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Jana Hančlová

Lucie Chytilová

Jaroslav Ramík

Mehdi Toloo

František Zapletal

Seddigheh Babaee

Yongjun Shen

Elke Hermans

OPTIMIZATION PROBLEMS IN ECONOMICS AND FINANCE

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Jana Hančlová, Lucie Chytilová, Mehdi Toloo, František Zapletal
Department of Systems Engineering
Faculty of Economics
VŠB-Technical University Ostrava
Sokolská 33
701 21 Ostrava, CZ
jana.hanclova@vsb.cz, lucie.chytilova@vsb.cz, mehdi.toloo@vsb.cz,
frantisek.zapletal@vsb.cz

Jaroslav Ramík
School of Business Administration
Karviná
Univerzitní náměstí 1934/3
733 40 Karviná, CZ
ramik@opf.slu.cz

Seddighed Babaee, Yongjun Shen, Elke Hermans
Transportation Research Institute (IMOB), Hasselt University,
Wetenschapspark 5 bus 6,
3590 Diepenbeek, BE
seddigheh.babaee@uhasselt.be, yongjun.shen@uhasselt.be,
elke.hermans@uhasselt.be

Reviews

Miloš Kopa, Charles University, Prague
Martin Dlouhý, University of Economics, Prague

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Preface

Each optimization model is a simplification of some real system and it consists of objective function(s) of many variables and limiting constraints. Optimization models differ in a number of objective functions and constraints as well as in their types. A solution of each optimization model can be obtained by analytical, numerical or graphical method. Optimization models are widely applicable in various fields of economics and finance and they help to support a decision making and thereby to save resources.

This book focuses on optimization models and methods for evaluating the effectiveness and efficiency of production units and it is divided into two parts and four chapters. The first part is devoted to Data Envelopment Analysis (DEA) models and the second part contains optimization models involving the uncertainty given by (intuitional) fuzzy quantities.

Chapter 1 focuses on the optimization models with specifications to assess banks effectiveness and efficiency in the countries of the Visegrad Four (V4) in time period 2004-2013. This chapter is not just about obtaining the order of surveyed decision making units - banks in the Visegrad Four (V4). It is about deeper analysis of traditional and specific factors affecting the efficient and inefficient behaviour of the units, which should lead to improve their management behavior. Understanding the current state of development and transformation of the banking system in the V4 countries, defining analytical methods and measuring the effectiveness are used to address four research questions: (i) What is the role of non-traditional activities in evaluating the efficiency of banks? (ii) Is the size of banks influencing factor for efficiency? (iii) Are there regional differences in the assessment of the technical, cost and allocation efficiency? (iv) What is the trend of development and what are the main factors affecting the development of efficiency and productivity change of banks in the period under review? Empirical findings of this research provide an important insight for bank managers and policy makers, and the general professional public.

Chapter 2 aims to develop an appropriate screening tool to evaluate older drivers' performance. It first gives an overview of the multiple layered DEA based Composite Indicator model (MLDEA-CI) developed by Shen et al. Next, it incorporates the concept of a layered hierarchy into the DEA model with common set of weights (CSW). This way, the existing hierarchical structure between the indicators is reflected in the model and all of the decision making units (DMUs) under study can be compared directly. As a numerical example, aforementioned methods are applied in a case study to evaluate the overall driving performance of a sample of older drivers using data from an assessment battery and a fixed-based driving simulator. By using 16 hierarchically structured indicators from three different subcategories - psychological, physical and driving ability - the optimal driving performance of individual older drivers is computed and further the weights allocated to each layer of the hierarchy are analyzed for the case of the worst driver. While in the MLDEA-CI model each DMU obtains its own best possible indicator weights, the MLDEA-CSW model determines a set of weights to get the highest performance for all DMUs simultaneously leading to a more fair comparison among the DMUs. Results show that this methodology could be used as an effective screening tool as it can assist the elderly driver evaluator to gain insight in the driver's overall performance by combining the outcome of various assessment tools. This screening could become a part of the regular process of license renewal.

Chapter 3 contains an application of fuzzy linear programming. In particular, fuzzy model optimizing the profit margin of an industrial company under economic and ecological constraints is built. Using the fuzzy parameters in optimization models allows a decision-maker to involve the uncertainty in that model and in the resulting decision. This fact can help to make a model more realistic. Many approaches how to solve the fuzzy optimization models as well as types of uncertainty which are taken into account exist. Particularly, in this chapter, a level sets approach has been chosen to solve the proposed model and the uncertainty in the form of ambiguity is only considered (i.e. the uncertainty caused by one-to-many relations where the choice between two or more alternatives are left unspecified). The model built in this chapter is verified using the data of one metallurgical company in the Czech Republic which has a duty of covering its carbon dioxide emissions by emissions allowances.

