16%	11,16%	15,00%	-7,07%	3,74%	2,46%	-2,70%	1,43%	0,94%
72	9,00%	-1,00%	-2,24%	11,16%	15,00%	-6,45%	3,29%	2,16%	-2,80%	1,43%	0,94%

*not available due to zero relative change of the base ratio

Performance Evaluation of individual branches of the Czech Republic economy

Dagmar Richtarová [1] Martina Borovcová [2]

Abstract

The aim of this paper is to analyse the financial performance of the individual sectors of the economy of the Czech Republic over the period 2012 - 2019. A static approach is used for performance analysis. First, the economic added value is determined and a pyramidal decomposition is applied to quantify the influences of the component ratios on the economic added value of the individual sectors of the Czech economy. Next, factors affecting the Economic Value Added are analysed using pyramidal decomposition approach. In the end of the paper, comments are provided.

Key words

Financial performance, economic value added, pyramidal decomposition

JEL Classification: C1, C5, C58, G3, G30

1. Introduction

The financial performance of individual sectors of the Czech economy affects the size of a country's gross domestic product. The financial performance of a sector or a company is a random process that can be divided into individual indicators. It can also be evaluated by accounting, economic or market indicators and is addressed by many authors, see Copeland (1994), Vernimmen (2005) or Brealey (2014).

Economic value added is one of the most preferred modern economic indicators which is used because of their ability to take into account the costs of capital, factor of time or factor of risk. The preference of this approach can be found in many publications, see Ehrbar (1998), Young (2001), Vernimmen (2005) or Ross (2013). Most of the authors solve problems of company performance valuation but only in few publications the performance valuation of industry can be found, see Dluhošová (2004) or Dluhošová, Ptáčková, Richtarová (2018). When assessing financial performance, it is also necessary to identify the factors and influences that affect it. The effects can be determined using static or dynamic methods.

The aim of this paper is to analyze the financial performance of individual sectors of the economy in the Czech Republic using Economic Value Added indicator during the analyzed period 2012 - 2019. The proposed pyramid decomposition is used to quantify indicators that affect the economic value added of individual sectors of the Czech economy.

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2. Methodology

This part of the paper is divided into two subchapters. Firstly, economic value added as a measurement of financial performance of industries is characterized. Then pyramidal decomposition of economic value added is proposed as one of the static methods for quantifying the effects of selected ratios.

2.1 Economic value added

Economic value added is based on the concept of the economic profit. When the economic profit is positive, it means that company earns more than the weighted average costs of capital, which also means that some wealth for the shareholders is created.
There are many ways how economic value added can be expressed, see Ehrbar (1998), Young (2001) and Dluhošová (2010). It can be distinguished EVA – Entity, EVA – Equity or relative economic value added. In this paper financial performance of branches of manufacturing industry is analyzed according to EVA – Equity. Economic value added is expressed as

$$EVA = \left(ROE - R_E\right) \cdot E, \tag{1}$$

where ROE is return on equity, E is equity and R_E are costs of equity.

The difference between ROE and Re is called spread. Spread is very important parameter influencing EVA. If this spread is positive, it means that industry or company earns more than the costs of equity are.

2.2 Static approach – method of pyramidal decomposition

Static approach is based on a longer time series of the performance indicator. For in-depth analysis of the impact of component ratios on the base ratio, it is useful to apply the analysis of deviations. According to this analysis it is possible to quantify the impact of the changes on the component indicators on the base indicator, Zmeškal (2013). Among the ratios it is possible to distinguish two operations - additive relationship and multiplicative relationship, Zmeškal (2013).

Additive relationship can be expressed as

$$x = \sum_i a_i = a_1 + a_2 + \ldots + a_n \,. \tag{2}$$

Quantification of the impact under the additive relationship is generally applicable and the total impact is divided in proportion to the changes in the component ratio as

$$\Delta x_{a_i} = \frac{\Delta a_i}{\sum_i \Delta a_i} \cdot \Delta y_x \,, \tag{3}$$

where $a_{i,0}$ and $a_{i,1}$ are the values of the i-th indicator for the initial period and the subsequent period $\Delta a_i = a_{i,1} - a_{i,0}$.

Multiplicative relationship can be expressed as

$$x = \prod_i a_1 = a_1 \cdot a_2 \cdot \ldots \cdot a_n \,. \tag{4}$$

According to the way in which the multiplicative relationship is handled, we can distinguish five basic methods: a method of gradual changes, a decomposition method with a residue, a logarithmic method, functional method or the integral method, Zmeškal (2013).
In this paper, integral method is used for quantification of the impact of component indicators on a base indicator.

Integral method (Zmeškal, 2013) is derived as follows

$$\Delta x'(a_{1,0}, a_{2,0}, a_{3,0}) = a_{2,0} \cdot a_{3,0} \cdot \Delta a_1 + a_{1,0} \cdot a_{3,0} \cdot \Delta a_2 + a_{1,0} \cdot a_{2,0} \cdot \Delta a_3,$$
$$\frac{\Delta x'}{x_0}(a_{1,0}, a_{2,0}, a_{3,0}) = \frac{\Delta a_1}{a_{1,0}} + \frac{\Delta a_2}{a_{2,0}} + \frac{\Delta a_3}{a_{3,0}}. \tag{5}$$

By substituting to

$$\Delta y_x = \frac{\Delta a_1}{a_{1,0}} \cdot \frac{\Delta a_2}{a_{2,0}} \cdot \frac{\Delta a_3}{a_{3,0}} \cdot \frac{x_0}{\Delta x'} \cdot \Delta y_x, \tag{6}$$

for any three factors,

$$R_{a_j} = \frac{\Lambda a_j}{a_{j,0}}, R_{x'} = \frac{\Delta x'}{x_0}, \Delta y_x = (R_{a_1} + R_{a_2} + R_{a_3}) \cdot \frac{1}{R_{x'}} \cdot \Delta y_x. \tag{7}$$

Then the impact of component indicator to base indicator is quantify as follows

$\Delta x_{a_1} = \frac{R_{a_1}}{R_{x'}} \cdot \Delta y_x$ and $\Delta x_{a_2} = \frac{R_{a_2}}{R_{x'}} \cdot \Delta y_x$, then $\Delta x_{a_3} = \frac{R_{a_3}}{R_{x'}} \cdot \Delta y_x$,

where $R_{a_i} = \frac{\Delta a_i}{a_{i,0}}$ and $R_{x'} = \sum_{i=1}^{N} R_{a_i}, R_x$.

(8)

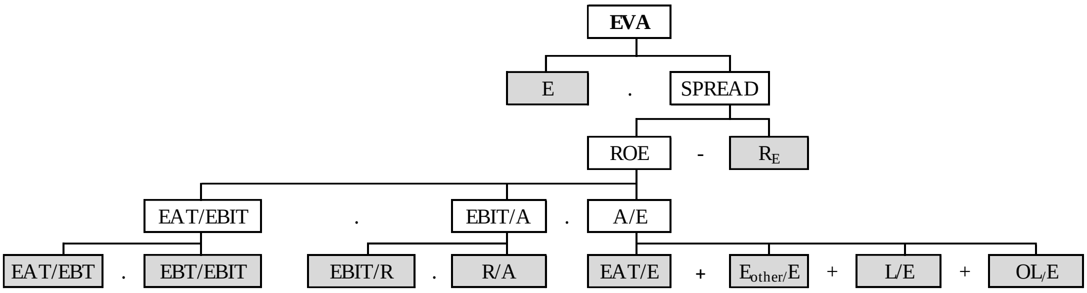
3. Application part

For the analysis of financial performance are chosen the individual sectors of the Czech economy. Financial performance of these sectors is analyzed according to economic value added and this indicator is decomposed to component financial indicators using pyramidal decomposition method,

There are many ways, how economic value added can be decomposed. The pyramidal decomposition of the EVA indicator is shown in Figure 1. The primary pyramidal decomposition factors are highlighted. An integral method is used to quantify the effects of partial indicators.

In this paper, there is proposed possible pyramidal decomposition of economic value added to four levels as

Figure 1: Economic value added – pyramidal decomposition



Source: own calculation

where E is equity, EAT is earnings after taxes, $EBIT$ is earnings before interests and taxes, EBT is earnings before taxes, A are assets, R are revenues, E_{other} is other equity (E - EAT), L are liabilities, OL is other liabilities and R_E are costs of equity which are determined according to a build-up model of Ministry of industry and trade of the Czech Republic.

3.1 Input data

The financial performance of individual sectors of the Czech economy is analyzed according to economic value added during the period 2012 to 2019. The pyramidal decomposition method is used and individual indicators are calculated on the basis of input data, which are taken from the website of the Ministry of Industry and Trade (MIT). Annual data are used for the analysis.

Table 1: Economic value added (thousand CZK) – individual sectors

	2012	2013	2014	2015	2016	2017	2018	2019
A	-1 469 857	-1 198 941	758 752	1 276 342	393 784	978 536	-3 812 340	-10 237 019
B	-3 646 256	-9 104 210	-2 582 442	-7 368 162	-2 538 243	-2 518 789	-422 718	-4 999 826
C	-2 929 976	-15 837 647	46 077 390	64 551 675	92 911 102	64 989 986	14 461 921	8 457 629
D	47 972 199	36 423 448	-26 039 950	-11 485 269	7 751 915	-7 262 573	-9 272 062	2 397 718
E	-5 076 948	-5 159 691	-2 856 946	-4 773 070	-1 232 321	-3 454 008	-5 291 258	-6 606 615
F	-4 477 547	-6 279 252	-4 388 976	-2 139 605	-907 407	-208 978	-7 914 837	-2 269 144
G	-22 330 128	-23 010 144	-11 061 288	-2 680 275	4 196 440	-1 674 066	1 378 230	-2 979 427
H	-20 469 529	-25 985 477	-20 945 556	-14 781 518	-9 373 824	-7 485 897	-14 264 407	-9 166 875
I	-2 215 030	-1 742 605	-1 292 627	-32 283	-18 276	348 868	-937 436	-67 680
J	8 686 491	4 359 303	1 619 586	4 471 679	7 453 279	12 124 521	12 146 604	15 899 921
L	-18 719 948	-22 626 468	-23 137 691	-13 667 396	-16 651 798	-15 119 953	-25 694 885	-21 046 665
M	-19 504 324	9 805 013	8 874 790	7 051 488	10 556 248	-1 188 621	-1 990 085	3 552 707
N	-486 206	1 687 849	-963 388	3 215 059	3 868 886	-1 670 076	-1 654 360	-1 275 291

Source: self-elaboration based on MIT data, own calculation

Individual sectors are marked according to the classification CZ_NACE. There are **A** Agriculture, Forestry and Fishing, **B** Mining and quarrying**,** **C** Manufacturing, **D** Electricity, Gas, Steam and Air Conditioning Supply, **E** Water Supply; Sewerage, Waste Management and Remediation Activities, **F** Construction, **G** Wholesale and Retail Trade; Repair Of Motor Vehicles And Motorcycles, **H** Transportation and Storage, **I** Accommodation and Food Service Activities, **J** Information and Communication, **L** Real Estate Activities, **M** Professional, Scientific and Technical Activities, **N** Administrative and Support Service Activities.

Table 1 presents values of EVA of individual sectors during the analyzed period 2012-2019. Negative values of EVA are highlighted. Most sectors generated negative values of EVA.

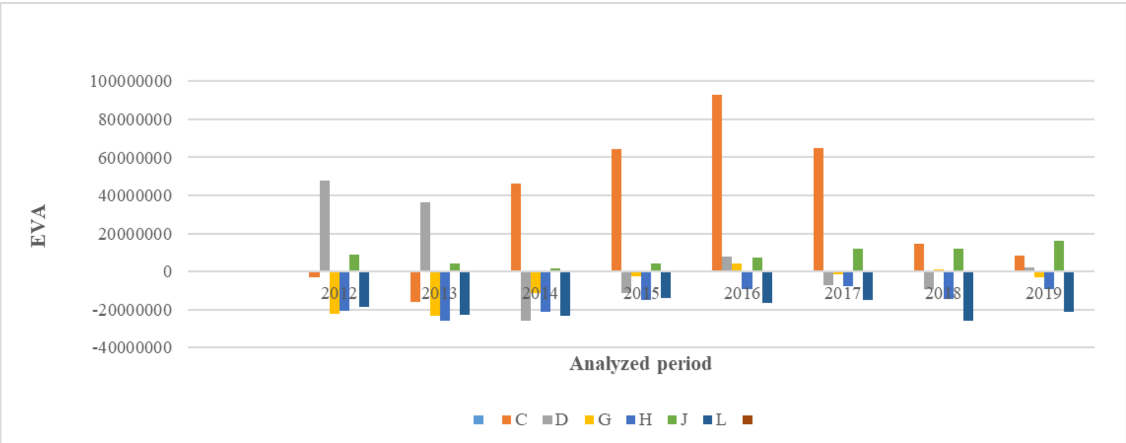
Sectors B, E, F, H and I had negative EVA throughout the analyzed period. Sector C (Manufacturing) created the highest economic added value in the years 2014 to 2018. It can be described as the driving force of the Czech economy. On the contrary, sector J showed positive economic value added throughout the period under review. Since 2016, economic value added has been increasing and the highest value was reached in 2019. This section includes the production and distribution of information and cultural products, the provision of the means to transmit or distribute these products, as well as data or communications, information technology activities and the processing of data and other information service activities.

The economic value added is calculated for each sector. Based on the determination of the mean value of the EVA indicator, the individual sectors are ranked. For the deeper analysis, 3 sectors with the highest positive EVA and 3 sectors with the highest negative EVA are selected.

The sectors with the largest negative mean value of EVA include sectors L, H and G. This is because of the low value of equity and the negative value of spread. The return on equity was much lower than the cost of the capital. By contrast, the sectors with the highest positive mean values of EVA are C, J and D.

In the Figure 2 there are shown values of EVA selected sectors, which has higher positive (C, J and D) or negative (L, H and G) value of the EVA indicator during the period 2012 – 2019.

Figure 2: Economic value added (thousand CZK) – selected sectors



Source: own calculation

3.2 Pyramidal decomposition of EVA

The pyramidal decomposition method is used for a deeper analysis of the factors influencing economic value added of selected industries. Pyramidal decomposition is performed according to Figure 1. An integral method is used to quantify the effects according to formula (8). Table 2 shows the change in the EVA indicator, equity and margins of selected divisions between years 2012 and 2019. The first level of this decomposition quantifies the impact on equity and the spread of economic value added. The largest increase of economic value added between 2012 and 2019 was found at the sector G, when indicator EVA was decreased from CZK -22.3 to CZK -2.9 billion. On the contrary, the largest decrease was recorded in sector D, when economic value added decreased from CZK 47.9 to CZK 2.4 billion.

Table 2: First level of decomposition of EVA – selected sectors

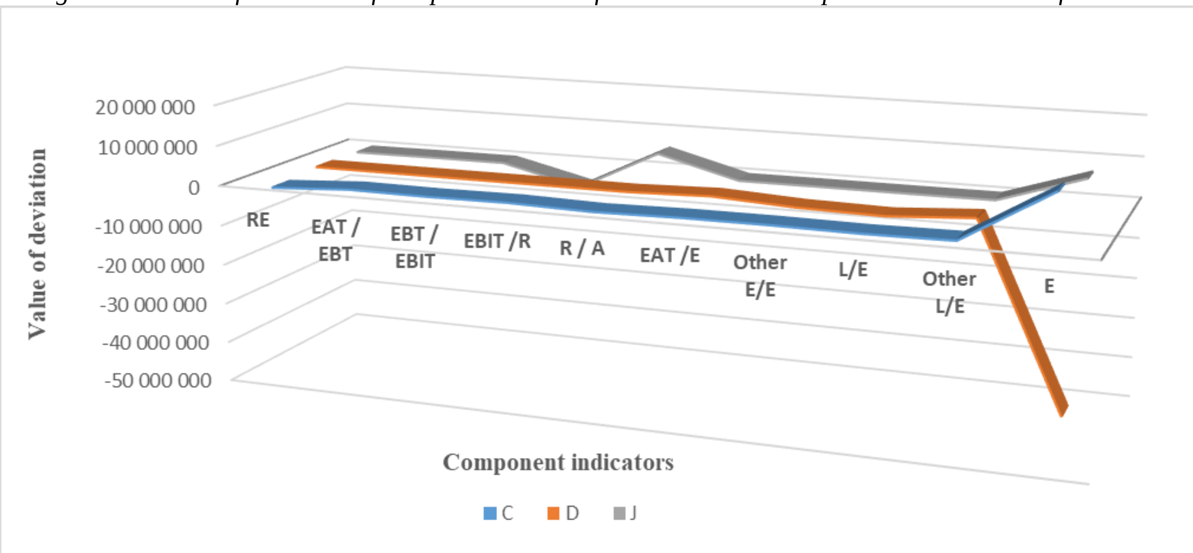
Indicators/CZ-NACE sectors	C	D	J	G	H	L
EVA	**11 387 605**	**-45 574 482**	**7 213 430**	**19 350 701**	**11 302 654**	**-2 326 717**
E	12 649 607	-45 033 439	6 824 913	28 686 715	14 047 142	2 305 399
Spread	-1 262 002	-541 042	388 517	-9 336 014	-2 744 488	-4 632 117

Source: self-elaboration based on MIT data, own calculation

After applying method of pyramidal decomposition, economic value added of selected sectors is distributed to 10 proposed component financial ratios (according to the Figure 1).

In the Figure 3 there are presented values of deviation of selected component indicators of EVA for sectors with positive mean value of the EVA in the analyzed period. The pyramidal decomposition revealed that the order and magnitude of the effects of the component indicators for individual sectors are changing. Positive influence means that if influence of component indicator is growing, then the value of EVA is increasing. On the other hand, negative influence works in reverse.

Figure 3: Values of deviation of component ratios of EVA – sectors with positive mean value of the EVA



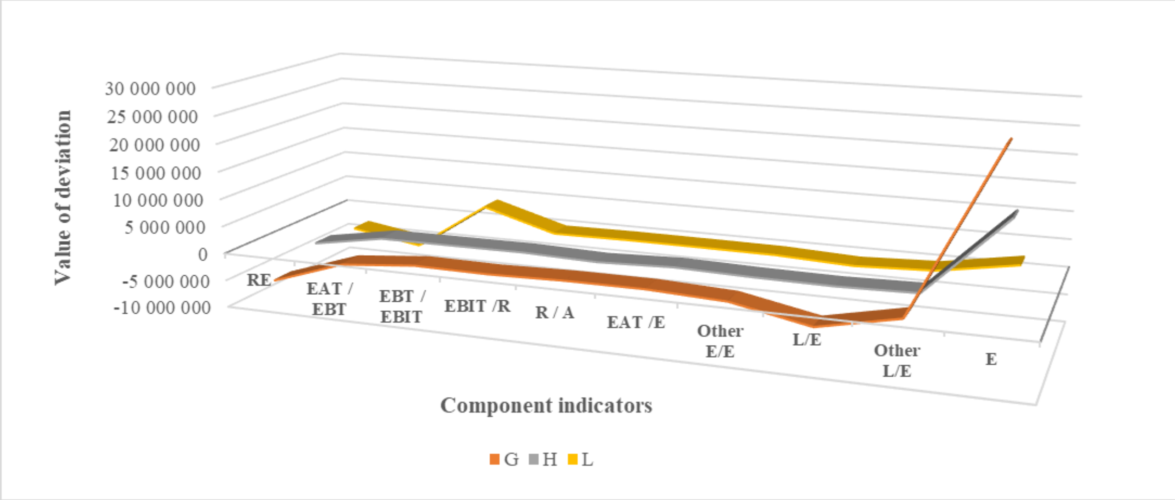
Source: own calculation

The value of equity has the biggest influence on the change of EVA, which increased in sectors C and J and decreased in sector D. The ratio of liabilities and equity caused decrease of the economic value added of these sectors.

Although sector C showed the largest increase of EVA indicator between 2012 and 2019, the performance has been declining since 2016. This decrease was caused due to a decline in the return on equity.

In the Figure 4 are presented values of deviation of selected component ratios of EVA for sectors with negative mean value of the EVA in the analyzed period. The analysis of deviations revealed that component indicators have the same effect on economic value added. The amount of equity has the positive effect on the economic value added, conversely, the spread has a negative effect. The negative effect of the spread is caused due to the low value of return on equity and higher cost of equity. Other indicators with a negative influence also include the share of liabilities and equity and the share of EAT/ EBT. In sector L, the indicator of the EBT/EBIT share had the greatest positive effect on economic value added. This effect was due to a significant increase in EBT and EBIT. In contrast, the EAT/EBT ratio negatively affected the EVA indicator.

Figure 4: Values of deviation of component ratios of EVA – sectors with negative mean value of the EVA



Source: own calculation

4. Conclusion

The contribution was focused on financial performance evaluation of the individual sectors of the economy of the Czech Republic over the period 2012 - 2019. The aim of this paper was analyzed the financial performance of individual sectors of the economy in the Czech Republic using the Economic value added during the analyzed period 2012 - 2019. The proposed pyramidal decomposition of economic value added was used to quantify indicators that affect the economic value added of individual sectors of the economy. Based on the determination of the mean value of the EVA indicator, the individual sectors were sorted. For a deeper analysis, 3 sectors with the highest positive and negative average EVA value were selected. Sectors C, D and J are the sectors that generate the largest positive mean of the EVA indicator. Only sector J showed positive economic value added throughout the period under review. Sector C (Production) is one of the main generators of economic value added. Among the indicators that had the greatest impact on the EVA indicator was the value of equity. Sectors G, H and L are the sectors that generate the largest negative average value of economic value added. The analysis of deviations revealed that 5 indicators have a positive effect and 5 a negative effect on economic added value. The value of equity has the greatest positive effect on economic value added, while the spread has a negative effect. The negative effect of the margin is due to the low return on capital and higher costs of this capital.

Acknowledgments

This paper was supported by the SGS Project VŠB – TU Ostrava SP2020/124 "Finanční modelování a rozhodování podniků a finančních institucí za rizika, flexibility a interakce”.

References

[1] Brealey, R. A., S. C. Myers, F. Allen. (2014). Principles of corporate finance. 11th global ed. Maidenhead: McGraw-Hill Education. ISBN 978-0-07-715156-0.

[2] COPELAND, T. E. – KOLLER, T. - MURRIN. (1994): Valuation: Measuring and Managing the Value of Companies. 2nd ed. New York: Wiley. ISBN 0-471-00993-8.

[3] Dluhošová, D. (2004). Přístupy k analýze finanční výkonnosti firem a odvětví na bázi metody EVA – Economic Value Added. Finance a úvěr, ročník 54, 2004, číslo 11-12

[4] Dluhošová, D. (2010). Finanční řízení a rozhodování podniku. Praha: EKOPRESS, 2010. ISBN 978-80-86926-68-2.

[5] Ehrbar, A. (1998). EVA: the real key to creating wealth. New York: Wiley, c1998. ISBN 0-471-29860-3.

[6] Dluhošová, D., Ptáčková, B., Richtarová, D. Financial performance evaluation of metallurgy of the Czech Republic and the role of metallurgy in manufacturing industry sector. In: METAL 2018. Brno: Tanger, 2018.

[7] Ross, S. A., Westerfield, R., Jordan, B. D. (2013). Fundamentals of corporate finance. New York: McGraw-Hill/Irwin, c2013

[8] Vernimmen, P. (2005). Corporate finance: theory and practice. Chichester: Wiley, c2005.

[9] Young, D., O'byrne, S. (2001). EVA and value-based management: a practical guide to implementation. New York: McGraw-Hill, c2001.

[10] Zmeškal, Z. (2015): Finanční modely. Praha: Ekopress. 2013. ISBN: 978-80-86929-91-0.

[11] Analytical material and statistics – Analysis of Czech economic development according to Ministry of industry and trade (2012 – 2019). Retrieved from: http://www.mpo.cz/ministr-a-ministerstvo/analyticke-materialy.

Impact of Volume on Portfolio Optimization

Kamonchai Rujirarangsan[1], Sergio Ortobelli Lozza[2]

Abstract

This study explores the use of volumes of stock returns in portfolio problems. In the analysis, we consider different portfolio strategies applied to the portfolio returns conditional the portfolio of transaction volume using two different estimators of the conditional expectation based either on the Gaussian kernel density function or the Epanechnikov one. In addition, we value some strategies based on penalized returns. To compute the optimal portfolios, we implemented the Sharpe ratio, global minimum $CVaR_{5\%}$, and Rachev ratio optimization, and we found that taking into account volume has an impact on the ex-post wealth.