The final chapter (Chapter 4) presents the fuzzy optimization method where fuzzy quantities are extended to intuitional fuzzy quantities. That extension helps to define models describing the real-world situations in a more precise way by using a non-membership function in addition to a membership function which describes each fuzzy set. This gives a decision-maker more freedom how to express the non-belongingness. Because when using the *ordinary* fuzzy sets, the non-membership degree is taken as one minus the membership degree. As well as in the Chapter 3, a level sets approach has been applied. However, unlike that chapter, the author presents the original method allowing the use of intuitionistic fuzzy sets in optimization models. A numerical example of a production planning problem is given at the end of the chapter.

The authors have written the chapters as follows:
Chapter 1: J. Hančlová and L. Chytilová
Chapter 2: S. Babaee, Y. Shen, E. Hermans and M. Toloo
Chapter 3: F. Zapletal
Chapter 4: J. Ramík.

This book is intended for postgraduate or graduate students in the areas of operations research, systems engineering and fuzzy sets. The book will also be useful for the academic community researchers, as well as public institutions with an interest in these respective areas.

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Jana Hančlová, Ostrava, June 2015

Contents

List of Abbreviations

ADS	Amsterdam Dementia Screening
ADV	Loans and Advances to Banks
AE	Allocative Efficiency
BOD	Benefit Of the Doubt
BCC	Banker, Charnes and Cooper
BCC-I	Input-oriented BCC Model
CCR	Charnes, Cooper and Rhodes
CCR-I	Input-oriented CCR model
CE	Cost Efficiency
CER	Certified Emission Reduction
CI	Composite Indicator
CRS	Constant Returns to Scale
DEA	Data Envelopment Analysis
DEA-WEI	DEA Without Explicit Inputs
DEP	Deposits & Short Term Funding
DMU	Decision Making Unit
DSF	Digit Span Forward
EFFC	Efficiency Change
EMP	Number of Employess
EUA	European Union Allowance
EU ETS	European Trading Scheme of the European Union
FA	Fixed Assets
FLP	Fuzzy Linear Programming
IFLP	Intuitionistic Fuzzy Linear Programming
IFS	Intuitionistic Fuzzy Set
LCSSDI	LCS'Scale Deficiency Index
MCS	Mean-Complete Stop
MLDEA-CI	Multiple Layer DEA-based Composite Index
MLDEA-CSW	Multiple Layer DEA with Common Set of Weights

MMSE	Mini Mental State Examination
MPI	Malmquist Productivity Index
NEA	Non-Earning Assets
OECD	Organization for Economic Co-operation and Development
PTE	Pure Technical Efficiency
RTS	Return to Scale
RSR	Road Sign Recognition
SBM	Slack-Based Measures
SE	Scale Efficiency
SEC	Securities
SFA	Stochastic Frontier Analysis
TE	Technical Efficiency
TECH	Production Technology
TECHC	Technical Change
UFOV	Useful Field Of View
VRS	Variable Returns to Scale

Part I

Optimization Problems and DEA Models

Chapter 1

Efficiency Evaluation of Banks in the Visegrad Group Using a Non-parametric Approach

1.1 Introduction

This chapter is devoted to evaluating the banks' efficiency of Visegrad Four (V4) countries in the time period 2004-2013. The economies of these countries have in common that after communism's collapse, there have been some changes - the transition to a market economy, joining the European Union in May 2004 and especially the transformation of banking system. The transition from centrally planned economy to market economy had been accompanied with restructuring and liberalization of the banking system. It had been associated with the privatization of some banks, the entry of foreign-owned banks, deregulation of interest rates and changes in legislation.

If the evaluation of banks efficiency has to be done, the examined banks have to be defined as a production units, which produce an intermediation services by channeling funds between depositors and creditors. The intermediation approach allows to involve the total banking cost. It includes deposits as inputs and also it allows to monitor the qualitative data. The first research question examines the role of the inclusion of non-traditional activities (non-interest income) into the output of banks production.

The process of restructuring and liberalization of the banking system also brings changes in the efficiency of banks according to their size and countries where the banks operate. Is bank's efficiency determined by the size of the bank? Is the level and the efficiency development same for all banks in the V4 countries? Finally, it is important to monitor the development trends of efficiencies and to evaluate the productivity changes including all components which are causing them.

The search for answers to the above research questions, first we have to summarize the current state of development and transformation of the banking system in the countries of the V4. It will be followed by a part which devotes the formulation of models, analyze data and introduce all used analytical tools for measuring the technical, cost, allocative and scale efficiencies under conditions of constant and variable return to scale. The dynamics of efficiency development is monitored through windows analysis. The productivity changes are monitored by Malmquist index and its components.

The next section provides an overview of scientific literature that examines the impact of deregulation on the efficiency. It is mainly focused on European banks from the perspective of the efficiency type, parametric or non-parametric approaches and framework of the production model which evaluates the bank efficiency.

The empirical part of the chapter is based on the specification and data analysis. Also the formulation of the basic variants of models to assess the efficiency in terms of (non)inclusion of non-traditional activities is done. First, the input-oriented model of data envelopment analysis is examined under the assumption of constant return to scale. Technical, cost and allocative efficiencies are evaluated for the model and its development is observed in the time period 2004-2013. The model for the assessment of efficiency is further modified by the assumption of variable return to scale. The banks efficiency is examined not only in cross-section, but also in terms of dynamics and productivity changes. During the analysis of banks efficiency, the attention is given to the size of banks and countries where the banks operate. The conclusion of the chapter is devoted to a summary of the results.

1.2 Development and transformation of the banking system in the countries of the Visegrad Group

The last hundred years have been dramatic and turbulent for most of the European countries. There have been wars, various political mindset, the European Union (EU) was established and several economical crises have passed. The European countries have changed demographically, politically and economically. Since the World War II. countries as Czechoslovakia, Hungary and Poland were part of so-called an Eastern Block. The Eastern Block was influenced by the Soviet Union. Economically it was characterized by a centrally planned economy with a focus on economic relations of countries within the block. The Eastern Block had fallen apart in 1989. This resulted in changes in all directions - political, sociological, demographical and mostly economical.

Countries from the Eastern Block had to change their established attitudes. The current economy had to be changed to a market economy. These countries had begun to slowly open up to the world. Considering the complexity of the situation, some countries from the Eastern Block tried to cooperate to succeed in the new environment. For example, Czechoslovakia, Hungary and Poland, countries of Central Europe, continued the declaration from the 14th century. They had formed so-called the Visegrad Group (the Visegrad Four - V4). The Visegrad Group was established in February 1991. Since 1993, after the split of Czechoslovakia, the Visegrad Group consisted of four countries - the Czech Republic, Hungary, Poland and Slovakia.

The main and the essential point of the economic transformation was the transformation of the banking system. Banks have a crucial role as financial intermediaries in the market economy. In 1990, the banking system of all V4 countries was at the same level. The basic step was to lead the commercial activities from the state-owned banks to the central bank. The primary reform of the banking institutional arrangements was completed between 1992 and 1993 for all countries in the V4. The achievement of so-called fourth degree -the highest degree of transformation[1] was completed in different time for each country. The Czech Republic has undergone a gradual long-term development and full liberalization of the banking sector was reached in 2005. The fastest full liberalization was achieved in Hungary. It was in 1997. Problems of the year 2010 are justified by imposing the high banking taxes. Also, the state intervention into the pension system and its full nationalization in 2011 brought problems for Hungary which put their economy down. The development of Polish banking market slightly lags behind the Czech Republic and Hungary. It is cost by the similar problems as in Hungary, in 2010 the government had intervention in the pension system. Also Polish banking system is very turbulent - in the past there were many banks that now are beginning to wear off or to associate. Slovakia has experienced a fundamental shift in the institutional arrangements after the arrival of Dzurinda's government in 1998. Since 2000, Slovakia has significantly improved the banking system according to the EBRD. Also its economy is growing since then.

In May 2004, all V4 countries joined the European Union. They are included into a common internal market and the financial sector. This had help them to develop the better financial institutions. On the other hand, this also had brought a fear - new competitors. In 2009, Slovakia joined the third stage of European Monetary Union - they adopted the common European currency (euro). This may as well help to its development.

Although, the nowadays world is characterized by globalization, there are still differences between countries which influence the structure, development and stability of the financial system of each country. However, the V4 countries are geographically and historically close so we should assume that their financial and banking systems do not show the substantial differences. Identification of the possible differences is one of the goals of the chapter. Also, the situation of the individual banking system within the V4 region for the period 2004-2013 is described.

[1] scale of EBRD - European Bank for Reconstruction and Development

1.3 Analytical methods for measuring the efficiency of banks

This section is devoted to a review of different methodological approaches that may be applied for measuring and evaluating the banks efficiency. Many research articles evaluate the performance of banks through different variants of financial accounting ratios. The use of these ratios is very simple, but the main criticism of the approach is that they do not reflect different combinations of inputs and outputs (Tripe, 2004). The frontier techniques measure banks efficiency with respect to the efficient frontier. It includes the dominant banks for the examined area. Individual banks may be organized according to the inefficiency of their performance. The relative distance from the efficient frontier may be examined as well as the possible causes of inefficiency. Many research articles also discuss the development of these relationships over the time and in terms of the structural changes.