Key words

Conditional Expectation, Behavioral Finance, Portfolio Optimization

JEL Classification: C61, G12, G41

1. Introduction

The relationship between the stock price and trading volume has been studied in several financial works of literature. In early studies, Ying (1966) and Westerfield (1977) found positive relationships between the absolute value of price changes and volume. Further, the evidence of the price-volume relationship can be explained by the rate of information flow into the stock market, as documented by Karpoff (1987). The results provide the behavior of contemporaneous relations. However, the prediction powers have not been investigated. After that, Gervars, Kaniel, and Mingelgrin (2001) revealed that the large of trading volumes tend to induce the large changes in the stock prices in the next future period.

In the dynamic relationship scheme, the stock returns contribute a positive correlation to volume. The Granger causality tests also show the persistence of its lagged relations; see Chen, Firth, and Rui (2001). Taking a volatility approach to stock return, Lee and Rui (2002) showed that the return volatility reacts to a causal relationship to the trading volume. Moreover, considering the volume as additional information, the forecast volatility model can be explained appropriately by the behavior of the stock returns (Lamoureux and Lastrapes 1990; Gallant, Rossi, and Tauchen 1992).

The change in stock return tends to occur on a high-volume day than a low-volume day, as suggested by Campbell, Grossman, and Wang J. (1993). The results underlying this work explained that the buying or selling volume is associated with the stock return changes. Thus, the basic idea of this work is to implement the effects of volume returns and stock returns in portfolio strategies based on the conditional expectation.

Inspired by taking the volume as information to return, we investigate how the stock returns conditional volumes information impacts the portfolio performance. To do so, we apply the conditional expectation using Gaussian and Epanechnikov kernel density function in which the stock returns are conditional to the volumes. The false information may generate if the stock returns are negatively decreasing, while the volume returns are positively increasing. Thus we use penalized stock returns to compensate for this effect. We then optimize the portfolio performance by using Sharpe Ratio, global minimum $CVaR_{5\%}$, and Rachev Ratio applied to the penalized returns.

2. Methodology

In this section, we apply the conditional expectation of returns using Gaussian and Epanechnikov kernel functions to the returns and the penalized returns. Then, we use different portfolio optimization models to find optimum choices.

In particular, we use the conditional expectation to approximate the returns from the volumes of portfolio. We set the Nadaraya-Watson as a kernel density estimator:

$$\mathbb{E}(y|Vol\ =\ x) = \frac{\sum_{n=1}^{N} y_n K(\frac{x - x_n}{h(N)})}{\sum_{n=1}^{N} K(\frac{x - x_n}{h(N)})}$$

where Vol is volume, y is return, $K(.)$ is kernel density function, and $h(.)$ is the bandwidth function defined following the Scott rule (see Scott (2015)) as $h(N) = 3.5N^{-1/3}std$ for Gaussian and $h(N)\ =\ 0.32N^{-0.8}$ for Epanechnikov kernel function.

As kernel function, we use either the univariate Gaussian:

$$K(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

or the Epanechnikov one :

$$K(z) = \frac{3}{4}(1 - z^2)I(|z| \leq 1)$$

where $I(|z| \leq 1)$ is the indicator function that indicates the value outside [-1,1] are zero.

As the "bad" news observed, the stock returns tend to be a large decrease. Meanwhile, the volume returns react to the highly incre

asing trend. This result may influence the choice of optimized returns. To overcome this drawback, we penalize the return as -1 when the stock return is negative with the positive volume return. Otherwise, we apply the stock return conditional the volume return as:

$$y_{m,(n)}\ =\ \begin{cases} -1 & ,for\quad y_{m,(n)} < 0\ \&\ Vol_{\ m,n-1} > 0 \\ \mathbb{E}(y_{m,(n)}\ |Vol_{\ m,n-1}) & \qquad otherwise \end{cases}$$

The basic idea of this penalization is that we want to avoid speculation because we assume that no short sales are allowed. To assess the optimum performance of a portfolio, we apply the Sharpe ratio, global minimum $\mathrm{CVaR}_{5\%}$, and Rachev ratio. The risk-free rate of the Sharpe ratio defines as the 13-week of daily U.S. treasury bill.

Recall that Sharpe ratio is given by:

$$\begin{aligned} &\underset{(w)}{Max} && \frac{w^T\mu - r_f}{\sqrt{w^T\Sigma w}} \\ &s.t. && w^T 1 = 1 \\ & && w \geq 0 \end{aligned}$$

global minimum $\mathrm{CVaR}_{5\%}$ is given by

$$
\begin{array}{ll}
\underset{(w,\gamma,z_n)}{Min} & \gamma + \dfrac{1}{(\alpha)N}\displaystyle\sum_{n=1}^{N} z_n \\
s.t. & z_n \geq -w^T y_{(n)} - \gamma \\
 & w^T 1 = 1 \\
 & w \geq 0 \\
 & z_n \geq 0 \\
 & n = 1,2,3,\ldots,N
\end{array}
$$

and Rachev Ratio portfolio optimization problem is given by:

$$
\begin{array}{ll}
\underset{(w,a,\lambda,z_n,\gamma)}{Max} & \dfrac{1}{(\alpha)N}\displaystyle\sum_{n=1}^{N} z_n \\
s.t. & z_n \geq B\lambda_n \\
 & z_n \geq w^T y_{(n)} - B(1-\lambda_n) \\
 & z_n \leq w^T y_{(n)} + B(1-\lambda_n) \\
 & \gamma + \dfrac{1}{(\alpha)N}\displaystyle\sum_{n=1}^{N} z_n \leq 1 \\
 & z_n \geq -w^T y_{(n)} - \gamma \\
 & w^T 1 = t \\
 & w \geq 0 \\
 & z_n \geq 0 \\
 & t \geq 0 \\
 & \lambda^T 1 = [\alpha N] \\
 & n = 1,2,3,\ldots,N
\end{array}
$$

3. Empirical Analysis

In the analysis, we select 30 companies among the components of the S&P500. The adjusted closing price of daily data and volume retrieve from 01 January 2004 to 31 May 2020. We then convert the data into the log-returns form. To obtain the persistence length of observations, we use backtesting data preparation by setting the in-sample and out-of-sample as 1-year and 1-month. Thus, the dataset contains 250-day for each observation point and rebalances every 20-day before the next analysis.

In the following step, we estimate the data by using the conditional expectation. The stock returns are conditional to the volume returns with Gaussian and Epanechnikov kernel density functions. Furthermore, to compensate for the stock returns by the volume information, we apply the penalization method. We thus have five different returns to analyze. Finally, we optimize the portfolio by Sharpe ratio, global minimum $CVaR_{5\%}$, and Rachev Ratio methods.

Table 1 shows that:

- The conditional expectation of penalized returns using the Gaussian kernel density function shows relatively high performance.

- If we use the conditional expectation without penalization of returns, the performances is generally lower than the optimization done on the historical return.

In particular, by considering Figure we find that the global minimum $CVaR_{5\%}$ optimization shows a steady increase in performances compared to the other models.

Table 1: The ex-post annual returns of portfolio performance using different optimization models and approximated returns

Optimization/Approximation	Historical Return	Gaussian	Penalized Gaussian	Epanechnikov	Penalized Epanechnikov
Sharpe Ratio	18.79 %	11.82 %	**24.97 %**	13.27 %	13.98 %
global minimum $CVaR_{5\%}$	13.65 %	18.47 %	**22.31 %**	14.34 %	13.09 %
Rachev Ratio	14.49 %	12.08 %	**24.45 %**	13.33 %	20.05 %

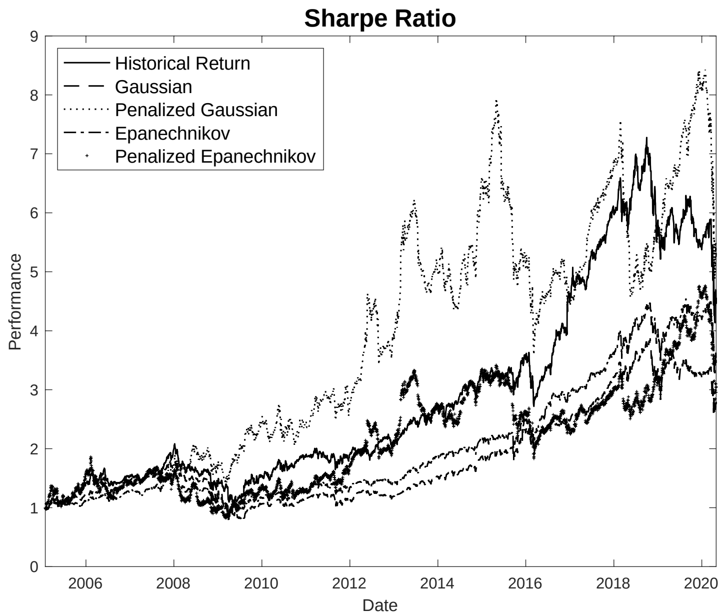


Figure 1: The ex-ante performance of Sharpe ratio optimization among the different of return conditions.

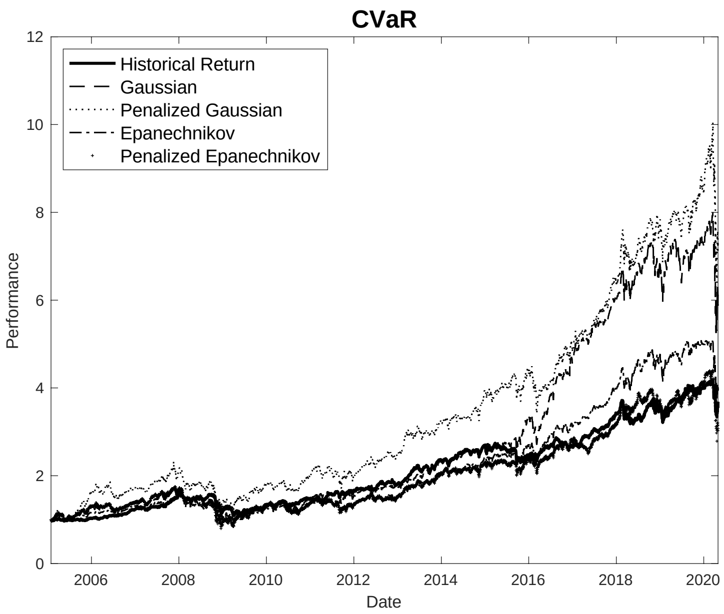


Figure 2: The ex-ante performance of global minimum $CvaR_{5\%}$ optimization among the different of return conditions.

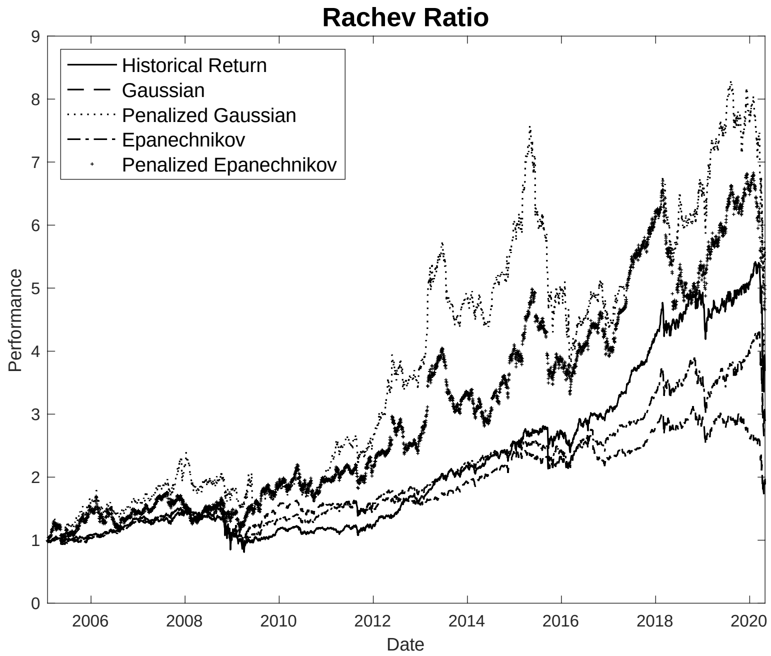


Figure 4: The ex-ante performance of Rachev ratio optimization among the different of return conditions.

4. Conclusion

In this study, we explore the impact of volume information on the stock returns. In particular, we apply the conditional expectation and penalization of returns to investigate the behavior of information. To assess the results, we use different portfolio optimizations.

We set the difference of return conditions by using the conditional expectation with Gaussian and Epanechnikov kernel density function and penalized returns. We then compute the ex-ante performance by the Sharpe ratio, global minimum $CVaR_{5\%}$, and Rachev ratio optimizations. From the results, there is evidence that the volume provides some information to the stock return, in particular when we penalize the returns to avoid the speculative strategies.

In sum up, the stock return conditional volume information with penalized return can be able to use as a profitable model.

Acknowledgements

The authors thank for Italian grants ex-MURST 60% 2020 and acknowledges the support of the Czech Science Foundation (GACR) under Project 20-1674S. SGS project of VSB-TUO under number SP2020/11 is acknowledged as well.

References

[1] Ying, C.C. (1966). Stock Market Prices and Volumes of Sales. *Econometrica*, vol. 34, 676-686.

[2] Westcrfietd, R. (1977). The Distribution of Common Stock Price Changes: An Application of Transactions Time and Subordinated Stochastic Models. *Journal of Financial and Quantitative Analysis*, vol. 12, 743-765.

[3] Karpoff, J. (1987). The relation between price changes and trading volume: A survey. *Journal of Financial and Quantitative Analysis*, vol. 22, 109-125.

[4] Gervais, S., Kaniel, R. & Mingelgrin, D.H. (2001). The high-volume return premium, *Journal of Finance*, vol. 51, 877-919.

[5] Chen, G., Firth, M. & Rui, O. (2001). The Dynamic Relation between Stock Returns, Trading Volume, and Volatility, *The Financial Review*, 36, issue 3, p. 153-73.

[6] Lee, B., & Rui, O. (2002). The dynamic relationship between stock returns and trading volume: Domestic and cross-country evidence, *Journal of Banking & Finance*, Volume 26, Issue 1, Pages 51-78.

[7] Lamoureux, C., & Lastrapes, W. (1990). Heteroskedasticity in Stock Return Data: Volume versus GARCH Effects. *The Journal of Finance, 45*(1), 221-229.

[8] Gallant, A., Rossi, P., & Tauchen, G. (1992). Stock Prices and Volume. *The Review of Financial Studies, 5*(2), 199-242.

[9] Campbell, J.Y., Grossman, S.J., & Wang J. (1993). Trading Volume and Serial Correlation in Stock Returns. *Quarterly Journal of Economics*, 108(4), 905-939.

[10] Scott, D.W. (2015). *Multivariate density estimation: Theory, practice, and visualization.* 2nd ed. Wiley.

Securitization of longevity risk in Slovakia

Lenka Smažáková, Ľudovít Pinda[1]

Abstract

The aim of this paper is to design and evaluate catastrophic bond linked to longevity risk in the environment of pension insurance in Slovakia. For the evaluation the knowledge of future mortality will be needed and we use Booth-Maindonald-Smith mortality forecasting method. To calculate market price of risk, the one factor Wang transform will be used. Subsequently, we will construct and evaluate a longevity bond for persons aged 65 +, where the annual payment of the coupon will depend on a predetermined number of persons reaching a given age and we will test sensitivity of this bond, when selected parameters are changed.

Key words

longevity bond, Wang transform, BMS method

JEL Classification: G22

1. Introduction

The valuation of life insurance products is based on expectations of mortality. If the probability of death increases, then insurers, whose portfolio is mostly made up of death insurance, may suffer a loss. Those who provide pensions will make a profit because they will pay annuities shorter. However, as life expectancy is on the rise due to better health care, we can expect this trend to have a negative impact on pension plans. These will have not enough credit to pay annuities out, so they will have either higher-than-expected loss levels or they will pay lower pensions.

In the case of state-owned pension plans longevity risk is getting more significant and countries are trying to solve it as effectively as possible. One of the solutions is a gradual increasement of the retirement age, which is already approaching 65 years in most of the EU countries. Otherwise, it would be necessary to either increase the incomes of pension institutions or reduce the amount of paid annuities. As an example of the impact of longevity risk we can mention a constitutional law, passed in 2019, on capping the retirement age to 64 years in Slovakia. This law can lower the future pensions of today's people in age 30 years by approximately 12 %.

Therefore, commercial insurers providing annuity payments and also state insurers are facing a serious problem with longevity risk (Páleš and Kaderová, 2018). This risk, unlike the standard notion of mortality risk, is not diversifiable and can be estimated using the law of large numbers. In order to minimize the risk it is necessary to estimate future mortality as best as possible and use an appropriate mortality modelling method for this estimation.

An effective way to hedge longevity risk is to transfer it to the capital market via longevity bond, a special type of catastrophe bond. The bond must be designed to eliminate the impact of increasing life expectancy on insurers and also has to be attractive to investors.

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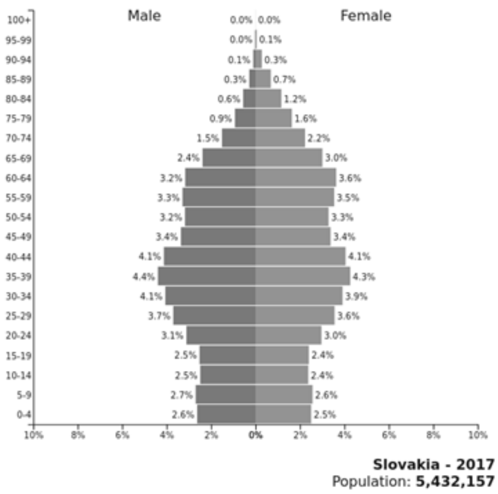
2. Demographic trends in Slovakia

Between the years 1950 and 1989 Slovakia experiences a demographic boom with a high birth rate and more than 80,000 children being born each year. Natural population growth is a desirable phenomenon for the country's economy, as it brings with it new consumption, increasing demand leads to increasement of supply, which leads to higher production and investments, which is subsequently reflected in GDP growth and in collected taxes. Slovakia is currently in a situation where the inhabitants born in these years form an economically active part of the population. The age structure of the population in Slovakia in 2017 can be seen in Figure 1.

Figure 1: Population pyramid of Slovakia in year 2017.
(Source: www.populationpyramid.net)



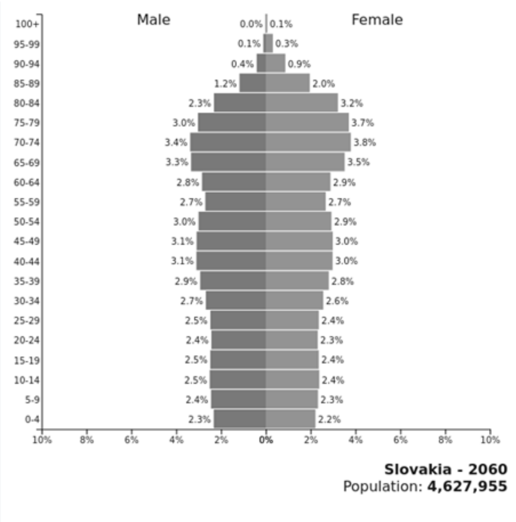
The high number of people in working - age and their effect on the economy is referred to as the demographic dividend. Demographic dividend means the economic growth potential that can result from shifts in a population's age structure, mainly when the share of the working-age population (15 to 64) is larger than the non-working-age share of the population (14 and younger, and 65 and older). As this share increases, the index of economic dependence decreases, which is the ratio of the economically dependent population to the economically active. In 2018, the dependency index of young people (aged 0 to 14) was 23.08% and the dependency index of older people (aged 65 and over) was 23.52%[2].

After passing major part of the population to the post-productive age, the population will beaging and the benefits of the demographic dividend will expire since the low population of young people cannot replace their places. Currently the share of the population in the productive age to the total population is at the level of 68.22% in Slovakia.

In developed countries the number of newborns declines, which also has a negative impact on the age groups in the population and may even lead to its decline. Between the years 1980 and 2000, there was an almost 50 percent decrease in the number of newborns in Slovakia (from 95 000 per year to 55 000), and this trend continues. The negative trend in number of births per year will cause a decrease in the number of children born from 55 000 to 32 000 in 2060 and this fact will affect the size of the Slovak population. The estimated population in year 2060 is 4,6 million.

[2] source: www. statdat.statistics.sk

Figure 2: Population pyramid of Slovakia in year 2060.
(Source: www.populationpyramid.net)

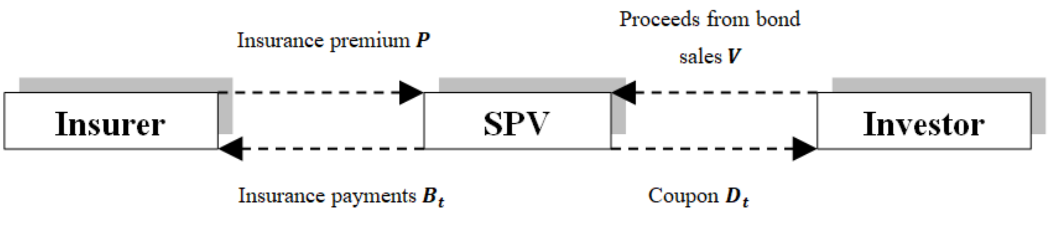


3. Construction of longevity bond

In this section we will present the construction and valuation of a longevity bond. We will use the models described in Lin and Cox (2003) and Lin (2006). Subsequently, we will evaluate longevity bond using specific data from Slovak insurance market.

Let us assume that the insurer took out insurance with l_x annuitants at time $t = 0$ and insurer will pay each annuitant 1 000 m.u. if he is still alive on the end of the year t. The insurer pays $1000 \cdot l_{x+t}$ to its annuitants, where l_{x+t} stands for a random variable of the number of survivors to time t from the initial number of annuitants l_x. The cash flows in securitization process between insurer, SPV and investors can be seen in Figure 3.

Figure 3: Cash flows in securitization.
(Source: Lin (2006))



The insurer buys insurance from its SPV for a premium P at time $t = 0$. If the number of survivors in year t exceeds the trigger level X_t, SPV will pay amount B_t to insurer, which is bounded from above in our case $1000 \cdot C$, where C is a pre-agreed non-negative constant by which we limit the upper amount of the bond payout. Then the insurance payments will be

$$B_t = \begin{cases} 1\,000 \cdot C & \text{if} \quad l_{x+t} > X_t + C, \\ 1\,000 \cdot (l_{x+t} - X_t) & \text{if} \quad X_t < l_{x+t} \leq X_t + C, \\ 0 & \text{if} \quad l_{x+t} \leq X_t. \end{cases} \tag{1}$$

The insurer, on one hand, pays annuities to annuitants and, on the other hand, receives payments from the SPV. His cash flow will then be

$$1\,000 \cdot l_{x+t} - B_t = \begin{cases} 1\,000 \cdot (l_{x+t} - C) \text{if} l_{x+t} > X_t + C, \\ 1\,000 \cdot X_t \text{if} X_t < l_{x+t} \leq X_t + C, \\ 1\,000 \cdot l_{x+t} \text{if} l_{x+t} \leq X_t. \end{cases} \quad (2)$$

SPV issues a bond with face value F with market price V. By purchasing a bond, the investor will be entitled to an annual coupon D_t in the amount of

$$D_t = \begin{cases} 0 & \text{if} \quad l_{x+t} > X_t + C, \\ 1\,000 \cdot C - B_t, & \text{if} \quad X_t < l_{x+t} \leq X_t + C, \\ 1\,000 \cdot C & \text{if} \quad l_{x+t} \leq X_t. \end{cases} \quad (3)$$

SPV's expenses consist of payments $B_t + D_t = 1\,000 \cdot C$, which are repeated annually, and the payment of a face value of the bond F at the end of the contract. Then the value of $P + V$, where P denotes the reinsurance premium and V the market price of the bond, should be at least equal to the price of a bond with a fixed coupon of $1\,000 \cdot C$ and the face value of F, i.e.

$$P + V \geq F \cdot d^c(0, T) + \sum_{k=1}^{T} 1\,000 \cdot C \cdot d^c(0, k), \quad (4)$$

where $d^c(0, k)$ are discount factors used in the bond market at the issue date. To evaluate the bond, we use the one-factor Wang transformation described in Wang (2002) with the distribution function $F^*(t)$ of a random variable t, which is defined as

$$F^*(t) = \Phi\left[\Phi^{-1}\left(F \quad (t)\right) - \lambda\right], \quad (5)$$

where λ is the market price of risk. According to Kaderová et al. (2018), "*market risk is the exposure to potential loss that would result from changes in market prices or rates*".