There exist two approaches for the evaluation of the efficiency frontier. They are classified as parametric and non-parametric. **Parametric techniques** require the specification of the functional forms of the production functions, e.g. a Cobb-Douglas function or translog function. The parameters of these functions are further estimated by many econometric methods. On the other hand, **non-parametric approaches** do not require the specification of the functional forms of the production functions. They involve certain assumptions about the structure of the production technology, e.g. convexity (i.e. efficient frontier includes all linear combination of dominant production unit). Non-parametric approach are solved by mathematical programming tasks.

The next part of this subsection includes the definition of the basic method for the efficiency evaluation - data envelopment analysis (DEA) for evaluating cross-sectional data. Consequently, a discussion on DEA models is dedicated to measuring the efficiency units not only in cross-section, but also in time (Moving window analysis and Malmquist productivity index).

1.3.1 Data envelopment analysis (DEA)

DEA is the non-parametric approach based on mathematical programming. This approach has been historically formulated by Farrell (1957) as a single-output or single-input model with radial measure of technical efficiency. Charnes et al. (1978) extended the original approach to the multiple-output or multiple-input case. Further development of DEA models was very intense and was influenced by Seiford, Zhu, Cooper and many other researchers.

The classic DEA approach determines the level of technical efficiency (TE) as an estimation of discrete piecewise frontier (i.e. **efficient frontier**) known as Pareto-efficient decision making units (DMUs). Pareto-efficient DMUs use a minimum of productive input sources to generate the outputs (case input-oriented model), or maximize the outputs for the input sources (case output-oriented model). These Pareto-efficient DMUs have a **benchmark efficiency** score of unity. They are not able to improve any input or output without deterioration of other input or output. DMUs which are not at efficient frontier are inefficient relative to other DMUs. They have the technical efficiency score between 0 and 1.

The criterion for the classification of DEA models is possible orientations in DEA models. As it was already mentioned, **input-oriented models** represent models where the DMUs produce a given quantity of outputs with the a minimum possible amount of controllably inputs. Inefficient DMUs may be moved to the effective border by reducing the consumption of the certain inputs. **Output-oriented models** are models where the DMUs production for a given amount of inputs produce the maximum amount of controllably outputs. In this case, inefficient DMUs may be shifted to efficient frontier by increasing of the certain outputs. The last type of orientation in the DEA models are a non-oriented (additive) models. These models are based on the optimal mix of inputs and outputs. In input-oriented DEA model, the technical efficiency (TE) of production units (banks) is measured as:

$$\Theta_{\text{input}} = \frac{\text{minimum input}}{\text{actual input}}. \tag{1.1}$$

DEA models may be divided into two categories: allocation DEA models and non-allocation DEA models. The **non-allocation DEA models** determine the technical efficiency of individual DMUs without using any information on prices of inputs or outputs. **Technical efficiency** evaluates the physical transformation of production inputs to outputs relative to other DMUs with the use of the certain technologies. TE of banks with the ability to transform multiple resources into multiple financial services.

1.3.2 Non-allocation DEA models

In the calculation of the technical performance of the non-allocation DEA models are distinguished two types of models - CCR model and BCC model according to the criteria of return-to-scale. Charnes et al. (1978) developed the DEA model based on the assumption of **constant return-to-scale** (CRS) with the name of CCR DEA model. CCR model assumes a strong disposability of inputs and outputs and convexity of the production possibility set. The application of CCR model provides not only the technical score of each unit. It also provides information on input and output slacks and reference set for inefficient units. This publication is focused on the input-oriented CCR model, which is identified as **CCR-I model**.

For the definition of CCR-I model we consider n banks ($j = 1, 2, ...,$ n), which use quantities of inputs $x \in R_+^m$ to produce quantities of outputs $y \in R_+^s$. x_{ij} are denoted as the amount of the i^{th} input used by the bank j ($i = 1, 2, ..., m$) and y_{rj} express the amount of the r^{th} output produced by the bank j ($r = 1, 2, ..., s$). A linear combination of the multiple-inputs and multiple-outputs for each bank allows to obtain the technical efficiency for the target 'o' by solving the fractional programming model:

$$
\begin{aligned}
&\max_{u,v}\ h_o(u,v) = && \frac{\text{virtual output}_o}{\text{virtual input}_o} = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} && \\
&\text{s.t.} && \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, && j = 1, ..., n, \\
& && u_r \ge \varepsilon, && r = 1, ..., s, \\
& && v_i \ge \varepsilon, && i = 1, ..., m,
\end{aligned}
\tag{1.2}
$$

where y_{ro} is the amount of the r^{th} output produced by the bank 'o', x_{io} is the amount of the i^{th} input used by the bank 'o', u_r is the weight given to output r, v_i is the weight given to input i and ε is a non-Archimedean (infinitesimal) constant.