In case of life annuities the usage of Wang transform is based on the idea that their market price should reflect the uncertainty contained in the mortality table, which was used to calculate life annuities.

Next we set $F(t) = {}_tq_x$, where ${}_tq_x$ is a probability that x year old person will die before reaching age $x + 1$. In order to determine the market price of risk λ, it is necessary to know the market price of annuities, which is equal to the sum of payments discounted at the risk-free interest rate and the price of the risk contained in the probability distribution by the given distribution function $F(x)$. From the observed market price of annuities, we can then determineλ by solving the equation

$$l_x \cdot a_x = \sum_{t=1}^{\infty} E^*[l_{x+t}] \cdot d(0,t), \tag{6}$$

where $l_x \cdot a_x$ is the total immediate annuity premium net of the insurer's expenses from an initial number of annuitantsl_x, $d(0,t)$ is the discount factor based on the risk free interestrate term structure at the time the bond is issued and $E^*[l_{x+t}]$ is the transformed expected number of survivors to time$t$. The bond price is then given by

$$V = F \cdot d^c(0,T) + \sum_{t=1}^{T} E^*[D_t] \cdot d^c(0,t), \tag{7}$$

in which $d^c(0,t)$ denotes discount factor based on a coupon rate of an equivalent bond with fixed copon payment of $1000 \cdot C$. To calculate $E^*[D_t]$ we adjust the annual coupon D_t to

$$\frac{1}{1\,000} \cdot D_t = \begin{cases} 0 & \text{if} \quad l_{x+t} > X_t + C, \\ C + X_t - l_{x+t} & \text{if} \;\; X_t < l_{x+t} \leq X_t + C, \\ C & \text{if} \quad l_{x+t} \leq X_t, \end{cases} \tag{8}$$

what can be rewritten into

$$\frac{1}{1\,000} \cdot D_t = C - \max(l_{x+t} - X_t, 0) + \max(l_{x+t} - X_t - C, 0), \tag{9}$$

and so we get

$$\frac{1}{1\,000} \cdot E^*[D_t] = C - E^*[\max(l_{x+t} - X_t, 0)\;\;] + E^*[\max(l_{x+t} - X_t - C, 0)]. \tag{10}$$

Suppose we know the survival distribution for the pool of l_x annuitants upon which the bond is based, so we know the transformed (using one factor Wang transform) survival probability ${}_tp_x^*$. Then the distribution of the number of survivors l_{x+t} has a binomial distribution with number of trials l_x and success probability ${}_tp_x^*$. Sincel_x is rather large, we can use the normal approximation with mean $E^*[l_{x+t}] = \mu_t^* = l_x \cdot {}_tp_x^*$ and variance $Var^*[l_{x+t}] = {\sigma_t^*}^2 = l_x \cdot {}_tp_x^* \cdot \left(1 - {}_tp_x^*\right)$.

Let $\phi(y)$ denote density and $\Phi(y)$ distribution function of normalized normal distribution.Afterwards we get

$$E[\max(X - k, 0)\;\;] = \int_k^{\infty} [1 - F(t)dt]\,, \tag{11}$$

which results from per-parties integration for a random variable $X$with $E(X) < \infty$ and by using appropriate adjustments we can express the transformed mean value of coupon payments as

$$E^*[D_t] = 1\,000 \cdot \left\{C - \sigma_t^* \cdot \left[\Psi\ \ (k_t) - \Psi\left(k_t + \frac{C}{\sigma_t^*}\right)\right]\right\}, \tag{12}$$

where

$$\Psi\ \ (k_t) = \int_k^\infty [1 - \Phi(t)]dt = \phi(k) - k \cdot [1 - \Phi(k)]$$

and

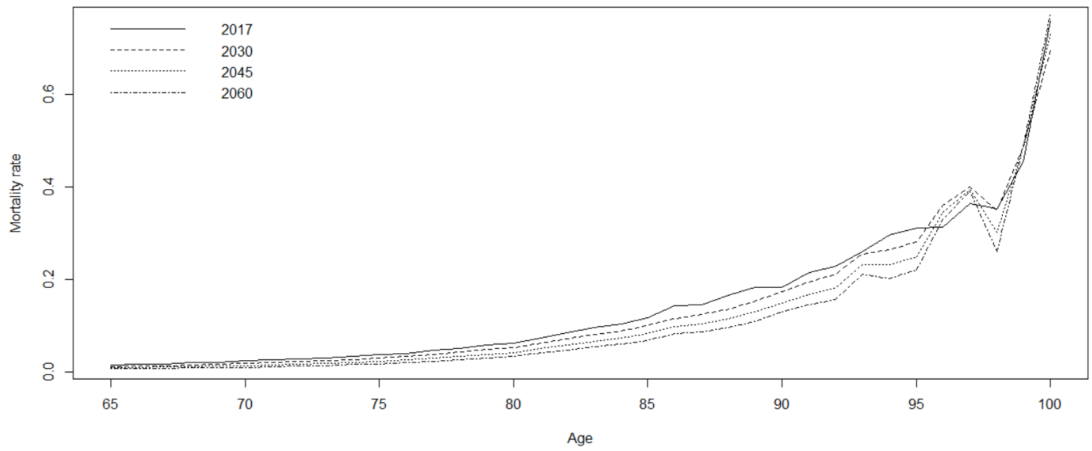
$$k_t = \frac{X_t - \mu_t^*}{\sigma_t^*}.$$

3.1 Mortality prediction

First of all it is necessary to predict future mortality. There are several methods used for mortality predictions and the best known and most commonly used is the Lee-Carter model (LC model) described in Lee and Carter (1992). This model uses an exponential function that best replicates the declining trend in mortality. However, its disadvantage is that the values of the age parameter α_x are influenced by the trend of an uncertainty at age x, which may lead to an underestimation of the uncertainty in mortality, especially in higher ages. Considering that for our longevity bond we will need a prediction of mortality for ages 65+ we will, rather than LC model, use Booth-Maindonald-Smith method (BMS method) which is a modification of LC model and has better properties for higher ages.

Historical data needed for mortality forecasting were taken from a website of Human Mortality Database (HMD), which is a database containing data on the population, mortality and birth rate of 40 countries, including Slovakia. In this database we can find data in the original version (raw data) and also in the modified version, such as cohort mortality tables. They are divided by country, gender, age and year of origin. In this database we have data for the Slovak Republic from 1950 to 2017 and ages from 0 to 110+ years. We will work with mortality rates m_x, using the years 1962 to 2017 to predict the future mortality, because the quality of data between 1950 and 1961 is lower than data in later years. In addition, we will consider ages from 0 to 100+, which are the ages for which our mortality tables are compiled by default. We will perform the estimation in the environment of R language[3].

Figure 4: Estimated mortality rates by BMS model for ages 65+ and selected years compared to year 2017. Source: Author's work according to HMD and Charpantier (2016).



[3]For the BMS model code for R see Charpentier (2016), p. 329

3.2 Pricing longevity bond on Slovak insurance market

We will consider the construction of the catastrophic bond mentioned above, i.e. a bond without loss of nominal valueF, with an annual coupon payment. For our model we will assume that the persons entering the insurance will be at the age of 65 and their number will be expressed as l_{65}, we will not distinguish between the sexes. Annuitants buy insurance from the insurer on January 1, 2018 and this is also a date when bond is issued. It matures on January 1, 2043. If a person of age x will survive to age $x+1$, he or she will receive an annual payment of 5 010,96 €[4]. When discounting, we will use risk-free interest rate forecasts published by European Insurance and Occupational Pension Authority (EIOPA) for the Slovak Republic, as these are used in the calculation of life annuities. Mortality forecasting will be based on BMS method and we will use the estimates from previous section.

First of all it is necessary to estimate the market price of risk λ and for the estimation the one factor Wang transform will be used. Let us define our transformed distribution function as

$$F^*(t) = \Phi[\Phi^{-1}({}_tq_{65}) - \lambda]. \tag{13}$$

For the values of distribution function $F(t) = {}_tq_{65}$ we will use data from a lifetable of Slovak Rebublic for year 2017 published by HMD.

Considering the insurer's expense factor equal to 4 %, then we determine the market price of risk from the adjusted relationship (6) as

$$(1-0{,}04)\cdot l_{65}\cdot a_{65} = \sum_{t=1}^{35} E^*[l_{65+t}]\cdot d(0,t), \tag{14}$$

where $l_{65}\cdot a_{65}$ is the total immediate annuity premium net of the insurer’s expenses from an initial number of annuitantsl_{65} and $E^*[l_{65+t}]$ is the transformed expected number of survivors to time$t$.

We determine the left side of the equation by using estimates obtained in BMS forecasting and for discounting we consider the estimates of risk - free interest rates published by EIOPA. The value of the left side of equation we get from

$$(1-0{,}04)\cdot l_{65}\cdot a_{65} = (1-0{,}04)\cdot l_{65}\cdot \sum_{t=1}^{35} {}_tp_{65}{}^{BMS}\cdot d(0,t),$$

where ${}_tp_{65}{}^{BMS}$ are survival probabilities counted from BMS method estimates. Then we are are able to calculate the market price of risk from equation (14) and it's value will be $\lambda = 0{,}060157$.

To evaluate the bond we need to set the bond strike levels X_t at time t. Consider a trigger defined as difference between the initial mortality rates (from life table for year 2017) and the mortality rates estimated by the BMS method, which reflects the trend in decreasing mortality over time. Strike levels will then be defined as

$$X_t = l_{65}\cdot\ {}_tp_{65}\cdot e^{-\sum_{s=1}^{t}(m^{BMS}{}_{65+s} - m_{65+s})} \text{ for } t = 1,2,\dots,35.$$

[4] 5 010,96 € represents the amount of average pension paid from social insurance in Slovakia for year 2017. Source: https://www.socpoist.sk/priemerne-vysky-dochodkovych-davok-pre-potreby-valorizacie---k-306-/56026s

Insurer's payments B_t and coupons D_t for years $t = 1,2, \dots ,35$ will be

$$B_t = \begin{cases} 5\,010{,}96 \cdot C & if\ l_{65+t} > X_t + C, \\ 5\,010{,}96 \cdot (l_{65+t} - X_t) & if\ X_t < l_{65+t} \leq X_t + C, \\ 0 & if\ l_{65+t} \leq X_t. \end{cases}$$

$$D_t = \begin{cases} 0 & if\ l_{65+t} > X_t + C, \\ 5\,010{,}96 \cdot C - B_t & if\ X_t < l_{65+t} \leq X_t + C, \\ 5\,010{,}96 \cdot C & if\ l_{65+t} \leq X_t, \end{cases}$$

where as the maximum payment of SPV in year t is limited by value $B_t + D_t = 5\,010{,}96 \cdot C$. Subsequently, we can value such a bond using (7), while the coupon interest rate$c$, which will be needed to discount the cash flows, will be equal to the share of the maximum payout of the bond and its nominal value, i.e.

$$c = \frac{5\,010{,}96 \cdot C}{F}.$$

3.2.1 Example of bond valuation for specific parameter values

Consider an initial cohort of 10 000 annuitants all the same sex, $l_{65} = 10\,000$. Let the annual payment of life anuity be $5\,010{,}96 \cdot C$. The face value of bond will be determined as $F = 5\,010{,}96 \cdot l_{65} = 50\,109\,600$ €. The coupon rate will depend on the parameter$C$.

We will compare the mortality bond constructed in this way with a straight bond with the same face value, coupon interest rate and with a constant coupon paid annually, regardless of whether the event triggering the mortality bond occurs or not. While in the case of a straight bond the investor is not exposed to the risk of a coupon shortening, the risk arising from holding a mortality bond must be offset by an above-average return.

In Table 1 we can see what effect the change of parameter C will have on the price of a longevity bond. The differences are due to the fact that the coupon payment of a bond increases as the value of parameter C increases, and that the higher the parameter C, the lower the probability of exceeding the bond trigger threshold value resulting in a coupon payment forgiveness.

Table 1: Price of straight and longevity bond at different levels of C.

C	Coupon rate		Straight bond	Longevity bond
450	4,50%	annual coupon	2 254 932 €	$\langle 0; 2\,254\,932$ €$\rangle$
		bond price	50 109 600 €	**49 457 981,19 €**
600	6,00%	annual coupon	3 006 576 €	$\langle 0; 3\,006\,576$ €$\rangle$
		bond price	50 109 600 €	**49 490 372,88 €**
750	7,50%	annual coupon	3 758 220 €	$\langle 0; 3\,758\,220$ €$\rangle$
		bond price	50 109 600 €	**49 520 366,47 €**

From (4), if equality were to apply, we can also determine the reinsurance premium that the insurer pays for this protection. For example, for $C = 600$, the price of reinsurance will be equal to 619 227,20 €, which represents 0.0754 % of the total premiums collected by the insurer.

The mortality bond is also very sensitive to changes in the insurer's expense rate. As the expense rate decreases, the market price of risk λ increases, which means that a higher survival rate ${}_tp_x^*$ is expected, which means that a bond is expected to be triggered. As a result the bond price decreases because the expected coupon payments are lower and, in addition, the price of the reinsurance increases. In Table 2 can be seen how the decrease/increase in the insurer's expense rate will affect the price of the bond and reinsurance.

Table 2: Effect of changing expense rate on λ, bond and reinsurance price.

Expense rate	$\boldsymbol{\lambda}$	Bond price	Reinsurance price
2 %	0,107021	47 468 628,52 €	2 640 971,48 €
4 %	0,060157	49 490 372,88 €	619 227,12 €
6 %	0,013554	50 051 120,44 €	58 479,56 €

4. Conclusion

In recent decades, the securitization of insurance risks has proven to be an effective alternative to traditional reinsurance (Krčová, 2017). While the insurance market is limited in capacity, the capital market offers sufficient resources to cover even catastrophic claims, while investors look for such securities in their portfolio, which on the one hand offer above-average returns and on the other hand the correlation with other held assets is low. The goal of the insurer is to achieve several years of reinsurance protection at a reasonable price.

Longevity risk can be considered as a catastrophic risk because of the development of people in post-productive age mortality and the subsequent economic impact. In Slovakia, this trend, with the unchanged method of calculating pensions paid from social insurance, could lead to the collapse of the state pension system, which must already be funded by the government.

As a solution, we propose securitization, specifically in the form of a longevity catastrophic bond. In this way, the Social Security Administration could receive the capital needed to pay pensions while minimizing government funding. Our proposed bond includes the calculation of the market price of risk using a one factor Wang transform and mortality forecasting by the BMS method. In stress-testing, we can see that the sale price of the bond is, among other things, sensitive to changes in the values of parameter limiting the maximum payment C and the expense rate of the insurer. As parameter C increases, the price of the bond increases, which is mainly due to the decreasing probability of complete forgiveness of the coupon payment to investors. At the expense rate, we observe a similar trend, namely that if the expense rate increases, the price of the bond also increases, i.e. the calculated market price of risk λ decreases, and thus a lower mortality in the post-productive ages is expected.

This paper is supported by the Grant Agency of Slovak Republic – VEGA, grant no. 1/0166/20 Establishing a capital requirement to cover selected catastrophic risks in life and non-life insurance.

References

[1] Barlák, J. (2014). *Predikcia Úmrtnosti na Slovensku.* BIATEC Vol. 22, No. 4, pp. 18-23.

[2] Booth, H., Maindonald, J. and Smith, L. (2002). *Applying Lee-Carter Under Conditions of Variable Mortality Decline.* Population Studies 56(3), pp 325-336.

[3] Bruchánik, R. (2015). *Demografický Vývoj na Slovensku.* Bencout Weekly report. No. 4, pp 1-5.

[4] Charpentier, A. (2016). *Computational Acturial Science with R.* Chapman and Hall, pp. 319-341.

[5] EIOPA (2020). *Risk-free interest rate term structures.* Available online: https://www.eiopa.europa.eu/tools-and-data/risk-free-interest-rate-term-structures_en?fbclid=IwAR0xUgK04ystb6mLk5Kq0BBDNbmMc5ZU3cbK8vDp14waLytzGFuP5eCiqkk

[6] HUMAN MORTALITY DATABASE. 2019. Available online: www.mortality.org, 27.5.2020.

[7] Kaderová, A., Krátka, Z. and Páleš, M. (2018). *Volatility Modelling in Market Risk Analysis.* Journal of Applied Economic Sciences. - Craiova : Spiru Haret University, pp. 787-796.

[8] Krčová, I. (2019). *Securitization of Mortality Risk in Life Insurance*. Investment Modelling in the Environment of Catastrophic Insurance Risk. Ostrava, pp. 38-43.

[9] Lee, R.D. and Carter, L. (1992). *Modeling and Forecasting the Time Series of US Mortality*. Journal of the American Statistical Association. Vol. 87, pp.659-671.

[10] Lin, Y. and Cox, S.A. (2003). *Securitization of Mortality Risk in Life Annuities. Actutial Research Conference.* University of Michigan, pp.227-252.

[11] Lin, Y. (2006). *Mortality risk management*. Dissertation. Georgia State University, pp. 30-61.

[12] Páleš, M. and Kaderová, A. (2018). *Stress Tests in Actuarial Practice. In Managing and Modeling of Financial Risks.* International Scientific Conference. Managing and Modeling of Financial Risks. Ostrava, pp. 388-396.

[13] Sociálna Poisťovňa (2020). *Priemerné Výšky dôchodkových Dávok pre Potreby Valotizácie (k 30.6.).* Available online: https://www.socpoist.sk/priemerne-vysky-dochodkovych-davok-pre-potreby-valorizacie---k-306-/56026s

[14] Verejná Databáza Údajov. (2020). *Demografické a Sociálne Štatistiky*. Available online: https://www. statdat.statistics.sk

[15] Wang, S.S. (2002). *A Universal Framework for Pricing Financial and Insurance Risks.* ASTIN Bulletin. Vol. 32, No. 2, pp. 213-234.

Findings of Normally Distributed Series inside Market Price Development[1]

Bohumil Stádník [2]

Abstract

In this financial engineering study we try to answer the question whether certain Gaussian periods of time exist inside price/yield development in the case of liquid investment instruments, and, if so, how often they appear, how long those periods are, and how their existence may influence our point of view regarding price/yield behavior?
In our research we really establish the existence of the Gaussian periods and, consequently, we assess their role in the following issues: Are the departures from normality inside the yield series caused by a mix of volatility clusters, in which each of them is created only by a Gaussian process (Gaussian mixture)? And, how to interpret financially (using realistic market processes) these different Gaussian periods, especially inside a high frequency series?
All the problematics in the research have had a significant impact on the opportunities for price direction prediction, and we also assess the problematics in their entirety in the light of Gaussian and non-Gaussian processes.

Key words

Gaussian processes; price/yield development; volatility clustering; departures from normality

JEL Classification: G1, G10, G12, G14

1. Introduction

There is a large amount of empirical evidence that the distribution of the yield of a liquid investment instrument is non-Gaussian, which implies that the process behind the price/yield development is not purely chaotic (for example, an independent random walk).

In spite of this fact, we try to discover, in this research, if certain Gaussian periods of time exist inside the price/yield development of liquid investment instruments, and if so, how often and when they appear, how long these periods are, and how their existence may influence our point of view on price/yield behavior? The main research hypothesis could, then, be formulated as follows: We expect Gaussian periods of time (with a Gaussian distribution of yield over this period) in the price/yield development. As there is a large amount of empirical evidence that a financial time series with volatility clusters is non-Gaussian, we have to try to find Gaussian periods inside the volatility cluster. In our point of view, any attempt to find a series which contains clusters and which is Gaussian makes no sense.

The contribution of this research is that we really confirm the research hypothesis of the existence of the Gaussian periods, and, consequently, we assess their role in the following issues:

[1] This research has been supported by the VSE institutional grant IP 100040 and VSE internal grant IG102027.
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1. Are the departures from normality inside the yield series caused only by a mix of volatility clusters, in which each of them is created by a certain Gaussian process (this concept is also well-known as a Gaussian mixture)? Or, is it also by the coaction of a certain mechanism which is non-Gaussian. To answer this question, we have to assess how big a part of all the data series is represented by the Gaussian periods.
2. How to interpret financially (using realistic market processes) the different types of Gaussian periods, especially inside high frequency series.

2. Literature Review

Volatility clustering is nowadays considered to be the main cause of departures from normality (leptokurtic features, etc.), and the clusters themselves are usually considered to be caused by a certain dependence in the volatility, which means that the price direction is independent of the past but that the volatility is dependent. Such a process does not allow directional forecasting, but it is closely connected to the size of the price steps or their number (trading intensity) over the given time period. There are several theories of basic research in the area of volatility dependence. One of the simplest is the Gaussian mixture distribution. The simplest, and also the original Gaussian mixture has an acceptable interpretation: a financial market occurs in two regimes with either high or low volatility. We can model many non-normal distributions whose characteristics depend on the probability of both regimes and their parameters. If the regimes have a Markov law of motion, the mixture is, then, a hidden Markov model (Baum, Petrie 1966), which is also known as the Markov regime switching model. We find many extensions of the Markov switching model (Krolzig 1997; etc.). Other well-known work in this area was done by Bollerslev 1986, on the GARCH process, and Engle 1995, on the ARCH process. Some new research in the area of volatility dependence was done by Roch 2011. While GARCH, ARCH and other volatility models propose statistical constructions based on volatility clustering in financial time series, they do not provide any financial explanation. The financial explanation of volatility clustering is quite difficult. The simplest possible financial clustering mechanism is simply the switching of the market between periods of high and low trading activity, or the clustering of economic news. Another idea was competition between several trading strategies, but the simulation does not allow us to confirm that the mechanism is responsible for volatility clustering (Cont 2005). Some economic works contain examples where the switching of economic agents between two behavioral patterns leads to great volatility. Volatility clustering should also arise from the switching of market participants between fundamentalist and chartist behavior (Lux, Marchesi 2000). Chartist traders evaluate their investments using historical development, whereas fundamentalists evaluate their investment opportunities according to the difference between the market price and the fundamental valuation. According to the Lux-Marchesi model, the market price development follows a Gaussian random walk until the moment when some chartist traders using certain techniques surpass a certain threshold value, and at that moment, a volatility outbreak occurs. According to Cont 2005, the origin of volatility clustering can also be caused by the threshold response of investors to news arrivals. Other new research connected to volatility clustering has been done by Jianga, Lia, Caia 2008 and Tsenga Jie-Jun, Sai-Ping Lia 2011.

Instead of using volatility dependency effects, we are able to explain non-normalities using purely directional dependency effects. This approach considers the price development direction to be dependent on the past and allows certain forecasting opportunities, unlike volatility dependency. There are many case studies based on the directional dependency, but the comprehensive modeling of the departures from normality in this way is not so frequent. For example, the commonly used technical trading rules are based on market price direction forecasting based on the past. We can consider technical analysis to be a prediction tool, but its benefit is still under discussion. We encounter many other interesting detailed works or case studies in the area, such as Henriksson, Merton 1981; Anatolyev, Gerko 2005; Primbs, Rathinam 2009; Gontis Ruseckas, Kononovičius 2010; Lux 2011; Dzikevičius Vetrov 2012. Price direction development dependence also takes place in the basic feedback process according to the behavioral finance concept, where an upward trend is more likely to be followed by another upward movement (Schiller 2003) or in other research, such as, for example, momentum or jump-difussion studies (Pesaran, Timmermann 1995; Stankevičienė, Gembickaja 2012; Janda, Kourilek 2020, a short term trend trading strategy on the futures market based on chart pattern recognition (Masteika, Rutkauskas 2012), or in the development of the concept of a strategy for sustainable return investment decisions on the capital and money markets (Rutkauskas, Miečinskiene, Stasytyte 2008). We have to mention also the work of Larrain 1991, which states that long term memory exists within the financial market, and other similar works by Hsieh 1991, Peters 1989, 1991, and 1994, which focus mainly on the measurement of the probability of diversions from normality.

According to the Dynamic Financial Market Model (Stádník 2011 [1]; 2011[2]) we are able to cause sharpness and fat tails in the distribution. Feedback increases the value of the probability of an up or down direction of the next price step (from 50/50 for a purely symmetric random walk to, for example, 51/49) depending on the previous development. The idea of feedback processes is based on empirical observations that traders, investors and other market participants not only watch present or historical data, but according to them, they also place buy or sell orders and thus influence future development. Feedback which keeps the movement in a certain direction is described in the model as trend stabilizer feedback. For example, traders participating in "momentum trading" try to find instruments that are moving significantly in one direction, and in order to realize financial profit on the movement, they basically prolong the short-term trends. The other important feedback is price inertia feedback, which pushes the market price back to a certain level and which results from "level or mean reversion trading", where traders believe the price will return to the level which was set after the last economic news of great importance, for example.

3. Methodology

The whole methodology is divided into the following steps:

1. As is stated above, the only possibility of discovering the Gaussian periods is to carry out normality tests within volatility clusters. To do this, we separate clusters with a similar volatility from the yield series, or, in other words, we decompose the time series into "similar" clusters. Then we carry out a normality test for these similar clusters, which we compose into one series. We also define d as the minimum length of one cluster. If we succeed in resolving all the volatility series into clusters with Gaussian distribution, we may conclude that the departures from normality are the result only of a Gaussian mixture. If not, we have to conclude that there may also be present another non-Gaussian process causing the departures.
2. If Gaussian periods are found, we attempt to propose a realistic financial mechanism responsible for the clusters in both the daily and high frequency time series.
3. We make a summary of prediction opportunities using realistic processes from the financial markets in the light of the existing Gaussian and non-Gaussian processes.

As the measure of non-normality, we assess the value of kurtosis, which is 3 for Gaussian distribution and also the Shapiro–Wilk test. All algorithmic tasks are implemented in the Matlab programming environment from MathWorks.Inc.

The methodology for finding the Gaussian periods can be divided into the following steps:

a. The definition of "big", "small" and "mid" cluster groups.
b. The separation of clusters from a time series into groups according to point a.
c. The study of groups, which we do on the Nikkei 225 index, SP500 index and Crude Oil commodity.

1.1

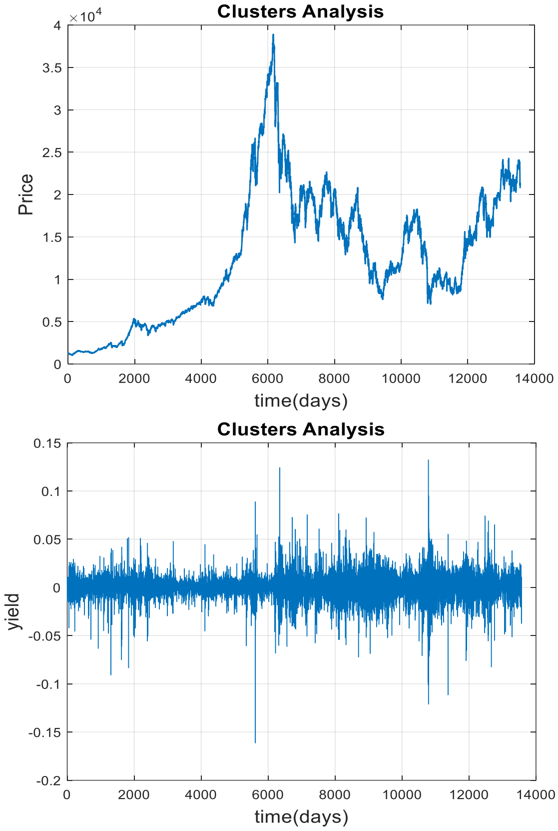

Fig. 1 Nikkei 225 index daily development (left), Nikkei 225 daily yield development (right)

To explain this better, let us give an example using the Nikkei 225 index. In figure 1 there is the development of the index going back 14 000 days (figure1-left), while figure 2-right shows its yield development. We may observe volatility clusters, and the yield distribution is leptokurtic (figure 2).

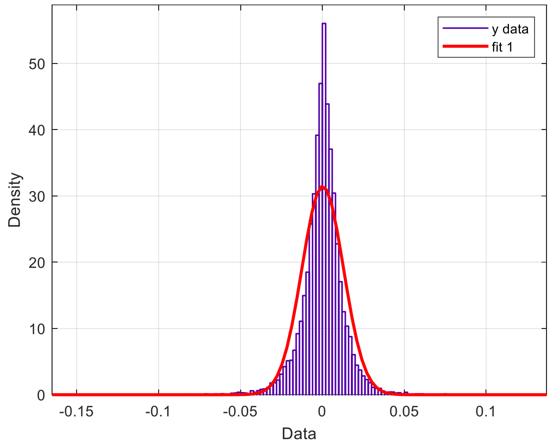

Fig. 2 Nikkei 225 daily yield distribution, kurtosis 12.53

The definition of clusters:

- A big cluster is defined as the part of a time series of yields where the absolute value of the moving average of the value of yields (the thick line in figure 3) is higher than a certain level (the horizontal line in figure 3) which is above the zero level. The moving average is calculated over a sliding window of length *m* up to the last *m* neighboring elements of the series.
- A small cluster is analogically defined as the part of the time series of yields where the absolute value of the moving average of yields is lower than a certain level which is above the zero level.

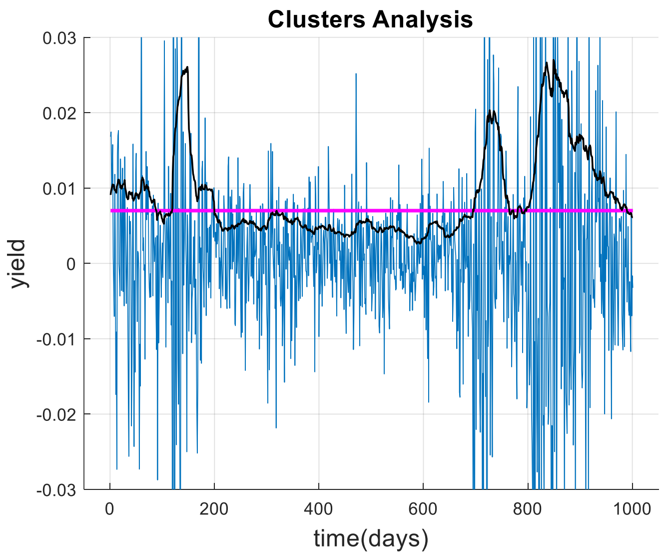

Fig. 3 Example of separating the clusters using the moving average and one level, level=0.0065, m=30, d=30

- A mid cluster is defined as the part of the time series of yields where the absolute value of the moving average of yields is between two limits (the horizontal lines in figure 8)

The other condition is the minimum length of the time period, denoted as *d*, which is connected to the number of days (in the case of daily series) when we observe a certain type of clusters. In our research its minimum value is limited to 5.

There is an example of separated big clusters from the time series of Nikkei 225 in figure 4-left (level=0.0065, m=30, d=30), the time series of the big clusters is in figure 4-right.

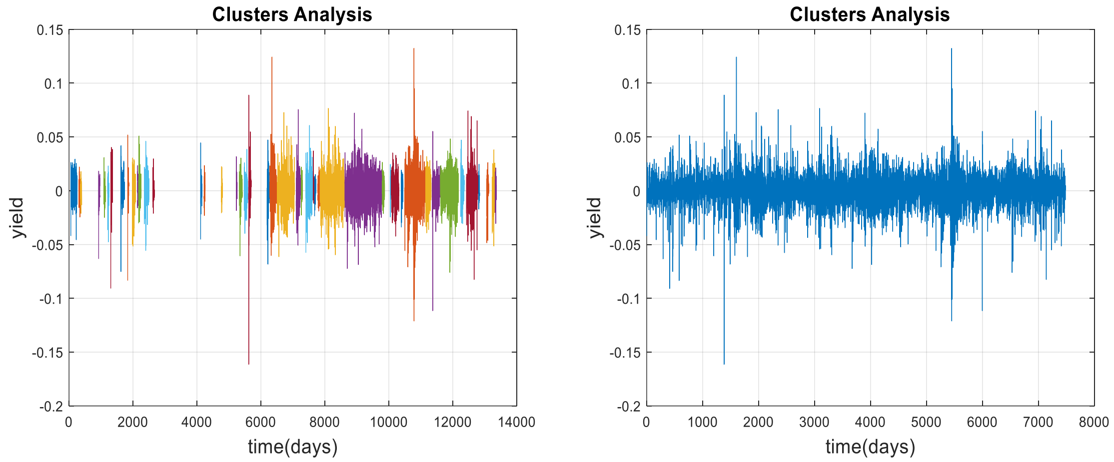

Fig. 4 Separated big clusters (left), series of big clusters (right)

The distribution inside big clusters is in figure 5. We can see that the distribution contains significant departures from normality.

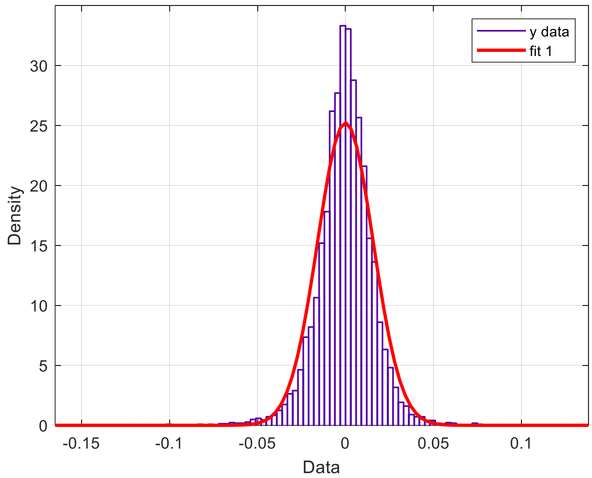

Fig. 5 Yield distribution inside big clusters, kurtosis 9.1

There is an example of separated small clusters in the time series of Nikkei 225 in figure 6-left (level=0.0065, m=30, d=30). The time series of big clusters is in figure 6-right.

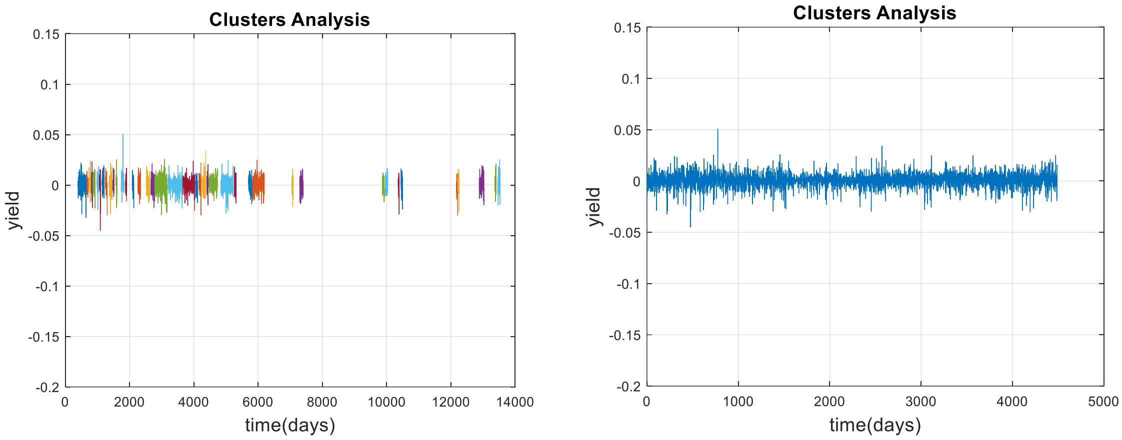

Fig. 6 Separated small clusters (left), series of small clusters (right)

The distribution inside small clusters is in figure 7. We can see that the distribution contains departures from normality.

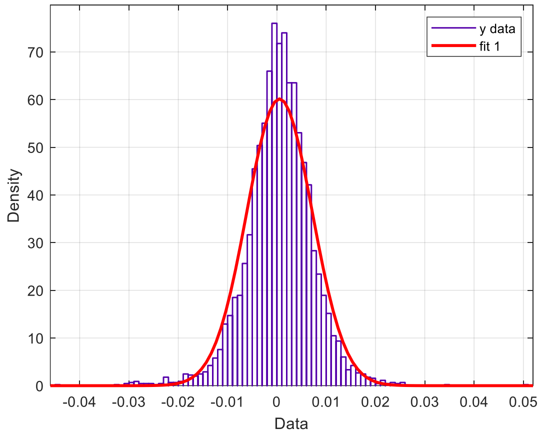

Fig. 7 Yield distribution inside small clusters, kurtosis 6.08

There is an example of the separation of mid clusters using two levels according to figure 8.

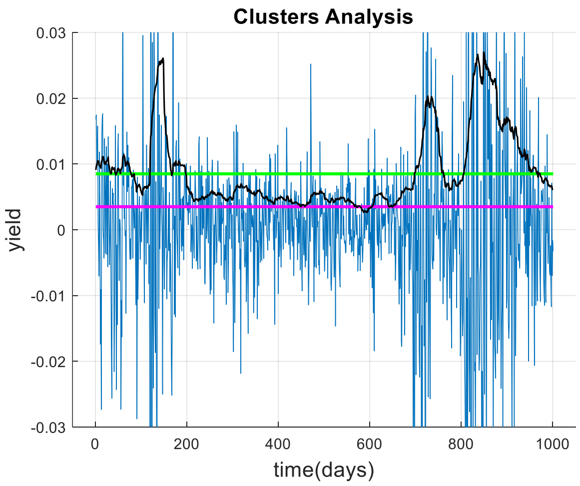

Fig. 8 Separating the clusters using moving average and two levels

There is an example of separated mid clusters in the time series of Nikkei 225 in figure 9-left (limit 1=0.004, limit 2=0.006, m=10, d=7). The time series of mid clusters is in figure 9-right.

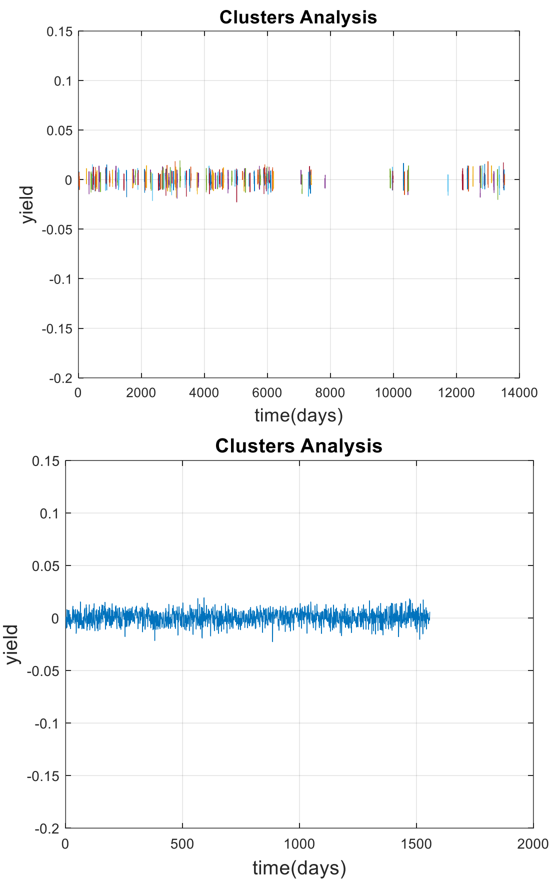

Fig. 9 Separated mid clusters (left), series of mid clusters (right)

The distribution inside mid clusters (series in figure 9-right) is in figure 10. We can see that the distribution does not contain significant departures from normality.

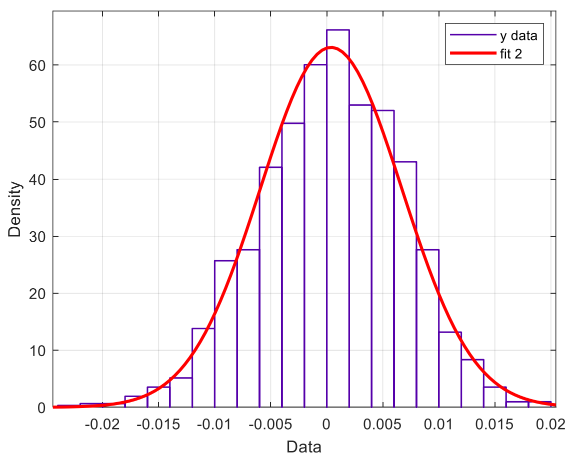

Fig. 10 Yield distribution inside mid clusters-Gaussian distribution, kurtosis=2.95

4. Results

4.1 Results of Testing of the Gaussian Periods Existence

In the first step we decompose the time series into big and small clusters. For the division into small and big clusters we use the yield limit. Then we compose only all the big clusters to one time series. We do the same in the case of small clusters. We carry out this procedure for the Nikkei 225 index, SP500 index and Crude Oil.

In all the cases of small and big clusters, we can state that we do not find any sequences with Gaussian periods.

In case of mid-clusters the situation differs. We can state that we find sequences with Gaussian distribution (utilizing the Shapiro-Wilk test) in the narrow part of the yield spread between limit 1 and limit 2 (table 1). The result of the separation of mid clusters from the Nikkei225 index volatility time series is in figure 11, with respect to the minimum length of cluster *d* and limits, limit2=limit 1+0.002, m=10.

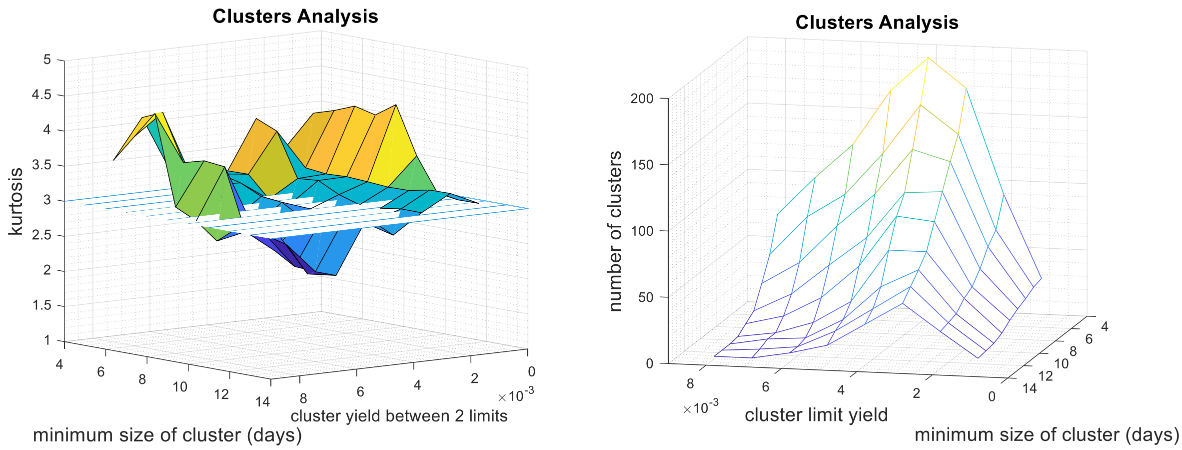

Fig. 11 Values of kurtosis (left) and number of clusters (right) with respect to d and limit 1 yield clusters (Nikkei 225)

Table 1 Values of kurtosis of mid clusters with respect to d and limit 1 yields

limit1\d	5	6	7	8	9	10	11	12	13
0,001	3,93	4,02	4,13	4,06	4,26	3,59	3,11	3,07	3,07
0,002	3,48	3,25	3,22	3,23	3,21	3,19	3,19	3,25	3,26
0,003	3,95	3,82	3,20	3,22	3,08	2,98	2,99	3,03	3,03
0,004	3,25	3,25	2,95	2,93	2,92	2,86	2,96	2,90	2,76
0,005	3,26	3,31	3,23	3,09	3,06	2,89	2,97	2,98	2,99
0,006	3,34	3,31	3,26	2,80	2,75	2,48	2,30	2,28	2,28
0,007	4,15	4,28	3,62	3,70	3,66	2,79	2,79	2,71	2,35
0,008	3,59	3,98	4,36	3,08	2,94	2,71	2,95	2,95	2,95

From tables 2-4 we see that the length of the Gaussian period in the development is always a small part of the whole yield series. In the case of the Nikkei the total length is 3714 out of 13571, for the SP500 it is 8010/23068, for Crude Oil 2526/5001. Limit 1, limit 2 and their difference were chosen in order to maximize the total length of the period where kurtosis is close to 3.

Table 2 Nikkei 225 (length of all the periods/length of yield series: 3714/13571)

	limit 1	limit 2	kurtosis	length of periods	total length of series	number of clusters
-	0	0,002	3,588519	29	13571	4
-	**0,002**	**0,004**	**3,215914**	**991**	13571	**64**
-	**0,004**	**0,006**	**2,951236**	**1557**	13571	**135**
-	**0,006**	**0,008**	**3,256812**	**720**	13571	**75**
-	0,008	0,01	4,362939	266	13571	29
-	0,01	0,012	2,487685	119	13571	14
-	0,012	0,014	2,906833	32	13571	4
total	-	-	-	3714	-	-

Table 3 SP500 (length of all the periods/length of yield series: 8010/23068)

	limit 1	limit 2	kurtosis	length of periods	total length of series	number of clusters
-	0	0,002	3,736011	103	23068	10
-	**0,002**	**0,004**	**3,466017**	**2796**	23068	**185**
-	**0,004**	**0,006**	**2,977852**	**3260**	23068	**270**
-	**0,006**	**0,008**	**3,152405**	**1343**	23068	**127**
-	0,008	0,01	2,921744	350	23068	37
-	0,01	0,012	2,357475	142	23068	16
-	0,012	0,014	2,40784	16	23068	2
total	-	-	-	8010	-	-

Table 4 Crude Oil (length of all the periods/length of yield series: 2546/5001)

	limit 1	limit 2	kurtosis	length of periods	total length of series	number of clusters
-	0	0,006	3,145939	29	5001	3
-	**0,006**	**0,012**	**2,984881**	**879**	5001	**57**
-	**0,012**	**0,018**	**3,06816**	**1154**	5001	**91**
-	0,018	0,024	2,775712	382	5001	35
-	0,024	0,03	3,474687	102	5001	11
total	-	-	-	2546	-	-

4.2 Financial Interpretation of Gaussian Periods

Probably the most realistic and frequent process that explains Gaussian periods inside clusters of different volatility is a change in trading activity. There is quite extensive empirical evidence and also documented research in this area (Ping, Peijie,. & Aying 2005; Juchelka 2014). Activity is measured by traded volume, which means the number of trades per a certain unit of time. Each trade generates one or more minimum steps (according to market rules) in the price development.

In addition to a change in activity, we also recognize other realistic interpretations of Gaussian periods inside clusters:

- Incoming economic news. The direction of the market reaction is independent, but the magnitude of the price shift creates clusters of different volatility. An example of this is periods of changing economic or political stability, and uncertainty.
- According to Fama 1975, after the arrival on the market of unexpected information of weak-form efficiency, there is a delay between the moment of the arrival of the news and its pricing. During this period, we observe damped oscillations.

5. Scientific Discussions and Conclusions

In this financial engineering research, we confirm the existence of Gaussian periods of time in the price/yield development of liquid investment instruments. Gaussian periods were defined as the part of the series where we measure the Gaussian distribution of yield. According to our research, such periods are only a part of the whole series; approximately 25-50%.

In the case of the Nikkei 225, as an example, we find Gaussian periods which represent 27% of all historical yield series. Gaussian periods of a length of 5 days and more are inside volatility clusters and can be of different volatility. Each cluster has a yield of a certain value, and the deviation from the mean is +/- 0.1%. A study with similar results has been done on the Nikkei 225 index, SP500 index and Crude Oil commodity. We also assess how to explain their existence financially.

The question which logically follows is: "Are the rest of the time series non-Gaussian?" Due to the methodology we use, we can only state that we have found time periods longer than *d* days with the yield inside spread between limit 1 and limit 2, where the process is always Gaussian. This means that if we extend the yield spread, the distribution of yield may not be Gaussian; in other words, we can expect non-Gaussian periods in clusters which contains yield that is not within such narrow limits, or in periods which are not considered to be clusters ($d<5$).