The solution of the optimization problem (1.2) provides input and output weights that maximize the proportion of a virtual output to a virtual input for bank 'o'. The first constraint indicates that the ratio of virtual output to the virtual input for each bank must be less than or equal to 1. The maximum efficiency h_o^* is at the most equal to 1.

Charnes and Cooper (1962) developed a transformation of CCR-I model from the fractional programming problem (1.2) to an equivalent primal linear programming problem in **multiplier form** (1.3) or an dual program in **envelopment form** (1.4):

$$\begin{aligned}
\max_{\mu,v}\ f_o(\mu) = \quad & \sum_{r=1}^{s} \mu_r y_{ro} & \\
\text{s.t.} \quad & \sum_{i=1}^{m} v_i x_{io} = 1 & \\
& \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, & j = 1, ..., n, \\
& \mu_r \ge \varepsilon, & r = 1, ..., s, \\
& v_i \ge \varepsilon, & i = 1, ..., m,
\end{aligned} \tag{1.3}$$

and

$$\begin{aligned}
\min_{\theta^{CCR},\lambda,s^+,s^-}\ g_o(\theta^{CCR}, s^+, s^-) = \quad & \theta_o^{CCR} - \varepsilon\Big(\sum_{r=1}^{s} s_r^+ + \sum_{i=1}^{m} s_i^-\Big) \\
\text{s.t.} & \\
\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta_o^{CCR} x_{io}, \quad & i = 1, ..., m, \\
\sum_{j=1}^{n} \lambda_j y_{rj} - s_i^+ = y_{ro}, \quad & r = 1, ..., s, \\
\lambda_j \ge 0, \quad & j = 1, ..., n, \\
s_r^+, s_i^- \ge 0 & \\
0 < \varepsilon \le 1. &
\end{aligned} \tag{1.4}$$

The number of limitation for the primary model is $(n+s+m+1)$. The dual model has only $(m+s)$ limitations and thus it is easier to deal with the dual model (1.4). The value θ_o^{CCR} corresponds to the technical efficiency of the bank 'o'. It represents the largest possible radial contraction that is proportionally applied to the bank 'o's inputs in order to project it to a point to the efficient frontier. Inefficient banks have $\theta_o^{CCR} < 1$. s_i^- is the input slack (the input excess) and s_r^+ is the output slack (the shortfall in the production of output r).

The **BCC model** has been developed by Banker et a. (1984) as an extension of CCR model with assumption that the return-to-scale is variable (**VRS**). The BCC model allows to divide the technical efficiency (TE) into **pure technical efficiency** (PTE) and **scale efficiency** (SE). Input-oriented **BCC model** is referred to as BCC-I. BBC-I measures the pure technical efficiency of the bank 'o' by solving of primary optimization problem (1.5) in the multiplier form or dual form (1.6) expressed in envelopment form:

$$
\begin{aligned}
\max_{\mu,v}\ f_o(\mu,\mu_o) = \quad & \sum_{r=1}^{s}\mu_r y_{ro} - \mu_o \\
\text{s.t.} \quad & \sum_{i=1}^{m} v_i x_{io} = 1 \\
& \sum_{r=1}^{s}\mu_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - \mu_o \le 0, \quad && j = 1,...,n, \\
& \mu_r \ge \varepsilon, && r = 1,...,s, \\
& v_i \ge \varepsilon, && i = 1,...,m, \\
& \mu_o \text{ free in sign}
\end{aligned}
\tag{1.5}
$$

and

$$
\begin{aligned}
\min_{\theta^{BCC},\lambda,s^+,s^-} g_o(\theta^{BCC}, s^+, s^-) = \quad & \theta_o^{BCC} - \varepsilon\Big(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\Big) \\
\text{s.t.} \quad &
\end{aligned}
\tag{1.6}
$$

Optimization problems and DEA models

$$
\begin{aligned}
&\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta_o^{BCC} x_{io}, && i = 1, ..., m, \\
&\sum_{j=1}^{n} \lambda_j y_{rj} - s_i^+ = y_{ro}, && r = 1, ..., s, \\
&\sum_{j=1}^{n} \lambda_j = 1, && j = 1, ..., n, \\
&\lambda_j \geq 0 && \\
&s_r^+, s_i^- \geq 0. &&
\end{aligned}
$$