Based on empirical experience, we can state that some non-clustering mechanism that contributes to departures from normality, such as momentum, or the mean-reverting behavior of market price, exists based on feedbacks. The existence of the feedbacks is also supported by direct empirical observations and by statistical research (Stádník 2012). One type of such feedback is the typical directional dependency process, which is connected to better directional forecasting (Stádník 2013 [1]), but its practical value is still under discussion.

In the case of Gaussian periods, no improvement for directional forecasting can be expected.

References

[1] Anatolyev, S.; Gerko, A. 2005. A trading approach to testing for predictability, *Journal of Business and Economic Statistics* 23: 455-461.

[2] Baum, L., E.; Petrie, T. 1966. Statistical Inference for Probabilistic Functions of Finite State Markov Chains, *The Annals of Mathematical Statistics* 37(6): 1554–1563.

[3] Bollerslev, T. 1986. Generalized Autoregressive Conditional Heteroskedasticity, *Journal of Econometrics* 31: 307-327.

[4] Cont, R. 2005. Volatility Clustering in Financial Markets: Empirical Facts and Agent–Based Models, A Kirman & G Teyssiere (eds.): Long memory in economics, Springer.

[5] Dzikevičius, A.; Vetrov, J. 2012. Stock market analysis through business cycle approach, *Business: Theory and Practice* 13(1): 36-42.

[6] Fama, E.,F.(1970).The Journal of Finance, Vol. 25, No. 2, Papers and Proceedings of the Twenty-Eighth Annual, Meeting of the American Finance Association New York, N.Y. December, 28-30, 1969. (May, 1970), pp. 383-417

[7] Engle, R.F. 1995. *ARCH: Selected Readings*. Oxford, UK, Oxford University Press.

[8] Gontis, V.; Ruseckas, J.; Kononovičius, A. 2010. A long-range memory stochastic model of the return in financial markets, *Physica A: Statistical Mechanics and its Applications*: Volume 389, Issue 1: 100–106

[9] Henriksson, R., D.; Merton R.,C. 1981. On the Market Timing and Investment Performance of Managed Portfolios II – Statistical Procedures for Evaluating Forecasting Skills, *Journal of Business*: 513-533.

[10] Hsieh, D., A. 1991. Chaos and Nonlinear Dynamics: Application to Financial Markets. *Journal of Finance* 46: 1839–1877.

[11] Huang, J., Wang, J. (2010). Liquidity and Market Crashes, Review of Financial Studies.

[12] Jianga, J.; Lia, W.; Caia, X. 2008. Cluster behavior of a simple model in financial markets, Physica A: Statistical Mechanics and its Applications, Volume 387: 528–536

[13] Juchelka, T.(2014). Trading Volume and Volatility in the US Stock Markets, *Bachelor Thesis, Charles University in Prague, Faculty of Social Sciences, Institute of Economic Studies*

[14] Krolzig, H., M. 1997. International Business Cycles: Regime Shifts in the Stochastic Process of Economic Growth, *Applied Economics Discussion Paper 194*, University of Oxford.

[15] Larrain, M. 1991. Testing Chaos and Nonlinearities in T-bills Rates, Financial Analysts Journal, September-October: 51-62.

[16] Lux, T.; Marchesi, M. 2000. Volatility clustering in financial markets: a micro simulation of Interacting agents, *International Journal of Theoretical and Applied Finance*, 3: 675–702.

[17] Lux, T. 2011. Sentiment dynamics and stock returns: The case of the German stock market

[18] *Empirical Economics* 41 (3): 663-679.

[19] Janda, K.; Kourilek, J. (2020).Residual shape risk on natural gas market with mixed jump diffusion price dynamics, Energy Economics, Volume 85, January 2020, 104465.

[20] Masteika, S.; Rutkauskas, A., V. 2012. Research on futures trend trading strategy based on short term chart pattern, *Journal of Business Economics and Management* 13 (5): 915-930.

[21] Peters, E. 1989. Fractal Structure in the Capital Markets, *Financial Analysts Journal*/July-August: 32-37.

[22] Peters, E. 1991. Chaos and Order in the Capital Markets: A N*ew View of Cycles, Prices, and Market Volatility*, John Wiley & Sons, New York.

[23] Peters, E. 1994. Fractal Market Analysis: *Applying Chaos Theory to Investment and Economics*, John Wiley & Sons, New York.

[24] Ping, W., Peijie, W. & Aying L. (2005). Stock return volatility and trading volume: evidence from the Chinese stock market, Journal of Chinese Economic and Business Studies, 3:1, 39-54, DOI: 10.1080/14765280500040518

[25] Primbs, J., A.; Rathinam, M.2009. Trader Behavior and Its Effect on Asset Price Dynamics, *Applied Mathematical Finance 16/2: 151-181.*

[26] Rutkauskas, A., V.; Miečinskienė; A. Stasytytė, V. 2008. Investment Decisions Modelling along Sustainable Development Concept on Financial Markets, Technological and Economic Development of Economy, 14(3): 417–427.

[27] Roch, A., F.2011. Liquidity risk, price impacts and the replication problem, *Finance and Stochastics* 15/3: 399-419

[28] Schiller. 2003. *From Efficient Market Theory to Behavioral Finance*, Yale University.

[29] Stádník, B. 2011. [1]. Dynamic Financial Market Model and Its Consequences. Available at *SSRN*: http://ssrn.com/abstract=2062511.

[30] Stádník, B. 2011. [2]. Explanation of S&P500 Index Distribution Deviation from a Gaussian Curve (Dynamic Financial Market Model), *Journal of Accounting and Finance*, vol. 11(2), USA, North American Business Press, ISSN: 2158-3625.

[31] Stádník, B. 2012. Testing of Market Price Direction Dependence On US Stock Market, *Business, Management and Education*, vol. 10(2) 2012: 205–219

[32] Stádník, B. 2013. [1]. Market Price Forecasting and Profitability - How To Tame Random Walk?, *Verslas: Teorija ir Praktika / Business: Theory and Practice*, Volume 14, Issue 2, 2013: 166-176

[33] Stádník, B. (2013 [2]). Clustering Elimination, working paper, http://pozemstan.cz/ce.pdf

[34] Tsenga Jie-Jun; Sai-Ping Lia. 2011. Asset returns and volatility clustering in financial time series, *PhysicaStatistical Mechanics and its Applications*, Volume 390: 1300–1314

[35] Rey, D.,M., Schmid, M., M. (2007). Feasible momentum strategies: Evidence from the Swiss stock market, Financial Markets and Portfolio Management, Volume 21, Issue 3: 325-352

Minimum deviation enhanced portfolio replication with expectiles

Gabriele Torri[1]

Abstract

This work addresses the problem of enhanced portfolio replication, proposing a strategy based on the minimization of a novel deviation strategies based on expectiles. This measure allows to account asymmetrically for the differences between the portfolio and the benchmark, favouring positive deviations compared to negative ones. We show that the model nests the minimum TEV replication scheme. The empirical applications focuses on the Standard and Poor's 100 index, for which we create replicating portfolios with a positive expected excess return. The results show that the replication scheme proposed here allows to overperform the benchmark out-of-sample, and that portfolios with $\boldsymbol{\tau < 0.5}$ allow to reduce the lower tail risk measured in terms of CVAR compared to the minimum TEV portfolio. The resulting portfolios show a sufficient level of diversification, and a controlled turnover.

Key words

Tail risk; expectiles; portfolio optimization; enhanced portfolio replication

JEL Classification: C01, C44, C58

1. Introduction

Enhanced indexing strategies aim to replicate the performances of an index or a benchmark, while at the same time trying to minimize risks, looking for extra performances, or providing benefits to the investors. Such result can be obtained by applying simple heuristic rules (e.g. overweighting stocks with low variance), or by solving optimization problems, directly aimed at improving the distributional properties of the portfolio returns [1]. In this work, we aim to set up an investment strategy that aims to overperform over the benchmark, while minimizing the distance with the benchmark. The distance to the benchmark, is typically measured in terms of tracking error volatility, (i.e. the volatility of the difference between the returns of the portfolio and the ones of the benchmark). Here we use instead a deviation measure based on the expectile, that allows to treat asymmetrically the positive and negative deviations from the benchmark, thus allowing to better control for downward risk. This deviation measure has been used by [2] in the context of portfolio optimization. Based on such work, we set up the quadratic programming formulation for the problem, and we describe an implementation based on the replication of the Standard and Poor's 100 equity index. The paper is structured as follows: Section 2 describes the expectile deviation measure and introduces the optimization problem, Section 3 describes the data and the set up of the empirical analysis and presents the results, Section 4 concludes.

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2. Expectile deviation measure

[2] shows how to develop despersion measure using expectiles under the quadrangle framework in [3]. In particular, the dispersion measure can be computed as follows:

$$\mathcal{D}_\zeta(X) = \min_\xi \mathbb{E}[\eta_\tau(X-\xi)],$$

where $\eta_\tau(X)$ is the error function used in the expectile regression [4], defined as:

$$\eta_\tau(X) = (X-\xi)_+^2 + \frac{1-\tau}{\tau}(X-\xi)_-^2 \qquad (1)$$

with $\tau \in (0,1)$, and $X \in L^2$.
This measure, related to the loss function in the expectile regression proposed by [4], allows to account for asymmetry in the distribution of returns, overweighting the negative deviations instead of the positive ones. Intuitively, the measure in (1) generalizes variance in an asymmetric framework. Indeed, by setting $\tau = 0.5$, we have

$$\eta_\tau(X) = (X-\xi)_+^2 + (X-\xi)_-^2 = (X-\xi)^2 = \sigma^2(X).$$

The measure $\eta_\tau(X)$, is also related to the quantile deviation measure proposed by [5], with the difference of using a quadratic, instead of a linear function.
In a portfolio optimization context, the minimization of deviation from the benchmark with an excess return constraint can then be obtained using the specification of the optimization problem for the minimum deviation portfolio in [2]. Let us consider an n-variate random variable denoting stock returns where n is the number of assets, a scalar κ representing a target expected return, a vector of weights $\boldsymbol{w} \in \mathbb{R}^n$, and a univariate random variable R_w denoting the returns of a benchmark portfolio (e.g. a market index). We can address the enhanced portfolio replication problem by introducing new random variables $\tilde{R}_i = R_i - R_w \quad \forall\, i = 1, \dots, n$. Following [2], the problem is then:

$$\begin{aligned} \min_{\boldsymbol{w}\in\mathbb{R}_+^n,\xi\in\mathbb{R}} \; & \mathbb{E}\left[\eta_\tau\left(\tilde{R}\boldsymbol{w} - \xi\right)\right] \qquad (2) \\ s.t.\, & \boldsymbol{w}'1 = \mathbf{1} \\ & \mathbb{E}[\tilde{R}\boldsymbol{w}] = \kappa \\ & w_i \geq 0 \;\; \forall\, i = 1, \dots, n \end{aligned}$$

The vector of optimal weights $\boldsymbol{w}^*$ then aims to reduce as much as possible the deviation from the benchmark, while obtaining a superior excess return. The usage of the measure $\eta_\tau(X)$ instead of standard deviation or variance (as in the case of tracking error volatility), allows to differentiate between positive and negative deviations from the benchmark. We introduced a non-negativity constraint as a common practice in portfolio optimization, in order to increase out-of-sample portfolio stability, reduce turnover, and obtain portfolios more suitable for real-world investment. Alternatively, we could have used other techniques to regularize the portfolio, such as adding a penalty to the weights as in [2] or, in the case of minumum variance portfolio, regularize the covariance matrix of the data [6]. We opted for the non-negativity constraints as this approach yields non-leveraged portfolios, that are typically more appreciated by passive, unsophisticated investors. We leave the analysis and implementation of penalized enhanced portfolio replication problem to future works.

Concerning the practical implementation, following [Torri], it is possible to recast the problem as a quadratic program, that can be solved efficiently using standard optimization software:

$$\begin{aligned}
&\min_{\boldsymbol{w}\in\mathbb{R}_+^n;\ \xi\in\mathbb{R};\ \phi,\gamma\in\mathbb{R}_+^t} \mathbb{E}\left[\tau\sum_{i=1}^{t}\gamma_i^2+(1+\tau)\sum_{i=1}^{t}\phi_i^2\right]\\
&s.t. \qquad \gamma_i-\phi_i=\tilde{R}_i\boldsymbol{w}\quad \forall i=1,\dots,t\\
&\qquad\qquad \boldsymbol{w}'1=\mathbf{1}\\
&\qquad\qquad \mathbb{E}[\tilde{R}\boldsymbol{w}]=\kappa
\end{aligned}$$

where t is the number of discrete observations of the portfolios. For convenience, we refer to this problem as the TEEV portfolio (Tracking Error Expectile Deviation).
The computational time required to solve the problem is limited: considering 97 assets and windows of 250 observations, each portfolio optimization requires typically 0.05 seconds using the quadprog function in Matlab 2019b on an i7-9750H at 4.2 GHz. It is therefore suitable also for investment environments that require very frequent optimal portfolio rebalancement. We also underline that the framework could be enriched, for instance by including first and second order stochastic dominance constraints in the optimization problem.

a) Risk and deviation measures

Although the portfolio allocation techniques based on the mean-variance [7], or mean-deviation approach have a widespread popularity, recently the literature introduced enhanced portfolio replication approaches based on the *risk measures* rather than *deviation measures* (see [8]. [Rockafellar] introduces the risk quadrangle framework, that connects different measures used in portfolio theory, and allows to generalize many investment frameworks. In particular, given a *regular* deviation measure $\mathcal{D}(X)$ (see [3] for the formal definitions), it is possible to identify a regular risk measure $\mathcal{R}(X)$ as follows:

$$\mathcal{R}(X) = \mathcal{D}(X) - \mathbb{E}[X]$$

[2] shows that the minimization of portfolio's deviation $\mathcal{D}(X)$ under the presence of an expected return constraint is equivalent to the minimization of the associated portfolio risk $\mathcal{R}(X)$. Applied to the enhanced portfolio replication framework, this means that by minimizing the deviations from the benchmark, measured using $\mathcal{D}(\tilde{R}\boldsymbol{w})$, under an overperformance constraint, we also minimize the corresponding risk measure $\mathcal{R}(\tilde{R}\boldsymbol{w})$.

3. Empirical analysis

a) Data and set up

To test the empirical performances of the enhanced portfolio replication scheme proposed, we consider a dataset composed by all the constituents of the Standard &Poor's 100 index (SP100) in the period 01/01/2015-30/05/2020. Considering only the assets whose price is available for the entire period, the dataset is composed of 97 stocks. We used a rolling windows scheme for the investment, with estimation windows of 250 daily observations (approximately 1 year). The portfolio is calibrated every 20 trading days (approximately 1 month). The dataset includes a periods of relative calm in the market (until 2018, and periods of greater turmoil (e.g. the last quarter of 2018 in which the growing trend after the global financial crisis in 2008 saw the first main alt, and the first half of 2020, that due to the COVID 19 crisis witnessed the worse drop in market capitalization since the great depression.
We test the specification in (2) with different levels of the parameter τ that regulates the asymmetry in the deviation measure (0.05,0.1,0.2,0.5,0.9,0.95), and with two different target

excess returns for the portfolio (2.5% and 5% annualized expected excess return compared to the benchmark). We underline that the problem (2) with $\tau = 0.5$ corresponds to the mean-variance framework, that is, to the minimization of the tracking error volatility (TEV) subject to an excess return constraint. As pointed out before, we introduce a non-negativity constraint for the weights, to regularize the solution and to identify asset allocations more suitable to passive unsophisticated investors.

b) Results

We present here the results of the analysis. Table 1 reports the in-sample performances of the replicating portfolios with excess return of 5% per year, reporting Tracking Error Volatility (TEV), the excess return over the benchmark, Information Ratio (IR)[2], and the sample $CVAR_{95\%}$ of the portfolio. The reported values are the average across all the estimation windows. Table 2 reports the same results for the portfolios with an expected excess return of 2.5% per year. The portfolios with low τ should be able to better control for downward movements, the one with $\tau = 0.5$ corresponds to the minimum TEV portfolios, while the ones with $\tau > 0.5$ should reduce the upward deviations (for this reason the latters should not be considered in practical applications, we report them for comparison purposes). We see from Table 1 that the in-sample performances are relatively similar to each other in terms of excess return and CVAR. The TEV is lowest for $\tau = 0.5$, and assumes values only marginally higher for the portfolios with $\tau = 0.05$ and $\tau = 0.01$. The TEV is instead higher for portfolios with $\tau > 0.5$. Consequently, these differences are reflected by the information error. The results are qualitatively similar for the portfolios in Table 2, with lower expected excess return.

Table 1: In-sample performance of the portfolio. 250 observations in sample, 2017-20,k=5% annual, non neg.

Tau	TEV (%)	Excess return (%)	IR (%)	$CVAR_{95\%}$ (%)
0.05	2.43	4.99	205.35	2.20
0.1	2.40	4.97	207.45	2.19
0.5	2.35	4.97	211.05	2.20
0.9	2.68	4.94	184.08	2.21
0.95	2.84	4.96	174.56	2.21

Table 2: In-sample performance of the portfolio. 250 observations in sample, 2017-20,k=2.5% annual, non neg.

Tau	TEV (%)	Excess return (%)	IR (%)	$CVAR_{95\%}$ (%)
0.05	2.38	2.48	104.11	2.20
0.1	2.34	2.48	105.66	2.20
0.5	2.30	2.48	107.59	2.20
0.9	2.65	2.46	92.84	2.21
0.95	2.75	2.45	88.88	2.21

Looking at the out-of-sample results (Tables 3 and 4), we see that on average the excess return is lower compared to the in-sample and the risk (measures as either TEV or CVAR) is higher. Comparing asset allocations with different τ, we see that the portfolios with lowest taus (i.e. the ones that should better control for the lower tail risk), are indeed the ones the show the smallest $CVAR_{95\%}$. All the portfolios show positive excess return over the benchmark, although not as high as the in-sample, and rather similar between the portfolios with 2.5% and 5% expected excess return (with the exception of the portfolios with $\tau = 0.9$ and 0.95, where the excess return for the portfolios with target 2.5% is much smaller. Looking at Figure 1, reporting

[2] Information ratio is computed as: $IR = Excess\ return/TEV$.

the ex-post wealth of the portfolio computed using the TEED approach with different τ, as well as the performances of the benchmark, we see that all the approaches, especially the ones with τ equal or smaller than 0.5, outperform the benchmark in terms of total wealth. The enhanced tracking portfolios showed in particular positive over-performance until the end of 2019, while in the crisis period the replicating portfolios showed higher drops compared to the benchmark (still, the final wealth remained higher than the benchmark). Such sharper drop was not present in previous market downturns, and could be related to differences in sectorial exposures (different industrial sectors had very different reactions to the COVID 19 crisis, leading the replicating portfolio to have different effect compared to the benchmark. Moreover, the unpredictable changes in the economic context makes particularly difficult to optimize the portfolios for the crisis period using past data).

Finally, Table 5 reports two portfolio statistics relevant for investors: the first is the reciprocal of the Herfindahl–Hirschman Index, that approximates the effective number of stocks in the portfolio (a highly concentrated portfolio will result in a lower number of effective assets, while in an equally weighted portfolio the effective number is equal to the number of assets included), as well as the average portfolio turnover (the ratio of portfolio holdings sold at each rebalancing). We see that the number of effective assets in the portfolio is lower than 97, but still relatively high, being higher than 25 for all the portfolios. The turnover is in all cases smaller than 30%, and is higher for the portfolios with $\tau < 0.5$. Both the turnover and the concentration of the portfolio could be controlled by the investors by imposing additional constraints to the investment problem. Such contraints would be particularly useful in case on portfolios without non-negativity constraints.

Table 3: Out-of-sample performance of the portfolio. 250 observations, 2017-20, 5% annual, non negative

Tau	TEV (%)	Excess return (%)	IR (%)	$CVAR_{95\%}$ (%)
0.05	3.24	0.88	27.03	3.02
0.1	3.21	0.98	30.62	3.02
0.5	3.20	1.13	35.22	3.04
0.9	3.33	0.81	24.33	3.07
0.95	3.53	0.62	17.65	3.08

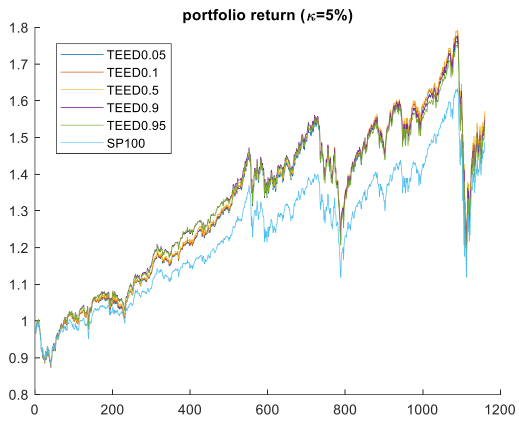
Table 4: Out-of-sample performance of the portfolio. 250 observations, 2017-20, 2.5% annual, non neg.

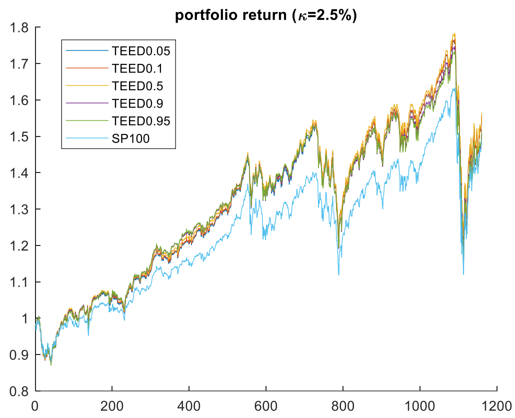
Tau	TEV (%)	Excess return (%)	IR (%)	$CVAR_{95\%}$ (%)
0.05	3.37	0.89	26.30	3.02
0.1	3.36	0.84	25.10	3.02
0.5	3.32	1.02	30.59	3.04
0.9	3.50	0.24	6.73	3.07
0.95	3.64	0.38	10.33	3.09

Table 5: Portfolio statistics. 1/HHI approximates the effective number of active assets. Turnover denotes the average percentage of assets changed at each iteration.

	κ = 5% annual		κ = 2.5% annual	
Tau	1/HHI	Turnover (%)	1/HHI	Turnover (%)
0.05	28.54	28.28	28.23	27.77
0.1	29.09	26.61	28.98	25.32
0.5	29.99	21.67	29.41	21.57
0.9	44.56	19.34	42.98	20.16
0.95	47.47	21.24	47.03	18.46

Figure 1: Ex-post portfolio wealth





c) Conclusion

This work addresses the problem of enhanced portfolio replication, proposing a strategy based on the minimization of a novel deviation strategies based on expectiles. This deviation measure, introduced by [2] in portfolio optimization, allows to account asymmetrically for the differences between the portfolio and the benchmark, favouring positive deviations compared to negative ones. We show that the minimization of the deviation measure under an excess return constaint corresponds to the minimization of the corresponding risk measure identified in the risk quadrangle framework of [3]. Moreover, the model proposed here nests the minimum TEV replication scheme, that is obtained using a value of $\mathbf{0.5}$ for the parameter $\boldsymbol{\tau}$. The empirical applications focuses on the Standard and Poor's 100 index, for which we create long-only replicating portfolios with a positive expected excess return. The results show that the replication scheme proposed here allows to overperform the benchmark out-of-sample, and that portfolios with $\boldsymbol{\tau < 0.5}$ allow to reduce the lower tail risk measured in terms of CVAR compared to the minimum TEV portfolio. Finally, the resulting portfolios show a sufficient level of diversification, and a controlled turnover. Future works may examine the effect of introducing penalization techniques on the portfolio to further improve the out-of-sample stability, or to introduce other constraints, such as first and second order stochastic dominance, to improve performances.

Acknowledgement

The author acknowledges the support of the Czech Science Foundation (GACR) under Project 19-11965S. SGS project of VSB-TUO under number SP2020/11 is acknowledged as well.

References

[1] Cassader, M. (2015). Benchmark Tracking Portfolio Problems with Stochastic Ordering Constraints.

[2] Giacometti, R., Torri, G., & Paterlini, S. (2020). Tail Risks in Large Portfolio Selection: Penalized Quantile and Expectile Minimum Deviation Models. *Quantitative Finance*, Forthcoming. *Available at SSRN 3587466*.

[3] Rockafellar, R. T., & Uryasev, S. (2013). The fundamental risk quadrangle in risk management, optimization and statistical estimation. *Surveys in Operations Research and Management Science*, *18*(1-2), 33-53.Boughton, J.M. (2002). The Bretton Woods proposal: an brief look. *Political Science Quarterly*, 42(6), p. 564.

[4] Newey, W. K., & Powell, J. L. (1987). Asymmetric least squares estimation and testing. *Econometrica: Journal of the Econometric Society*, 819-847.

[5] Bassett Jr, G. W., Koenker, R., & Kordas, G. (2004). Pessimistic portfolio allocation and Choquet expected utility. *Journal of financial econometrics*, *2*(4), 477-492.

[6] Torri, G., Giacometti, R., & Paterlini, S. (2019). Sparse precision matrices for minimum variance portfolios. *Computational Management Science*, *16*(3), 375-400.

[7] Markowitz, H. (1952). Portfolio selection. The Journal of Finance, 7(1):77-91.

[8] Guastaroba, G., Mansini, R., Ogryczak, W., & Speranza, M. G. (2020). Enhanced index tracking with CVaR-based ratio measures. *Annals of Operations Research*, 1-49.

Statistical Testing of Portfolio Performance Ratios

Anlan Wang, Aleš Kresta[1]

Abstract

Testing the performance of a portfolio is the significant step in the efficiency tests of the portfolio optimization. In this paper, we construct the efficient portfolios by applying the mean-variance model under the assumption of risk aversion of investors. The chosen dataset in the empirical analysis covers the 2007-2008 global financial crisis, which's background is that the financial institutions were damaged globally due to the excessive risk-taking by banks combined with the bursting of the housing bubble in the United States. The goal of this paper is to make the statistical testing of the obtained strategy portfolio's performance ratios, which included the return-measured ratios and the risk-measured ratios. According to the statistical testing, we conclude the contributions as well as the limiatations of the obtained strategies on the basis of the sample data.

Key words

Financial crisis, Mean-Variance model, statistical testing, Jensen's alpha, Treynor ratio, Sharpe ratio, maximum drawdown, mean absolute deviation, conditional value at risk

JEL Classification: G11, G17

1. Introduction

Financial portfolio optimization problems aim at finding the efficient investment strategies considering the optimal allocation of limited funds. Markowitz [10] proposed the Mean-Variance model which considered a portfolio's return and risk simultaneously. In financial markets, investors always balance the trade-offs between the risk and return of a portfolio according to their subjective preferences.

In this paper, the goal is to make the statistical testing of the obtained strategy portfolio's performance ratios, which included the return-measured ratios and the risk-measured ratios. In section 2, we make the literature review based on the pioneers' studies. The review is constructed from different aspects, which includes the limitations and extensions of the classical Markowitz model as well as the review of the applications of benchmarks in the previous portfolio optimization researches. In section 3, we describe the methodology applied in this paper with formulas. In section 4, we make the statistical testing of the obtained strategy portfolio's performance ratios based on the empirical analysis. The conclusion of this paper is made in section 5.

2. Literature Review

In the financial decision-making field, portfolio optimization involves the efficient investment strategies considering the optimal allocation of limited funds. On the premise of the liquidity and security of the investment funds, the common method of optimization is to balance the trade-offs between return and risk of a portfolio according to investors' subjective preferences. Markowitz

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[10] proposed the Mean-Variance model considering a portfolio's return and risk simultaneously, moreover, a higher degree of risk means a higher potential return.

In the last five decades, along with the considerations of real-life conditions and enhancements of algorithms, additional constraints have been developed to the early classical Markowitz model, for example, Fulga [3] presented an approach which incorporates the loss aversion preferences in the mean-risk frame-work, and the efficiency of the proposed approach is tested against the classical Markowitz model. Besides subjective preferences, more factors of portfolios are involved in later studies since 1950s, such as the liquidity of the portfolio, the transaction costs, the diversification degree of investments, the social responsibility and so on.

However, from later studies on portfolio optimization problems, the difficulties in the applications of the classical Markowitz model appeared. On one hand, there exists the computational complexity associated with the covariance matrix, so, as an alternative to the risk measure in the Markowitz model, the variance is replaced with mean absolute deviation (henceforth MAD) in the model proposed by Konno and Yamazaki [5], which requires no computation or inversion of a covariance matrix, it solves a linear optimization rather than the quadratic optimization in Markowitz model. While in practical analysis, many cases studied by pioneers indicated that the Mean-Variance model and Mean-MAD model usually generated similar portfolio optimization strategies, and the common disadvantage for both these two models is that the investments to assets in a portfolio are not well diversified.

On the other hand, according to several existing studies of portfolio optimization problems, we find the future returns of stocks in a portfolio are usually obtained by estimating the historical data of the stocks, and the uncertainty associated with the returns of stocks or portfolio is described as randomness. However, recently, more studies highlight the importance of considering the uncertainty of returns as fuzziness. What's more, in Mansour et al. [9], they pointed out that the investor is not able to define precisely his/her goals for stocks in his/her portfolio, and the fuzzy theory seems an alternative way to describe an imprecise or fuzzy environment, for example when the subjective behavior of financial investors is taken into account. In Tanaka and Guo [13], as extensions to Markowitz model, they proposed two kinds of portfolio optimization models, the first model is based on fuzzy probabilities and aims to minimize the variance of the portfolio return while the other one utilizes possibility distributions and minimizes the spread of the portfolio return.

Although more and more enhanced portfolio optimization models are proposed in recent years, at the point of scientific precision, measuring the performance of the proposed optimization model is just one side of the work, for another side, it's significant to involve a benchmark in the analysis to verify the efficiency of the models, see Kresta and Wang [6]. Various benchmarks can be found in scientific literature, however, there are few groups of benchmarks, which are generally applied. The most commonly applied benchmark is the 1/N strategy, which is easy to implement because it does not rely on estimations of the asset returns, the components assets of the naive portfolio are invested at equal weights. DeMiguel et al. [2] evaluated and compared the performance of several optimization methods with respect to the performance of the 1/N strategy, they found that the effect of estimation error on return probability distribution is large in those optimization models, but this type of error can be avoided by using the 1/N weights. Another commonly used benchmark is the indices, for example, Solares, et al. [12] used the prices of Dow Jones Industrial Average index (DJI) as the benchmark in their research. In the portfolio optimization researches, the classical Mean-Variance model or alternatively Mean-VaR model are also usually applied as benchmarks to test the efficiency of the new proposed approaches, see. e.g. Fulga [3], Rankovic et al. [11], Lwin et al. [8] or Babazadeh and Esfahanipour [1]. In this paper, we apply the 1/N strategy and the DJI index as the benchmarks.

3. Portfolio Optimization Methodology under Mean-Risk Framework

3.1 Mean-Variance model

In the Mean-Variance model, we analyze the inter-relationship between mean and variance of a portfolio's returns in a certain period. We denote x_i as the weight of asset i in a portfolio investment, and short sales are excluded, so the values of x_i satisfy $x_i \geq 0$ for all assets. We denote $E(R_i)$ as the expected return of asset i, and in our case we suppose that the expected stock return is identical to the average of the real returns within historical period, then the expected return of a portfolio $E(R_p)$ can be calculated as follow,

$$E(R_p) = \sum_{i=1}^{N} x_i \cdot E(R_i) = x^T \cdot E(R) \tag{1}$$

where $x = [x_1, x_2, \dots, x_N]^T$, $E(R) = [\,E(R_1), E(R_2), \dots, E(R_N)]^T$, the sum of x_i in a portfolio equals to 1, and the $E(R_p)$ is the weighted average of $E(R_i)$. The variance of a portfolio's returns in a certain period, which is denoted as ${\sigma_p}^2$, is regarded as the risk measure of a portfolio in the Mean-Variance model. ${\sigma_p}^2$ is calculated by the $N \times N$ covariance matrix $\boldsymbol{Q} = [\sigma_{i,j}, i = 1, 2, \dots, N, j = 1, 2, \dots, N]$ for all component asset pairs (i, j) in a portfolio, the calculation of ${\sigma_p}^2$ is shown in equation (2), where the σ_p in equation (3) is the standard deviation (henceforth STD) of a portfolio's returns.

$${\sigma_p}^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i \cdot \sigma_{i,j} \cdot x_j = x^T \cdot \boldsymbol{Q} \cdot x \tag{2}$$

$$\sigma_p = \sqrt{{\sigma_p}^2} \tag{3}$$

3.1.1 Fuzzy probability strategy

Tanaka et al. [13] proposed a Fuzzy Probability model, which combines the probability distribution of stocks returns in Markowitz's model with fuzzy theory, in our analysis, we apply this model to handle uncertainty in the stocks returns probability distribution. In the Fuzzy Probability model, we consider not only the historical stocks returns $\{r_{i,m}, i = 1, \dots, N, m = i, \dots, M\}$, but also possibility grades $\{h_m, m = i, \dots, M\}$, which reflects a similarity degree between the future state of the stock market and the state of m_{th} sample offered by experts, and according to Tanaka et al. [13], we define h_m as,

$$h_m = 0.1 + 0.3 \cdot (m - 1)/(M - 1) \tag{4}$$

these grades are applied to determine the fuzzy expected returns of stocks and fuzzy covariance matrix for a given data. We denote $E(r_i^F)$ as the expected return of stock i under Fuzzy Probability strategy, if given the historical stocks returns $r_{i,m}$ and possibility grades h_m, the fuzzy weighted expected return of stock i can be calculated as equation (5), where m is the number of historical observations of returns of stock i. The fuzzy weighted covariance matrix $\boldsymbol{Q^F} = [\sigma_{i,j}^F, i = 1, \dots, N, j = 1, \dots, N]$ can be defined by equation (6), we can see when the given data have the same possibility grades, the Fuzzy Probability model is just equivalent to the Markowitz's model.

$$E(r_i^F) = \frac{\sum_{m=1}^{M} h_m \cdot r_{i,m}}{\sum_{m=1}^{M} h_m} \tag{5}$$

$$\sigma_{i,j}^F = \frac{\sum_{m=1}^{M} \left(r_{i,m} - E(r_i^F)\right) \cdot \left(r_{j,m} - E(r_j^F)\right) \cdot h_m}{\sum_{m=1}^{m} h_m} \tag{6}$$

3.2 Mean-MAD model

Comparing with the Mean-Variance model, the risk measure of the portfolio is replaced by mean absolute deviation of the portfolio's returns in the Mean-MAD model, the calculation of mean absolute deviation is shown in equation (7), where T is number of observations, $R_{i,t}$ is the return of asset i for each time t.

$$MAD = \frac{\sum_{t=1}^{T} \sum_{i=1}^{N} x_i \cdot \left|R_{i,t} - E(R_i)\right|}{T} \tag{7}$$

3.3 Mean-CVaR model

Under the mean-risk framework, the Mean-Conditional Value at Risk (henceforth Mean-CVaR) model focuses on the measure of the expected shortfall for a portfolio. As we know, VaR is defined as the worst-case loss associated with a given probability and a time horizon. However, rather than the application of VaR, the CVaR which indicates the expected loss under the condition of exceeding VaR is applied as the risk measure in our analysis. The CVaR for a portfolio is defined as follow,

$$CVaR_\alpha(x) = \frac{1}{1-\alpha} \int_{f(x,y) \geq VaR_\alpha(x)} f(x,y) p(y) dy \tag{8}$$

where x is a portfolio satisfies $x \in X$ (X is the set of available portfolios), α is the probability level such as that $0 < \alpha < 1$, it implies that the probability of the portfolio's returns falling below the value $VaR_\alpha(x)$ is $1-\alpha$, and in our analysis we set $\alpha = 95\%$. $f(x,y)$ is a loss function for a portfolio x and asset return y, $p(y)$ is the probability density function for asset return y, VaR_α is the VaR of portfolio x at probability level α and it can be defined as follow.

$$VaR_\alpha(x) = \min\{\gamma : \Pr[f(x,Y) \leq \gamma] \geq \alpha\} \tag{9}$$

3.4 Portfolio optimization

Given the level of the portfolio's return, by minimizing the risk of the portfolio, we can get a set of efficient portfolios under the Mean-Variance framework, in our case, the portfolio risk is measured by ${\sigma_p}^2$, MAD and $CVaR_\alpha(x)$.

$$\text{minimize } {\sigma_p}^2 \text{ (or } MAD \text{ or } CVaR_\alpha(x))$$

$$\text{subject to} \tag{10}$$

$$\sum_{i=1}^{N} x_i = 1$$

$$x_i \geq 0, i = 1, \ldots, N$$

3.5 Hypothesis tests

In our analysis, to make the statistical testing of the obtained strategy portfolios' performance ratios, we generate 3,000 random-weights portfolios to make the hypothesis tests by comparing the performances of the random-weights portfolios with those of the strategy portfolios.

Random-weights portfolio, as it literally means, the weights of assets in each random portfolio are generated randomly, in our case we set up 3,000 random portfolios, and in each portfolio the sum of the weights x_i equals to 1. For the generation of random weights, we choose $y \in [0,1]^{N-1}$ uniformly by means of $N-1$ uniform reals in the interval $[0,1]$, and sort the coefficients so that $0 \leq y_1 \leq \cdots \leq y_{N-1}$, then x_i can be shown as in equation.

$$x_i = (y_1, y_2 - y_1, y_3 - y_2, \ldots, y_{N-1} - y_{N-2}, 1 - y_{N-1}) \tag{11}$$

As we know that a hypothesis test relies on the method of an indirect proof, that is, to prove the hypothesis that we would like to demonstrate as correct, we show that an opposing hypothesis is incorrect. In our case, the strategy portfolios under the optimization methods are more likely to be demonstrated as efficient, so according to the rule of hypothesis tests, we can make the null hypothesis and alternative hypothesis as follows:

null hypothesis—H_0: $P_s = E(P_r)$,

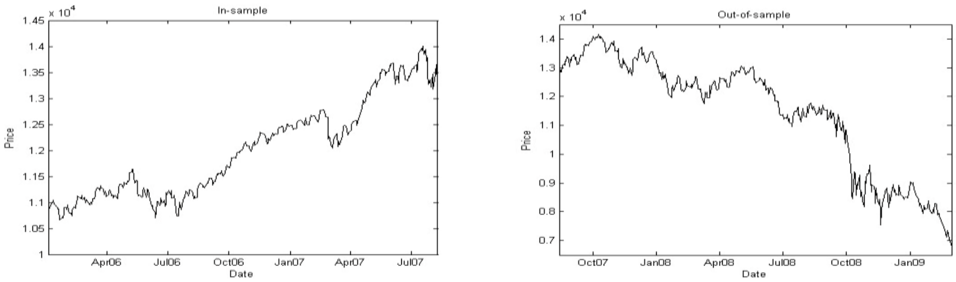
alternative hypothesis—H_A: $P_s > E(P_r)$

where P_s is the performance measure of the strategy portfolio, and $E(P_r)$ is the expected value of the performance measure of the random-weights portfolio. In our hypothesis test, the p-value is the proportion of the random-weights portfolios which meet the condition $P_S < P_r$ in the total number of random-weights portfolios. We set the significance level to 10%, when p-value is less than 10%, then we reject H_0, which means the performance of the strategy portfolio is better than that of the random-weights portfolio, so the strategy portfolio is efficient; when p-value is no less than 10%, then we fail to reject H_0, which means the performance of the strategy portfolio makes no difference from that of the random-weights portfolio, so the strategy portfolio is inefficient in this case.

The performance ratios of the statistical testing in our empirical analysis are Jensen's alpha, Treynor ratio, Sharpe ratio, STD, MAD, CVaR and MDD.

4. Empirical Analysis

Figure 1: Historical price evolutions of DJI



The chosen sample data is the daily closing prices of the components of Dow Jones Industrial Average index (DJI). In this case, there are 28 component stocks included, the two missing stocks is the stock of Visa Inc. and the stock of Dow Inc. due to the incomplete data in the chosen period. The chosen period in this case covers the 2007-2009 global financial crisis. We divide the whole sample data into two parts. The in-sample part is from January 3rd, 2006 to

August 10th, 2007 and the out-of-sample part is from August 13th, 2007 to March 2nd, 2009. In Figure 1 we show the price evolutions of DJI in these two periods, we can see in the in-sample period the price of DJI shows an increasing trend, however, in the out-of-sample period the price kept decreasing resulting from the influence of the global financial crisis.

4.1 Portfolios' selection

We obtain the strategy portfolios from the in-sample period, and then make the statistical testing of the performance ratios of the obtained portfolios in the out-of-sample period, in this sense, we can test the efficiency of the mean-risk strategies based on the background of financial crisis.

The strategy portfolios are obtained by applying the models introduced in section 3. For each obtained strategy portfolio, it lies on the efficient frontier of the corresponding mean-risk optimization model. Under each mean-risk model, from all the obtained efficient portfolios, we selected the maximum-return (the return is measured by mean of return, Jensen's alpha, Treynor ratio and Sharpe ratio) portfolios and the minimum-risk (the risk is measured by STD, MAD, CVaR and MDD) portfolios. We show the performance of the selected obtained portfolios in the in-sample period in Table 1.

Table 1: Selected strategy portfolios' performances in the in-sample period

	Portfoilio	**Mean of Reutun**	**Jensen's Alpha**	**Sharpe Ratio**	**Treynor Ratio**	**STD**	**MAD**	**CVaR**	**MDD**
Mean-Variance	Max-(Mean of Return&Jensen's Alpha&Treynor Ratio)	0.160%	0.102%	8.191%	0.117%	1.726%	1.191%	0.062%	21.529%
	Max-Sharpe Ratio	0.108%	0.058%	11.836%	0.095%	0.752%	0.563%	-0.023%	7.620%
	Min-(STD&MAD)	0.043%	0.001%	4.223%	0.034%	0.557%	0.413%	0.017%	5.418%
	Min-CVaR	0.095%	0.048%	11.459%	0.088%	0.662%	0.499%	-0.025%	6.065%
	Min-MDD	0.069%	0.025%	8.530%	0.065%	0.583%	0.437%	-0.008%	4.528%
Mean-Variance-Fuzzy	Max-(Mean of Return&Jensen's Alpha&Treynor Ratio)	0.157%	0.099%	8.076%	0.116%	1.705%	1.297%	0.042%	27.983%
	Max-Sharpe Ratio	0.104%	0.055%	11.393%	0.094%	0.742%	0.568%	-0.027%	8.259%
	Min-(STD&MAD)	0.040%	-0.003%	3.666%	0.029%	0.561%	0.420%	0.021%	5.507%
	Min-CVaR	0.104%	0.055%	11.393%	0.094%	0.742%	0.568%	-0.027%	8.259%
	Min-MDD	0.056%	0.013%	6.496%	0.050%	0.567%	0.423%	0.003%	4.855%
Mean-MAD	Max-(Mean of Return&Jensen's Alpha&Treynor Ratio)	0.160%	0.102%	8.191%	0.117%	1.726%	1.191%	0.062%	21.529%
	Max-Sharpe Ratio	0.108%	0.058%	11.676%	0.092%	0.765%	0.560%	-0.021%	7.447%
	Min-(STD&MAD)	0.043%	0.001%	4.326%	0.034%	0.561%	0.410%	0.016%	5.295%
	Min-CVaR	0.095%	0.047%	11.372%	0.087%	0.671%	0.497%	-0.025%	6.302%
	Min-MDD	0.082%	0.037%	10.191%	0.078%	0.621%	0.460%	-0.017%	4.637%
Mean-CVaR	Max-(Mean of Return&Jensen's Alpha&Treynor Ratio)	0.160%	0.102%	8.191%	0.117%	1.726%	1.191%	0.062%	21.529%
	Max-Sharpe Ratio	0.111%	0.061%	11.452%	0.099%	0.800%	0.600%	-0.014%	7.301%
	Min-(STD&MAD)	0.048%	0.006%	4.958%	0.041%	0.591%	0.444%	0.019%	5.419%
	Min-CVaR	0.098%	0.051%	11.216%	0.092%	0.705%	0.540%	-0.018%	5.481%
	Min-MDD	0.086%	0.041%	10.193%	0.086%	0.654%	0.506%	-0.015%	5.157%
	DJI	0.052%	-	4.778%	0.033%	0.687%	0.490%	0.022%	8.044%
	Naive	0.058%	0.006%	5.432%	0.038%	0.713%	0.514%	0.019%	7.417%

From Table 1, we find that the maximum-mean of return portfolio, maximum-Jensen's Alpha portfolio and maximum-Treynor ratio portfolio are the same one under each mean-risk model. Except for the minimum-STD&MAD portfolio under the Mean-Variance-Fuzzy model, in all the other selected portfolios, we also find that the values of all performance measures are positive, that is the result of the increasing trend of stocks price in the in-sample period. What's more, it's obvious that the portfolios which targeted at the return's maximization carry more risk in the meanwhile. Comparing with the DJI index and the Naive portfolio, the values of performance measures and the risk measures of strategy portfolios float up and down those values of the DJI index and Naive portfolio.

4.2 Statistical testing

The portfolios selected from the in-sample period are applied to make the back-tests based on the out-of-sample data, in the meanwhile, we also generate 3,000 random-weights portfolios to make the hypothesis tests by comparing the performances of the random-weights portfolios with those of the selected strategy portfolios, the key statistics of the 3,000 random-weights portfolios' performances are shown in Table 2, and the statistical testing results of the hypothesis tests are shown in Table 3.