Models (1.5) and (1.6) differ from CCR-I models (1.3) and (1.4) in one condition - the free variable μ_o in the primal model and one constraint $\sum_{j=1}^{n} \lambda_j = 1$ in the dual model. Since BCC-I model has the additional constraint, the feasible area of BCC models is a subset of CCR-I model. The relationship between the optimal objective values of CCR-I model and BCC-I model is $\theta_o^{*BCC} \geq \theta_o^{*CCR}$. The value of scale efficiency for the bank 'o' is given by relationship (1.7) as:

$$
\mathrm{SE} = \theta_o^{*CCR} / \theta_o^{*BCC}. \tag{1.7}
$$

The group of non-allocation models includes not just CCR and BCC models. It also includes additive models, multiplicative models and slack-based measures (SBM) models (Kumar and Gulati, 2014). **Additive models** or **Pareto-Koopmans** models are not based on the input or output orientation. They provides non-oriented measure that simultaneously reduces inputs and augments outputs by taking the slack. This group of models was first mentioned in the publication by Charnes et al. (1985). **Multiplicative models** differ from the additive models by the fact that efficiency is calculated as the ratio of the weighted multiplicative product of outputs divided by the weighted multiplicative product of inputs in order to account for interdependencies between input or output. These types of models were devoted by Charnes et al. (1982). The last type is represented by the **slack-based measures (SBM) models**. These models were mentioned in the publication by Tone (2001). Tone is examining inputs/outputs individually contrary to the radial approaches that assume proportional changes in inputs/outputs. The scalar measure deals directly with the input excesses and the output shortfalls increasing in each input and output slack. The measure is determined only by its reference set and it is not affected by statistics over the whole data set.

1.3.3 Allocation DEA models - cost efficiency

The allocation DEA models are used for cost minimization, revenue maximization and profit maximization. This book also focuses on the first group of models - **cost efficiency DEA models**.

In the calculation of traditional **cost efficiency** (CE) of each individual bank, it is assumed that the market prices of inputs are specified. If the objective of bank is to minimize the cost, then the measure of cost efficiency is provided by a ratio of the minimum cost to the observed cost (1.8). The value of the ratio is situated in the interval between 0 and 1.

$$\mathrm{CE}_o = \frac{\text{minimum cost}}{\text{actual cost}} = \frac{\sum_{i=1}^{m} p_i^o \ \tilde{x}_{io}^*}{\sum_{i=1}^{m} p_i^o x_{io}}, \tag{1.8}$$

where p_i^o is the unit price of i^{th} input for the bank 'o', $\tilde{x}_{io}^*$is the optimal quantity of i^{th} input for the bank 'o' that minimizes the cost, x_{io} is the actual value of the i^{th} input for 'o' bank.

Farrell's framework (1957) to measure the cost efficiency states that the input-oriented technical efficiency (TE) is just one component of the cost efficiency. In order to be cost efficient, first the bank must be technically efficient. The second component of CE is then input-oriented **allocative efficiency** (AE). AE reflects the ability of the bank to choose the inputs in optimal proportions, given their respective prices. Therefore AE expresses whether the examined bank uses the right mix of inputs due to their relative prices. Allocative efficiency is also described as a residual component of the cost efficiency of the bank. Therefore it is obtained as s share of cost and technical efficiency score:

$$\mathrm{AE} = \mathrm{CE}/\mathrm{TE}. \tag{1.9}$$

The linear programming problem for the calculation of the traditional cost efficiency is specified as below:

$$
\begin{aligned}
\min_{\lambda,\tilde{x}} \quad & \sum_{i=1}^{m} p_i^o \tilde{x}_{io} & & \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \tilde{x}_{io}, & & i = 1, ..., m, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, & & r = 1, ..., s, \\
& \lambda_j, \tilde{x}_{io} \geq 0 & & j = 1, ..., n, \\
& \Big(\sum_{j=1}^{n} \lambda_j = 1 \text{ in case of VRS}\Big) & &
\end{aligned}
\tag{1.10}
$$

where p_i^o is the unit price of i^{th} input for the bank 'o', $\tilde{x}_{io}$ is the quantity of i^{th} input for the bank 'o' that minimizes the cost, $\tilde{x}_{io}^*$ is the optimal value of $\tilde{x}_{io}$, x_{ij} is the actual value of i^{th} input for j^{th} bank. In the cost efficiency model the unit cost for the bank 'o' has to be fixed at p^o and cost-minimizing input-bundle $\tilde{x}^* = (x_{1o}^*, x_{2o}^*, ..., x_{mo}^*,)$ that produces the output y_{ro}t is to be found.

1.3.4 Panel data DEA models

The previous section of this subsection was devoted to DEA models which measure and evaluate the efficiency for cross-section data. If the above efficiency analysis would be extend over several years, efficiency trends of banks over time could be monitored with use of Moving window analysis and Malmquist productivity index and its components.