Table 2: Statistical testing of 3,000 random-weights portfolios

	Mean of Reutun	Jensen's Alpha	Sharpe Ratio	Treynor Ratio	STD	MAD	CVaR	MDD
mean	-0.11%	0.04%	-172.64%	-3.81%	2.23%	1.50%	0.39%	45.40%
standard deviation	0.02%	0.04%	9.07%	0.05%	0.12%	0.09%	0.03%	3.46%
skewness	-32.98%	-7.83%	-15.60%	-3.85%	55.27%	60.11%	23.68%	8.19%
kurtosis	3.34	3.04	3.01	3.19	4.01	4.14	3.41	3.21
J-B test	1	0	1	0	1	1	1	1

Table 3. Statistical testing of selected portfolios' performance ratios in the out-of-sample period

	Portfoilio	Mean of Reutun	Jensen's Alpha	Sharpe Ratio	Treynor Ratio	STD	MAD	CVaR	MDD	Number of stocks
Mean-Variance	Max-(Mean of Return&Jensen's Alpha&Treynor Ratio)	-0.157% (ns)	0.014% (ns)	-6.085% (ns)	-0.152% (ns)	2.820% (ns)	2.028% (ns)	0.502% (ns)	57.981% (ns)	1
	Max-Sharpe Ratio	-0.075% (*)	0.056% (ns)	-4.574% (ns)	-0.101% (*)	1.963% (**)	1.337% (*)	0.326% (**)	39.965% (ns)	10
	Min-(STD&MAD)	-0.070% (**)	0.037% (ns)	-5.061% (ns)	-0.114% (ns)	1.675% (****)	1.077% (****)	0.290% (****)	35.294% (***)	14
	Min-CVaR	-0.066% (**)	0.06% (ns)	-4.313% (*)	-0.096% (**)	1.868% (****)	1.262% (***)	0.308% (****)	36.571% (**)	10
	Min-MDD	-0.061% (***)	0.058% (ns)	-4.337% (*)	-0.097% (**)	1.74% (****)	1.162% (****)	0.287% (****)	34.096% (***)	14
Mean-Variance-Fuzzy	Max-(Mean of Return&Jensen's Alpha&Treynor Ratio)	-0.085% (ns)	0.052% (**)	-3.430% (***)	-0.093% (***)	2.905% (ns)	2.140% (ns)	0.427% (ns)	56.741% (ns)	2
	Max-Sharpe Ratio	-0.056% (****)	0.072% (ns)	-3.681% (**)	-0.082% (****)	1.919% (**)	1.317% (**)	0.299% (****)	37.436% (*)	10
	Min-(STD&MAD)	-0.078% (*)	0.027% (ns)	-5.656% (ns)	-0.128% (ns)	1.645% (****)	1.062% (****)	0.294% (****)	36.400% (**)	14
	Min-CVaR	-0.056% (****)	0.072% (ns)	-3.681% (**)	-0.082% (****)	1.919% (**)	1.317% (**)	0.299% (****)	37.436% (*)	10
	Min-MDD	-0.071% (**)	0.037% (ns)	-5.106% (ns)	-0.115% (ns)	1.673% (****)	1.110% (****)	0.288% (****)	35.148% (***)	13
Mean-MAD	Max-(Mean of Return&Jensen's Alpha&Treynor Ratio)	-0.157% (ns)	0.014% (ns)	-6.085% (ns)	-0.152% (ns)	2.82% (ns)	2.028% (ns)	0.502% (ns)	57.981% (ns)	11
	Max-Sharpe Ratio	-0.083% (*)	0.055% (ns)	-4.780% (ns)	-0.105% (ns)	2.038% (*)	1.383% (ns)	0.345% (*)	41.591% (ns)	11
	Min-(STD&MAD)	-0.069% (**)	0.041% (ns)	-4.914% (ns)	-0.110% (ns)	1.703% (****)	1.096% (****)	0.292% (****)	35.292% (***)	15
	Min-CVaR	-0.068% (**)	0.062% (ns)	-4.283% (*)	-0.094% (***)	1.924% (**)	1.29% (**)	0.317% (***)	36.955% (**)	12
	Min-MDD	-0.068% (**)	0.055% (ns)	-4.444% (ns)	-0.099% (**)	1.853% (****)	1.238% (****)	0.309% (****)	36.549% (**)	15
Mean-CVaR	Max-(Mean of Return&Jensen's Alpha&Treynor Ratio)	-0.157% (ns)	0.014% (ns)	-6.085% (ns)	-0.152% (ns)	2.820% (ns)	2.028% (ns)	0.502% (ns)	57.981% (ns)	11
	Max-Sharpe Ratio	-0.074% (*)	0.047% (ns)	-4.720% (ns)	-0.108% (ns)	1.887% (****)	1.310% (**)	0.318% (***)	39.816% (ns)	7
	Min-(STD&MAD)	-0.053% (****)	0.045% (ns)	-4.290% (*)	-0.099% (**)	1.588% (****)	1.052% (****)	0.261% (****)	32.301% (****)	8
	Min-CVaR	-0.063% (**)	0.049% (ns)	-4.342% (*)	-0.101% (*)	1.785% (****)	1.237% (****)	0.295% (****)	37.43% (*)	7
	Min-MDD	-0.042% (****)	0.061% (ns)	-3.274% (****)	-0.079% (****)	1.728% (****)	1.202% (****)	0.270% (****)	31.96% (****)	7
	DJI	-0.150%	-	-7.818%	-0.165%	2.107%	1.438%	0.412%	52.252%	
	Naive	-0.108%	0.047%	-5.586%	-0.119%	2.198%	1.478%	0.391%	44.945%	

*(ns p-value>0.05, * p-value<0.05, ** p-value<0.01, *** p-value<0.001, **** p-value<0.0001)*

In Table 2, we find that the distributions of the performance measures of random-weights portfolios are left-skewed while the distributions of the risk measures are right-skewed. What's more, based on the results of Jarque-Bera tests, we find that except for the Jensen' Alpha and Treynor ratio, the other performance ratios of the generated 3,000 random-weights portfolios are not fit to the normal distributions.

From Table 3, we can see that except for the Jensen's Alpha, the values of the other return-measured ratios of all selected portfolios are negative due the decreasing trend of stocks' prices during the out-of-sample period. What' more, comparing with the classical Mean-Variance strategy portfolios, we find that the values of return-ratios are higher in the maximum-return portfolios of the Mean-Variance-Fuzzy model. From the results of p-values of hypothesis tests, on one hand, we find the Jensen's Alpha is the most strict performance measure because the selected strategy portfolios are concluded as inefficient when the portfolios are measured by this ratio; on the other hand, under each mean-risk model, we find that the hypothesis results of the maximum-return portfolios are not good no matter on the basis of return performance nor the risk performance. Throughout Table 3, based on the p-values of the hypothesis tests, we find the selected minimum-CVaR portfolios under each mean-risk model have the best return performance as well as the risk performance.

5. Conclusion

The goal of this paper is to make the statistical testing of the obtained strategy portfolio's performance ratios, which included the return-measured ratios and the risk-measured ratios. The strategy portfolios are selected by applying the mean-risk models based on the in-sample data, from the results of the statistical testing results in the out-of-sample period, we make the following conclusions. Firstly, we find the financial crisis had a significant negative influence on the return of the portfolio investments; Secondly, we find the Fuzzy extension of classical Mean-Variance model has improved the return of portfolio to a certain extent, even though it doesn't help in the risk minimization. Thirdly, we conclude the minimum-risk portfolios we selected from the in-sample period work well at the point of risk minimization in the out-of-sample period, but the selected maximum-return portfolios perform not good at the point of return maximization in this period. Last but not least, according to the number of assets in each obtained portfolio, we conclude the portfolio investments are not well diversified.

Acknowledgements

The author was supported through the Czech Science Foundation (GACR) under paper 18-13951S and moreover by SP2020/11, an SGS research paper of VSB-TU Ostrava. The support is greatly acknowledged.

References

[1] Babazadeh, H. & Esfahanipour, A. (2019). A novel multi period mean-VaR portfolio optimization model considering practical constraints and transaction cost. Journal of Computational and Applied Mathematics, 361, 313–342.

[2] Demiguel, V., Garlappi, L. & Uppal, R. (2009). Optimal versus naive diversification: how inefficient is the 1/N portfolio strategy. Review of Financial Studies, 22, 1915–1953.

[3] Fulga, C. (2016). Portfolio optimization under loss aversion. European Journal of Operational Re-search, 251, 310–322.

[4] Konno, H. & Yamazaki, H. (1991). Mean-absolute deviation portfolio optimization model and its applications to Tokyo stock-market. Management Science, 37, 519–531.

[5] Kresta, A. & Wang, A. (2019). Efficiency test as the benchmark for minimum-risk portfolio optimiza-tion strategies. In Zadnik Stirn L. et al. (Eds.), Proceedings of the 15th International Symposium on Operational Research (pp. 417–422). Slovenian Society Informatika Press, Bled, Slovenia.

[6] Lwin, K. T., Qu, R. & MacCarthy, B. L. (2017). Mean-VaR portfolio optimization: A nonparametric approach. European Journal of Operational Research, 260, 751–766.

[7] Markowitz, H. (1952). Portfolio selection. The Journal of Finance, 7, 77–91.

[8] Rankovic, V., Drenovak, M., Urosevic, B. & Jelic, R. (2016). Mean-univariate GARCH VaR portfolio optimization: actual portfolio approach. Computers & Operations Research, 72, 83–92.

[9] Solares, E., Coello, C., Fernandez, E. & Navarro, J. (2019). Handling uncertainty through confidence intervals in portfolio optimization. Swarm and Evolutionary Computation, 44, 774–787.

[10] Tanaka, H., Guo, P. & Turksen, B. (2000). Portfolio selection based on fuzzy probabilities and possi-bility distributions. Fuzzy Sets and Systems, 111, 387–397.

Corporate life cycle, company profitability and corporate social responsibility

Xiaojuan Wu[1]

Abstract

It is generally believed that mature companies can better fulfil their corporate social responsibilities (CSR) due to their high profitability, big size and perfect corporate governance structure, etc. Empirical research in developed markets confirms this view, but there is little research in emerging markets. To fill this gap, this study examines the moderating role of the corporate life cycle in explaining the relationship between corporate profitability and CSR by using Chinese appliance listed companies as a sample. The regression results show that for mature companies, profitability has no significant impact on CSR; while the profitability of companies at other stages of the life cycle has a significant effect on CSR.

Key words

CSR, profitability, corporate life cycle

JEL Classification: C31, E32, M14

1. Introduction

In recent decades, as social and ecological problems have become more and more severe, the issue of corporate social responsibility (CSR) has attracted more and more public attention. A wide body of academic literature has emerged around CSR, and one of the focuses of these themes is the determinant of CSR. A large number of studies have shown that the larger the company's size and the stronger its profitability, the better its CSR. These characteristics of an enterprise are in line with the characteristics of a mature enterprise. Therefore, there is an opinion that mature enterprises may better implement CSR, which is confirmed by empirical research of Hasan and Habib (2017) based on American companies. Is this conclusion suitable for China, an emerging market? Little literature makes research. To close this gap, this study takes Chinese home appliance companies as the research object and explores the impact of their profitability on CSR at different stages of the corporate life cycle.

The study takes the Chinese appliance listed companies in 2018 as a sample and explore the moderating effect of the corporate life cycle on the relation between corporate profitability and CSR. First, the regression results show that corporate profitability has a significant positive correlation with CSR. Second, through hierarchical regression, we find that for mature companies, the impact of their profitability on CSR is not significant; on the contrary, the profitability of companies at other stages of the life cycle has a significant impact on CSR. Specifically, companies in the growth period are more willing to invest in CSR for better reputation and strategic value when the profit is relatively better, while companies in the introduction and recession periods have poor CSR due to poor profitability. This study contributes to the literature in two ways. First, it extends the CSR literature by examining the moderating role of the corporate life cycle in the link between profitability and CSR. Second, the results of this study break the inherent impression that mature companies have good CSR due to

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high profitability, and provide new empirical evidence for studying CSR in emerging markets like China.

2. Literature review and hypothesis development

2.1 Corporate life cycle

The theory of the corporate life cycle is derived from organizational science literature. Penrose (1959) provides a general theory of business growth and believes that business growth depends on their resources and production opportunities. Wernerfelt (1984) advances the resource-based theory that argues resources are the ultimate source of competitive advantage. Later, Helfat and Peteraf (2003) believe that the resource-based viewpoint must combine with the emergence, development and progress of organizational resources and capabilities over time. Therefore, they introduced the "resource-based dynamic theory" which documents that enterprises' combination of resources, capabilities and characteristics change with time at different stages of the corporate life cycle.

Recent empirical research has investigated how the evolution of capital structure decisions, financial leverage, and company risk-taking behavior occur at various stages of the corporate life cycle. La Rocca, La Rocca and Cariola (2011) report that small and medium-sized firms' capital structure changes over their life cycle. Rehman, Wang and Yu (2016) describe the application of low, high, and low financial leverage models in the growth, maturity, and recession stages of corporate life cycle. Shahzad, Lu and Fareed (2019) find that firm risk-taking is higher at the introduction and decline stages, and lower at the mature and growth stages.

2.2 CSR

Academic research on CSR began to take form in the 1950s. A lot of scholars and organizations strive to define the concept of CSR. So far, there is still no commonly accepted definition of CSR. This paper adopts the concept of CSR advocated by Elkington in 1997 based on the triple bottom line principle. He assumes that if an enterprise forms an economic and social system, then its development objectives should constitute a triple beam, which relates to the profit, the people associated with the company, and cares for the planet.

Existing CSR literature indicates CSR disclosure significantly reduces the firm-level cost of capital (Michaels and Grüning, 2017), and CSR has a positive effect on competitive advantage, reputation and customer satisfaction, further influence on firm performance positively (Saeidi et al.,2015).

2.3 Association between corporate profitability and CSR

Slack resource theory claims that better financial performance potentially leads to the availability of slack (financial and other) resources which provide the opportunity for companies to invest in social performance domains, such as community relations, employee benefits, philanthropic donation or environment. If slack resources are available, the allocation of these resources to the social domain will produce better social performance. Therefore, better financial performance will be a better predictor of better corporate social performance.

Much empirical evidence supports the positive relationship between CSR and financial performance. Based on Indonesia companies, Swandari and Sadikin (2016) got the result that profitability significantly influences CSR because 'companies with high profits have the fund flexibility to implement CSR programs that have been set'. Giannarakis (2014) took a sample consisting of 100 companies from the Fortune 500 list for 2011 and found that profitability is positively associated with the extent of CSR disclosure. The same result is obtained from the Chinese sample studied by Li and Zhang (2010). Hence, it is hypothesized that:

H1. There is a positive relationship between profitability and the level of CSR.

2.4 Corporate profitability and CSR: moderating role of corporate life cycle

Companies at different life cycle stages are associated with varying resource levels that shape their CSR behaviour. Firms in the introduction stage lack a stable customer base and suffer from potential danger about revenues, costs and industry dynamics (Jovanovic, 1982), and face the possibility of initial exit at any time. Despite the dramatic increase in sales and product numbers of growth companies, they are still subject to fierce market competition. Growth firms are more intent to invest more in product modification and improvement than in product differentiation (Hay and Ginter, 1979). Firms in the decline stages have limited or downgraded resources. These firms focus more on how to survive. Then , companies with such fragile financial performance may damage shareholder value if they invest in CSR. Therefore, the limited capacity and resource base restrict the profitability of these companies at introduction, growth and decline stages, further affecting their use of precious and scarce funds for CSR projects, finally greatly reducing the participation of CSR.

Mature companies have a sound customer base and pay more attention to product differentiation strategies (Hay and Ginter, 1979). The expertise and capabilities generated by organizational maturity enable these companies to make meaningful CSR investments. Firms with a larger scale of operations can focus on CSR activities by reorganizing or reallocating resources, thereby reducing their CSR costs (Udayasankar, 2008). Viewed from this perspective, mature companies have stable profitability due to their sufficient resource base, capabilities and excellent competitive advantages, so they are prone to better fulfil CSR. Hence, it is hypothesized that:

H2. When the company is at a mature stage, the impact of profitability on CSR is strengthened.

3. Data and methodology

3.1 Sample selection

This paper aims to explore the moderating effect of the corporate life cycle on the relation between corporate profitability and CSR in the Chinese appliance industry. Thus, the study selects the Chinese home appliance listed companies for the year 2018 as the research object. The companies list is taking over from the Iwencai database. After excluding the companies listed in 2018 and 2019 and ST companies, a sample of 56 listed companies is left. Precisely, it consists of 25 companies listed on the Main Board, 23 companies listed on the Small and Medium Enterprise Board, and 8 companies listed on the Growth Enterprise Market. All data are collected from Iwencai database, annual financial statements and CSR reports (if any).

3.2 Dependent variable

Vast previous literature on China's CSR research has selected the overall evaluation score published by Rankins CSR Ratings (RKS). It is a reliable CSR rating agency that measures the performance and disclosure of CSR in China. However, there is a flaw in its rating results that it cannot cover all listed companies. It is because RKS evaluates the CSR of listed companies based on the CSR reports issued by the companies, but not every listed company is able or willing to disclose its CSR report. Given the fact that the necessary sample data in this study cannot be obtained from this reliable rating report, this paper constructs an evaluation criteria system based on the characteristics of China's appliance industry to assess the CSR implementation status of the appliance companies. With the aid of the analytic hierarchy process, this study establishes an evaluation criteria system from the three aspects of economics, environment and society according to the aforementioned triple bottom line principle. Reasons and analysis for the criteria selection and the assignment of the specific weights for each criterion can be found in the author's another paper (Wu, 2019). The majority of the original data is manually collected by the author from the annual report and CSR report; the rest of the data used is from Iwencai database.

3.3 Moderator

This paper adopts the corporate life cycle proxies of Dickinson (2011) to capture the dynamic nature of the corporate life cycle. She believes that cash flow captures differences in company profitability, growth and risk, so people may use cash flow from operations (ONCF), investment (INCF) and financing (FNCF) to group companies into different life cycle stages. As shown in Table 1, the eight patterns are divided into five stages by Dickinson. Based on this division result, this paper further divides the three situations of the shake-out stage. Specifically, among the three situations of the shake-out stage, when ONCF > 0, it shows that firm operation activity is running normally, which is more close to the mature stage; when ONCF < 0, it shows that the company operational activities are abnormal and serious problems have occurred, which is more close to the decline stage. The final classification criteria of the sample companies in this study can be seen in the last row of Table 1.

Table 1: Division of the corporate life cycle in this paper

Patten	1	2	3	4	5	6	7	8
Dickinson's division	Introduction	Growth	Mature	Shake-Out			Decline	
ONCF	-	+	+	+	+	-	-	-
INCF	-	-	-	+	+	-	+	+
FNCF	+	+	-	+	-	-	+	-
Author's division	Introduction	Growth	Mature			Decline		

3.4 The research model of the study

Since data only has a cross-sectional dimension, a multiple linear regression model is proposed to test the hypotheses formulated in the previous section. Hence, this study develops the following regression specifications to test the association between corporate profitability and CSR (test of H1).

$$CSR = \beta_0 + \beta_1 ROE + \beta_2 FS + \beta_3 FR + \beta_4 EOC + \varepsilon, \tag{1}$$

where β_0 is the intercept, $\beta_{1,\dots,}\beta_n$ represent the regression coefficient, ε is the error term.

In the above equation, ROE denotes corporate profitability. Following the prior literature, we include a set of control variables which may be the determinants of CSR. Firm size (FS) is the natural logarithm of total assets. Financial risk (FR) is the ratio of total debt to total assets. Equity ownership concentration (EOC) is measured by the shareholding ratio of the largest shareholder.

To test H2, we regress CSR on corporate profitability, the life cycle proxy, their interactive terms and control variables in Eq. (2). This allows us to examine the moderating effect of the corporate life cycle on the association between profitability and CSR.

$$CSR = \beta_0 + \beta_1 ROE_C + \beta_2 CLC + \beta_3 ROE_C \times CLC + \beta_4 FS + \beta_5 FR + \beta_6 EOC + \varepsilon, \tag{2}$$

where ROE_C is the ROE after a mean centering treatment. Such processing does not affect the regression results of the model and eliminates multicollinearity between variables. Corporate life cycle (CLC) is a dummy variable. Its value 1 indicates that the company is in a mature stage, and its value 0 indicates that the company is in another stage.

4. Result and discussion

4.1 Descriptive statistics

Table 2 provides descriptive statistics for variables used in regression models. The CSR level varies from 0.06 to 0.66, with a mean of 0.4 and a standard deviation of 0.13. ROE ranges between -70.41% and 34.79%, with a mean of 3.42% and a standard deviation of 23.07. Statistics show that in 2018, the average value of the corporate life cycle is 0.55, which indicates that more than half firms in China's home appliance industry are in the mature stage.

Table 2: Descriptive statistics

	Minimum	Maximum	Mean	Std. Deviation
CSR	0.06	0.66	0.40	0.13
ROE	-70.41%	34.79%	3.42%	23.07
Corporate life cycle	0.00	1.00	0.55	0.50
Firm size	19.74	26.30	22.45	1.48
Financial risk	0.15	0.79	0.46	0.18
Equity ownership concentration	0.08	0.81	0.37	0.17

4.2 Correlation analysis

Table 3 presents Pearson's correlation analysis results among all variables with their significance level. When the correlation coefficient between regressors is above 0.80, it is considered as an indication of serious multicollinearity. In this study, the maximum coefficient of the Pearson correlations is 0.666; thus, multicollinearity does not exist in this sample data.

Table 3: Correlation analysis

	CSR	ROE	CLC	FS	FR	EOC
CSR	1.00					
ROE	0.321*	1.00				
CLC	0.02	0.09	1.00			
FS	0.497**	0.16	0.06	1.00		
FR	0.336*	-0.305*	-0.08	0.666**	1.00	
EOC	-0.16	0.453**	0.14	-0.11	-0.23	1.00

Note: *Significant at the 0.05 level (two-tailed); **Significant at the 0.01 level (two-tailed).

4.3 Regression results and discussion

Before testing the hypotheses, this study satisfied classical linear regression assumptions for cross-sectional data. Particularly test for homoscedasticity, the normality of the residual and multicollinearity. From Table 4 we can see the results of the LM statistic of Eq. (1) and Eq. (2) are not significant at the 5% level. It means, that there are no heteroscedasticity concerns. Figure 1 shows us intuitive answers that the residuals of Eq. (1) and Eq. (2) are normally distributed, respectively.

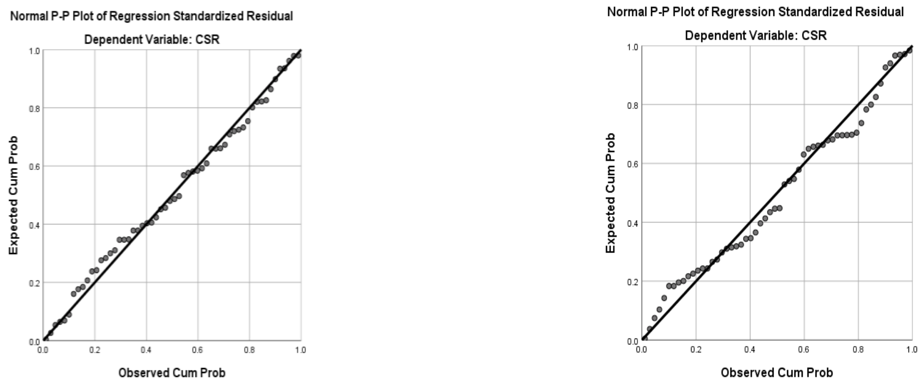
Table 4 presents the results of the two regression equations. R-squared of Eq. (1) is 0.407, which means that all the independent variables in Eq. (1) together explain about 40.7% of the variance in the CSR level of the Chinese appliance listed companies. For Eq. (2), the p-value of change in R-squared is significant at the 5% level, which means the moderator of the corporate life cycle plays an important role in explaining the association between corporate profitability and CSR. It is also verified by the coefficient of the interaction of ROE_C*CLC is significant at the 5% level.

Table 4: Regression results

	Eq. (1)				Eq. (2)			
Variables	Coefficient	SE	t	p	Coefficient	SE	t	p
Constant	0.014	0.314	0.045	0.964	0.028	0.305	0.090	0.928
ROE	0.003	0.001	3.587	0.001	0.005	0.001	4.346	0.000
CLC					0.006	0.029	0.197	0.845
ROE_C*CLC					-0.003	0.001	-2.349	0.023
FS	0.016	0.016	1.018	0.313	0.014	0.015	0.892	0.377
FR	0.233	0.131	1.777	0.082	0.272	0.129	2.109	0.040
EOC	-0.244	0.099	-2.464	0.017	-0.210	0.098	-2.139	0.037
R^2			0.407		0.468			
F-statistic			8.762**		7.185**			
Change in R^2 (ΔR^2)					0.061			
F-statistic (ΔR^2)					5.5162*			
White test for homoscedasticity			Obs*R^2 = 5.04		Obs*R^2 = 4.424.			
(the LM statistic)			p = 0.08		p = 0.098			

Note: *Significant at the 0.1 level (two-tailed); **Significant at the 0.01 level (two-tailed)

Figure 1: Normal P-P plot of residuals for Eq (1) and Eq (2).



The results reported in Table 4 indicate that in both regression models, corporate profitability is significant ($p < 0.01$), which reinforces the importance of the corporate profitability in explaining the variation in CSR. It supports Hypothesis 1. Profitable companies are able to afford to expenditure related to CSR, such as providing employees with higher salaries and wages, paying more taxes to the state, and donating to society, etc. This result is consistent with the findings of Swandari and Sadikin (2016) and Giannarakis (2014).

As shown in Table 4, there is a significant negative interaction between corporate profitability and corporate life cycle. This moderating effect of the life cycle is depicted in Figure 2. The last two columns of Table 5 report the results generated by using 5000 bootstrapping samples produced by the SPSS (PROCESS) procedure. The 95% confidence interval values of the conditional effects at CLC = 1 (companies in the mature stage and indicated by a dashed line in Figure 2) show a result of [-0.0002, 0.0038], which includes zero. This result suggests an insignificant conditional effect of profitability on CSR for firms at the mature stage. In other words, the CSR of mature companies will not increase significantly with the increase in profits. It is inconsistent with hypothesis 2. This situation may be related to the Chinese context. In China, many companies fulfil CSR mainly due to government regulatory requirements, especially for mature large companies, which are subject to stricter requirements. Therefore, when the CSR level of these mature enterprises reaches the necessary level of supervision, they lose the motivation to perform more CSR.