Charnes et al. (1985) had developed method known as a **Moving window analysis**. This method performs DEA analysis over time by using a moving average. The advantage of this method is in case, when cross-section observations are small then the discriminator power of DEA method is usually reduced (Coello et al. 2005). This approach also allows to obtain the trends of dynamic efficiency over the studied period and monitor the sensitivity analysis in time. If there are n banks over T $(t = 1, 2, ..., T)$ yearly periods, it may perform the analysis efficiency using a 3-year $(w = 3)$ window. For each bank is generated the first window as a rolling average efficiency for the first 3 years, the second window for the second till the fourth year, ..., and finally $(T-2)^{th}$

window as a rolling average over years $T-2$, $T-1$ and T.

The **Malmquist productivity index (MPI)** was first mentioned by Caves et al. (1982). This index was further developed by Färe and other authors (e.g. Färe et al. 1997, Ray and Desli 1997). Some research articles also monitor a conventional Törnqvist (1936) index.

The Malmquist productivity index in the case of input-oriented model is based on the definition of the distance function $D^t(\mathbf{x}^t, \mathbf{r}^t)$, which searches for the maximum quantity of inputs to reduce the given outputs. The Malmquist input-oriented productivity index, which is defined relatively to the initial technological period, is determined by the following relationship:

$$M^t(\mathbf{x}^t, \mathbf{y}^t, \mathbf{x}^{t+1}, \mathbf{y}^{t+1}) = \frac{D^t(\mathbf{x}^{t+1}, \mathbf{y}^{t+1})}{D^t(\mathbf{x}^t, \mathbf{y}^t)} \tag{1.11}$$

and

$$M^{t+1}(\mathbf{x}^t, \mathbf{y}^t, \mathbf{x}^{t+1}, \mathbf{y}^{t+1}) = \frac{D^{t+1}(\mathbf{x}^{t+1}, \mathbf{y}^{t+1})}{D^{t+1}(\mathbf{x}^t, \mathbf{y}^t)}. \tag{1.12}$$

The problem of the arbitrary choice of which technology to use is consist in the comparison of productivity changes in two examined periods t and $t+1$ as a geometric mean M^t and M^{t+1}:

$$\begin{aligned} \mathrm{MPI}^{t,t+1}(\mathbf{x}^t, \mathbf{y}^t, \mathbf{x}^{t+1}, \mathbf{y}^{t+1}) &= \sqrt{M^t M^{t+1}} = \\ &= \sqrt{\frac{D^t(\mathbf{x}^{t+1}, \mathbf{y}^{t+1})}{D^t(\mathbf{x}^t, \mathbf{y}^t)} \frac{D^{t+1}(\mathbf{x}^{t+1}, \mathbf{y}^{t+1})}{D^{t+1}(\mathbf{x}^t, \mathbf{y}^t)}}. \end{aligned} \tag{1.13}$$

The relation (1.13) can be rewritten in the following form:

$$\begin{aligned} \mathrm{MPI}^{t,t+1} &= \frac{D^{t+1}(\mathbf{x}^{t+1}, \mathbf{y}^{t+1})}{D^t(\mathbf{x}^t, \mathbf{y}^t)} \cdot \sqrt{\frac{D^t(\mathbf{x}^{t+1}, \mathbf{y}^{t+1})}{D^{t+1}(\mathbf{x}^{t+1}, \mathbf{y}^{t+1})} \cdot \frac{D^t(\mathbf{x}^t, \mathbf{y}^t)}{D^{t+1}(\mathbf{x}^t, \mathbf{y}^t)}} = \\ &= \mathrm{EFFC}^{t,t+1} \cdot \mathrm{TECHC}^{t,t+1}, \end{aligned} \tag{1.14}$$

where the first component $\mathrm{EFFC}^{t,t+1}$ in the equation (1.14) expresses the change in technical efficiency between periods t and $t+1$. If $\mathrm{EFFC}^{t,t+1} = 1$ then technical efficiency is same for the period. If $\mathrm{EFFC}^{t,t+1} < 1$, respectively ($> 1$), then the technical efficiency has decreased, respectively improved between periods t and $t+1$. The second component $\mathrm{TECHC}^{t,t+1}$ represents the change in production technology between periods t and $t+1$. It is also true, that if $\mathrm{TECHC}^{t,t+1} = 1$, then it represents no change in technology production. If $\mathrm{TECHC}^{t,t+1} < 1$ respectively (> 1), then the production technology has been worsened, respectively improved.