However, the 95% confidence interval values of the conditional effects at CLC = 0 (companies at other stages of the life cycle and indicated by a solid line in Figure 2) show a result of [0.0026, 0.007], which does not include zero. This result shows that when the enterprise is in other stages of the life cycle, the conditional impact of profitability on CSR is significant. Specifically, when corporate profitability is above average, the company is likely to be in the growth stage according to the characteristics of the corporate life cycle. Its CSR will increase significantly as the profit increases. When corporate profitability is below average, the company is likely to be in the stage of introduction or decline. Due to poor profit, the company does not have sufficient resources to invest in CSR activities, so the level of CSR is low. Figure 2 shows intuitively that in the Chinese home appliance industry, growing companies show better CSR than mature companies at the same profit level. This phenomenon can be explained by the view of the reputational and strategic values associated with CSR involvement mentioned by Hasan and Habib (2017). This view holds that early-stage firms are equally likely to invest in CSR activities. Younger companies need more support from stakeholders because they need external resources. CSR engagement can be an effective tool for obtaining such support. Despite the high cost of CSR, the marginal benefits of CSR investment in young companies may be greater than their mature counterparts.

Figure 2: Moderating effect of corporate life cycle on the relationship between profitability and CSR

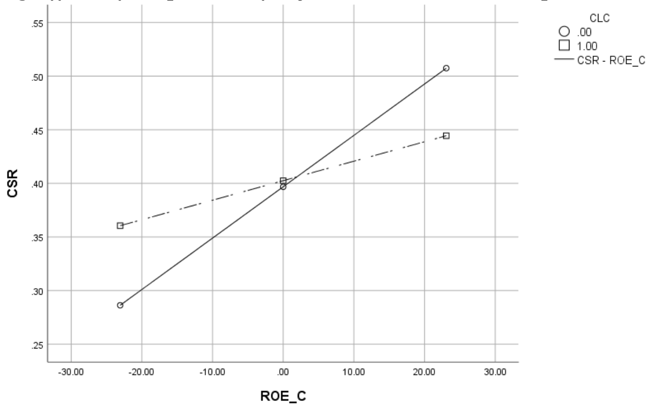


Table 5: Conditional effects of the focal predictor at values of the moderator

CLC	Effect	se	t	p	LLCI	ULCI
0	0.0048	0.0011	4.3462	0.0001	0.0026	0.007
1	0.0018	0.001	1.8414	0.0716	-0.0002	0.0038

5. Conclusion

This study explored whether corporate life cycle moderates the association between the corporate profitability and CSR. The regression results show that, first, profitability is positively related to CSR; second, corporate life cycle negatively moderates the relationship between corporate profitability and CSR. It means that when the company is in a mature stage, profitability has little effect on CSR, but when the company is in other stages of life cycle, profitability has a significant impact on CSR. These findings break the inherent view that mature companies have better CSR due to high and stable profitability, and provides new evidence to support the reputation and strategic value associated with CSR participation. For these values, young companies are more likely to invest in CSR.

This study suffers from some limitations, which could be overcome by further research. First of all, this article uses only cross-sectional data for analysis. Future research will extend the research period. Second, this research involves only one industry, and future research may include more industries to get wider evidence.

References

[1] Dickinson, V. (2011). Cash flow patterns as a proxy for firm life cycle. *Accounting Review*, 86(6), p. 1969–1994.

[2] Giannarakis, G. (2014). Corporate governance and financial characteristic effects on the extent of corporate social responsibility disclosure. *Social Responsibility Journal*, 10(4), p. 569–590.

[3] Hasan, M. M. and Habib, A. (2017). Corporate life cycle, organizational financial resources and corporate social responsibility. *Journal of Contemporary Accounting and Economics*. *Elsevier Ltd*, 13(1), p. 20–36.

[4] Hay, R.D.and Ginter, P. (1979). Strategies for maintaining a share of the market. *Annual meeting of the Southern Academy of Management*, Atlanta.

[5] Helfat, C. E. and Peteraf, M. A. (2003). The dynamic resource-based view: Capability life cycles. *Strategic Management Journal*, 24(10 SPEC ISS.), p. 997–1010.

[6] Jovanovic, B. (1982). Selection and the evolution of industry. *Econometrica*, 50(3), p.649–670.

[7] La Rocca, M. L., La Rocca, T. L. and Cariola, A. (2011). Capital Structure Decisions during a Firm Life Cycle. *Small Business Economics*, 37(1), p. 107–130.

[8] Li, W. and Zhang, R. (2010). Corporate Social Responsibility, Ownership Structure, and Political Interference: Evidence from China. *J. of Business Ethics*, 96(4), p. 631–645.

[9] Michaels, A. and Grüning, M. (2017). Relationship of corporate social responsibility disclosure on information asymmetry and the cost of capital. *Journal of Management Control*. Springer Berlin Heidelberg, 28(3), p. 251–274.

[10] Penrose, E.T. (1959). *The Theory of the Growth of the Firm*. Wiley, New York.

[11] Saeidi, Sayedeh Parastoo et al. (2015). How does corporate social responsibility contribute to firm financial performance? The mediating role of competitive advantage, reputation, and customer satisfaction. *Journal of Business Research*, 68(2), p. 341–350.

[12] Shahzad, F., Lu, J. and Fareed, Z. (2019). Does firm life cycle impact corporate risk taking and performance? *Journal of Multinational Financial Management*. *Elsevier B.V.*, 51, p. 23–44.

[13] Swandari, F. and Sadikin, A. (2016). The effect of ownership structure, profitability, leverage, and firm size on CSR. *Binus Business Review*, 7(3), p. 315.

[14] Udayasankar, K. (2008). Corporate social responsibility and firm size. *Journal of Business Ethics*, 83(2), p. 167–175.

[15] Ur Rehman, A., Wang, M. and Yu, H. (2016). Dynamics of financial leverage across firm life cycle in Chinese firms: an empirical investigation using dynamic panel data model. *China Finance and Economic Review*, 4(1).

[16] Wernerfelt, B. (1984). A resource-based view of the firm. *Strategic Manage*, 5(2), p.171–180.

[17] Wu, Xiaojuan. (2019). Research on the implementation of CSR of listed companies in the Chinese household appliance industry. *21st international conference Mekon 2019*. p.255–262.

Portfolio assets optimisation on the Shapley value and mean-variance criterion

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Abstract

The asset portfolio optimisation is important financial modelling problém. The goal of the paper is to propose a methodology and verify the possibilities of stating an optimal asset applying the Shapley value based on the mean-variance criterion. The classical mean-variance model is compared with Shapley model. The models are verified and compared portfolios from 2 to ten assets portfolios. The fundamental findings are that the Shapley methodology leads to more diversified portfolios allowing to immense resilience of the portfolio returns.

Keywords

Portfolio optimisation model, mean-variance criterion, Shapley value

JEL Classification: C4, C02, G 3, G11

1. Introduction

One of the crucial financial modelling problems is finding out of the asset optimal portfolio and determination of optimal asset portfolio. This problem is usually formulated as a stochastic one with random parameters (returns, covariances) in an objective function.

Various optimisation criteria are applied in portfolio optimisation models. We can distinguish two groups of optimisation criteria.

The first one includes the criterion of maximisation of the expected value of utility function. It is derived from axioms of comparability, transitivity, substitution and certainty equivalent. Examples of it are e. g, mean-variance maximisation and variance minimisation.

The second group includes so-called managerial criteria or safety first criteria. It stems from a managerial approach to managing risks. The goal is to find an optimal asset without significant or extreme losses stemming from a non-favourable scenario. We can introduce the following examples of criteria: the Value at Risk minimisation, the expected loss minimisation, the Conditional Value at Risk minimisation, and the risk-adjusted return on capital (RAROC) minimisation.

One of the innovative optimisation portfolio approaches is a usage of the Shapley value, which expresses by interaction a coalition players potential. Supposing assets are players the methodology should be applied for assets portfolio optimisation.

The objective of the paper is to propose a methodology and verify the possibilities of stating an optimal asset applying the Shapley value based on the mean-variance criterion. The classical mean-variance model is compared with Shapley model.

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2. Mean-variance model

Asset portfolio optimisation is one of the problém of financial modelling. In a case of an expected utility function $E\left[U(\cdot)\right]$, the problem under given conditions the problem is possible to transform in a mean-variance problem. Supposing the random variables are of Gaussian (elliptical) distribution, or utility function is of a quadratic shape or is transformed thru Taylor expansion of the second level, the problem can be formulated as a mean-variance model. So the distribution function is given the only ba two parameters; expected value and variance. The problém is to be formulated with random parameters in an objective function as follows.

Problem 1 mean-variance model

$$\max E\left[U(w_1)\right] = \max w_0\left[E(x) - k\cdot\sigma_P^2\right]$$

s. t.

(P1) $\sum_j x_j = 1,$

(P2) $x_j \in [0;1]$ pro $j = 1,2,\ldots,N,$

where $E(x) = \sum_j x_j \cdot E(R_j) = \vec{x}^{\mathrm{T}} \cdot E(\vec{R}),$

$$\sigma_P^2 = \sum_j \sum_k x_j \cdot \sigma_{jk} \cdot x_k = \vec{x}^{\mathrm{T}} \cdot C \cdot \vec{x},$$

$$\sigma_{ij} = \sigma_i \cdot \rho_{ij} \cdot \sigma_j,\ C = diag(\vec{\sigma}^{\mathrm{T}}) \cdot RR \cdot diag(\vec{\sigma}).$$

The objective function states expected utility function maximisation value. Simultaneously $E(x)$ is the expected utility of portfolio asset return value, *k* is the propensity to risk, σ_P^2 is the variance of portfolio asset return value, x_i is percentage invested to a particular asset, σ_{ij} is the covariance of assets *i* and *j*. Symbols $\vec{x}, E(\vec{R})$ are vectors and C are the covariance matrix, RR is the correlation matrix.

Constraint (P1) means that the value invested in particular assets must be equal to the assets portfolio w_0. Constraint (P2) reflects that percentage invested in particular assets are positive numbers, so short selling is not allowed.

3. Shapley value and portfolio optimisation

The Shapley value serves to the distribution of payoff in cooperative game theory on the coalition potential of the i-th player. For the game to be substantial (differentiate from the weighted average), then the necessary assumptions are the superadditivity or the subadditivity of coalitions payoffs. For a given number of players, N, the 2^{N-1} coalition combinations can be created.

The calculation of the i-th player potential is the following,

$$\phi_i = \sum_{i\in S} \frac{(s-1)!(N-s)!}{N!}\left[v(S) - v(S-\{i\})\right],$$

where S is coalition (players combination), s is the number of players in the coalition S, $s = |S|$, N is the number of players, $v(S)$ is coalition S value, $v(S-\{i\})$ is coalition S value with the exclusion of the i-the player. The formula implies the calculation is based on the idea of average contribution of i-th player to a coalition. Since the fraction determines probability and bracket means a contribution of i-th player to coalitions.

In a case of Shapley value, the portfolio optimisation stems from an idea that particular assets are players. Then due to Shapley value, ϕ_i it is stated the contribution of the given asset to portfolios (coalitions) on the interaction with other assets. By normalisation of Shapley values, the percentage in portfolio optimisation is computed.

A coalition value depends on the selected optimisation criterion. We can introduce the following criteria examples: the mean-variance, the variance, the Value at Risk, the expected loss, the Conditional Value at Risk and the risk-adjusted return on capital (RAROC).

Hodnota koalice závisí přitom na zvoleném kritériu optimalizace (maximalizace střední hodnoty funkce užitku, maximalizace mean-variance, minimalizace směrodatné odchylky, minimalizace value at risk, minimalizace střední hodnoty ztráty, minimalizace podmíněné (conditional) Value at Risk (CVaR, shortfall), minimalizace ukazatele RAROC (risk adjusted return on capital)).

In the case of the mean-variance criterion, the coalition S value is stated as follows,

$v(S)=E(x_S)-k\cdot\sigma_P^2(x_S)$, where $E(x_S)=\sum_j x_{Sj}\cdot E(R_{Sj})$, $\sigma_{SP}^2=\sum_j\sum_k x_{Sj}\cdot\sigma_{Sjk}\cdot x_{Sk}$, $x_{Sj}=1/s$.

Analogically for the coalition $S-\{i\}$, the value is determined in the way

$v(S-\{i\})=E\left(x_{S-\{i\}}\right)-k\cdot\sigma_P^2\left(x_{S-\{i\}}\right)$, where $E\left(x_{S-\{i\}}\right)=\sum_j x_{S-\{i\}j}\cdot E\left(R_{S-\{i\}j}\right)$,

$\sigma_{SP}^2=\sum_j\sum_k x_{S-\{i\}j}\cdot\sigma_{S-\{i\}jk}\cdot x_{S-\{i\}j}$, $x_{S-\{i\}j}=1/(s-i)$.

Algorithm steps of computations: (1) input data (RR, $\vec{\sigma}$, k), (2) calculation of the covariance matrix C, (3) stating of coalition quantity, (4) generation of assets combinations, (5) computation of the $v(S)$ and $v(S-\{i\})$ for every assets combination, (6) computation of a Shapley value of the particular asset ϕ_i, (6) computation of optimal asset portfolio composition, $v_i=\phi_i/\sum_i\phi_i$.

4. Illustrative example

We assume 10 assets (A, B, C to J). The goal is to find the optimal portfolio on the Shapley value basis for the mean-variance optimisation criterion, see algorithm steps introduced in section 3. Simultaneously, for comparison, the portfolio optimisation is calculated due to the mean-variance model, see Problem 1. We suppose the knowledge of the aversion coefficient $k=0.1$, furthermore correlation matrix, standard deviations and expected values of assets, see Fig. 1 and Fig. 2.

Fig. 1 *Correlation matrix RR*

RR	A	B	C	D	E	F	G	H	I	J
A	1	0.6	0.3	-0.2	-0.7	0.4	0.3	-0.9	0.5	0.2
B	0.6	1	-0.7	0.4	0.2	-0.2	-0.7	0.4	0.3	-0.9
C	0.3	-0.7	1	0.6	-0.9	0.4	0.8	0.6	0.3	-0.2
D	-0.2	0.4	0.6	1	0.2	0.6	0.3	-0.2	-0.7	0.5
E	-0.7	0.2	-0.9	0.2	1	-0.2	0.6	0.3	-0.2	0.3
F	0.4	-0.2	0.4	0.6	-0.2	1	-0.7	0.4	0.3	0.3
G	0.3	-0.7	0.8	0.3	0.6	-0.7	1	-0.1	0.4	0.4
H	-0.9	0.4	0.6	-0.2	0.3	0.4	-0.1	1	-0.2	0.6
I	0.5	0.3	0.3	-0.7	-0.2	0.3	0.4	-0.2	1	-0.5
J	0.2	-0.9	-0.2	0.5	0.3	0.3	0.4	0.6	-0.5	1

Fig. 2 Standard deviations and expected values of particular assets

	A	B	C	D	E	F	G	H	I	J
σ_i	0.3	0.35	0.5	0.4	0.45	0.25	0.55	0.7	0.6	0.35
E_i	0.02	0.05	0.03	0.04	0.05	0.025	0.065	0.075	0.07	0.045

Subsequently, the coalitions are generated. For instance, 4 assets $2^{4-1} = 8$ coalitions are created, see Tab. 1.

Tab. 1 Coalition compositions

koalice	A	B	C	D
1	1	1	1	1
2	0	1	1	1
3	1	0	1	1
4	0	0	1	1
5	1	1	0	1
6	0	1	0	1
7	1	0	0	1
8	0	0	0	1

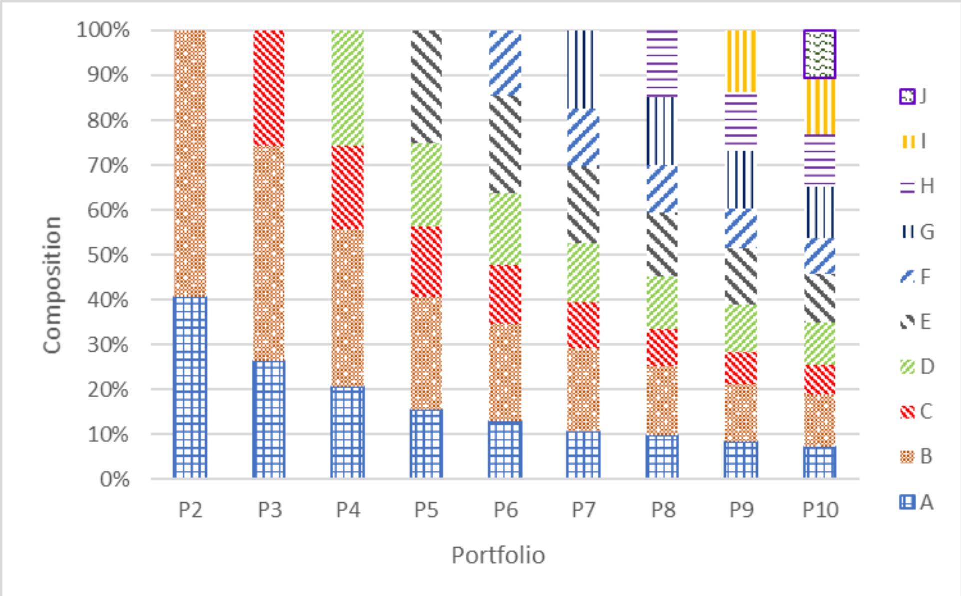
4. 1 Results

There is in Tab. 2 and on Fig. 3 are computed results presented for optimal portfolio due to Shapley methodology. Results for mean-variance model are presented in Tab. 3 and on Fig. 4.

Tab. 2 *Optimal portfolio composition due to Shapley methodology and assets quantity*

	A	B	C	D	E	F	G	H	I	J
P2	40.450%	59.550%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
P3	26.204%	48.054%	25.742%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
P4	20.577%	35.170%	18.415%	25.837%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
P5	15.532%	25.061%	15.634%	18.726%	25.047%	0.000%	0.000%	0.000%	0.000%	0.000%
P6	12.786%	21.939%	12.946%	15.990%	21.774%	14.565%	0.000%	0.000%	0.000%	0.000%
P7	10.586%	18.710%	10.037%	13.333%	17.133%	12.706%	17.495%	0.000%	0.000%	0.000%
P8	9.854%	15.349%	8.259%	11.800%	14.180%	10.637%	15.178%	14.742%	0.000%	0.000%
P9	8.227%	12.978%	7.117%	10.670%	12.335%	9.044%	12.847%	13.119%	13.663%	0.000%
P10	7.198%	11.825%	6.470%	9.383%	10.932%	7.954%	11.476%	11.609%	12.561%	10.593%

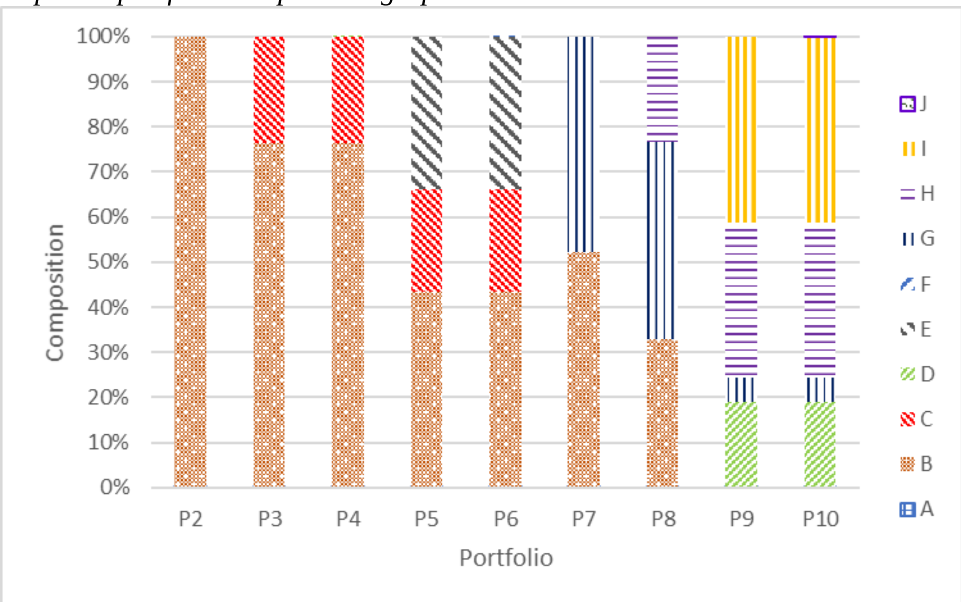
***Fig. 3** Graph of optimal portfolio composition due to Shapley methodology and assets quantity*



***Tab. 3** Optimal portfolio composition due to Mean -Variance model and assets quantity*

	A	B	C	D	E	F	G	H	I	J
P2	0.000%	100.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
P3	0.001%	76.516%	23.483%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
P4	0.001%	76.514%	23.481%	0.004%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
P5	0.002%	43.344%	22.879%	0.003%	33.773%	0.000%	0.000%	0.000%	0.000%	0.000%
P6	0.000%	43.348%	22.880%	0.000%	33.772%	0.000%	0.000%	0.000%	0.000%	0.000%
P7	0.000%	52.159%	0.000%	0.000%	0.000%	0.001%	47.839%	0.000%	0.000%	0.000%
P8	0.000%	32.999%	0.000%	0.000%	0.000%	0.000%	43.807%	23.194%	0.000%	0.000%
P9	0.001%	0.002%	0.001%	18.977%	0.005%	0.001%	5.440%	34.240%	41.333%	0.000%
P10	0.001%	0.002%	0.001%	18.975%	0.005%	0.001%	5.440%	34.238%	41.333%	0.003%

***Fig. 4** Optimal portfolio composition graph due to Mean-Variance model and assets number*



It is apparent that both methods lead to different results. The essential difference consists in greater diversification in case of Shapley method in comparison with the mean-variance method, even if the optimisation criterion is the same. Greater diversification means better risk distribution and resilience against unexpected shocks. The Shapley method has disadvantages for the growing number of combinations for more assets. určitou nevýhodou. Computation for 10 assets was very quickly.

5. Conclusion

The objective of the paper was to verify the Shapley methodology in portfolio optimisation of assets. The novelty methodology was described and explained. The criterion mean-variance was applied for portfolio optimisation. For comparison, the mean-variance model was applied as well.

The verification computation for 10 assets was presented. The fundamental conclusion is that the Shapley methodology leads to more diversified portfolios allowing to immense resilience of the portfolio returns. Next research should be focused on various optimisation criteria and dimension of the problem.

Acknowledgement

The paper was supported by the SGS Projects VŠB – TU Ostrava SP2020/124, SP2018/154, SP2019/132, Financial Modelling and Decision-making of Companies and Financial Institutions under Risk, Flexibility and Interaction

References

[1.] Colini-Baldeschi, R., Scarsini, M., Vaccari, S. (2018) Variance Allocation and Shapley Value. METHODOLOGY AND COMPUTING IN APPLIED PROBABILITY. Vol 20, Issue: 3, pp 919-933

[2.] Dowd, K. (1998), Beyond Value at Risk: The new Science of Risk Management. John Wiley and Sons.Chen Shu-Jen, Hwang Chin-Lai, (1992) Fuzzy Multiple Attribute Decision Making Methods and Applications. Springer-Verlag

[3.] Dluhošová, D. (2004). An analysis of financial performance using EVA method: Finance a úvěr – Czech Journal of Economics and Finance, 54(11-12), p. 541-559.

[4.] Fong, G.F., Vasicek, O.A. (1997): *Portfolio Risk Management*. In: Fabozzi, E.J.*: Advances in Fixed Income Valuation Modelling and Risk Management*. New Hope, Pennsylvania, 317-326

[5.] Inuiguchi, M. Ramik, J. (2000). Possibilistic linear programming: a brief review of fuzzy mathematical programming and comparison with stochastic programming in portfolio selection problem, Fuzzy Sets and Systems, 111, 3-28

[6.] Mussard, S., Terraza, V. (2008) The Shapley decomposition for portfolio risk. .APPLIED ECONOMICS LETTERS, Vol 15, Issue: 9, pp: 713-715

[7.] Simonian, J. (2019). Portfolio Selection: A Game-Theoretic Approach. Journal ofportfolio management . Vol 45, Issue: 6, pp 108-116

[8.] Zmeškal, Z. (2005). Value at risk methodology under soft conditions approach (fuzzy-stochastic approach). European Journal of Operational Research, 161 (2), pp. 337-347.

[9.] Zmeškal, Z. (2004). Finanční modely. 2. upravené vydání. Praha: Ekopress.

[10.] Zmeškal, Z., Dluhošová, D. (2015). Application of the advanced multi-attribute non-additive methods in finance distribution, Financial management of firms and financial institutions: 10th international scientific conference, PTS I-IV, Ostrava, Czech Republic pp 1439-1449