If we denote the production technology at time t as TECH^t and we define $\mathrm{TECH}^t = \{(\mathbf{x}^t, \mathbf{y}^t) : \mathbf{x}^t$ can produce $\mathbf{y}^t\}$. It includes all the input-output vectors are that technically feasibly at time t. The Shephard's (1970) input distance function can be defined on the technology TECH as:

$$D^t(\mathbf{x}^t, \mathbf{y}^t) = \sup\Big\{\theta : \Big(\frac{\mathbf{x}^t}{\theta}, \mathbf{y}^t\Big) \in \mathrm{TECH}^t\Big\}, \tag{1.15}$$

i.e. as a 'maximal' feasible contraction of $\mathbf{x}^t$. If the production function assumes the constant returns-to-scale (CRS), the Malmquist productivity index may be expressed:

$$\mathrm{MPI}^{t,t+1}(\mathbf{x}^t, \mathbf{y}^t, \mathbf{x}^{t+1}, \mathbf{y}^{t+1}) =$$

$$= \frac{D^{t+1}(\mathbf{x}^{t+1}, \mathbf{y}^{t+1}|CRS)}{D^t(\mathbf{x}^t, \mathbf{y}^t|CRS)} \cdot \sqrt{\frac{D^t(\mathbf{x}^{t+1}, \mathbf{y}^{t+1}|CRS)}{D^{t+1}(\mathbf{x}^{t+1}, \mathbf{y}^{t+1}CRS)} \cdot \frac{D^t(\mathbf{x}^t, \mathbf{y}^t|CRS)}{D^{t+1}(\mathbf{x}^t, \mathbf{y}^t|CRS)}} =$$

$$= \mathrm{EFFC}^{t,t+1} \cdot \mathrm{TECHC}^{t,t+1}. \tag{1.16}$$

$\mathrm{MPI}^{t,t+1}$ is expressed as the product of efficiency change $\mathrm{EFFC}^{t,t+1}$ and technical change $\mathrm{TECHC}^{t,t+1}$ in the time t and $t+1$. $\mathrm{EFFC}^{t,t+1}$ means how much closer a bank gets to the efficient frontier ($\mathrm{EFFC}^{t,t+1}$ > 1 is so called catching-up effect and $\mathrm{EFFC}^{t,t+1} < 1$ shows the falling behind). $\mathrm{TECHC}^{t,t+1}$ is technical change, which shows how much the benchmark production frontier shifts at each bank's observed input mix ($\mathrm{TECHC}^{t,t+1} > 1$ is the technical progress and $\mathrm{TECHC}^{t,t+1} < 1$ is the technical regress).

For the calculation $\mathrm{MPI}^{t,t+1}$ of o^{th} bank and input-oriented radial model (in the case of distance function, which is expressed as the reciprocal of the Farrell's (1957) measure of technical efficiency) are solved four optimization problems (1.17) - (1.20). The estimation of distance functions in the radial $\mathrm{MPI}^{t,t+1}$ are based on DEA model, developed by Charnes et al. (1978), which takes no account of slacks.

Distance function at time t using the reference technology for the period t:

$$\begin{aligned}
\left[\hat{D}_i^t(\mathbf{y}_o^t, \mathbf{x}_o^t)\right]^{-1} &= \min_{\theta,\lambda} \theta_o \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j \mathbf{x}_{ij}^t \le \theta_o \mathbf{x}_{io}^t \\
& \sum_{j=1}^{n} \lambda_j \mathbf{y}_{rj}^t \ge \mathbf{y}_{ro}^t \\
& \lambda_j \ge 0.
\end{aligned} \tag{1.17}$$

Distance function at time $t+1$ using the reference technology for the period $t+1$:

$$\begin{aligned}
\left[\hat{D}_i^{t+1}(\mathbf{y}_o^{t+1}, \mathbf{x}_o^{t+1})\right]^{-1} &= \min_{\theta,\lambda} \theta_o \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j \mathbf{x}_{ij}^{t+1} \le \theta_o \mathbf{x}_{io}^{t+1} \\
& \sum_{j=1}^{n} \lambda_j \mathbf{y}_{rj}^{t+1} \ge \mathbf{y}_{ro}^{t+1} \\
& \lambda_j \ge 0.
\end{aligned} \tag{1.18}$$

Distance function at time t using the reference technology for the period $t+1$:

$$\begin{aligned}
\left[\hat{D}_i^{t+1}(\mathbf{y}_o^t, \mathbf{x}_o^t)\right]^{-1} &= \min_{\theta,\lambda} \theta_o \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j \mathbf{x}_{ij}^{t+1} \le \theta_o \mathbf{x}_{io}^t \\
& \sum_{j=1}^{n} \lambda_j \mathbf{y}_{rj}^{t+1} \ge \mathbf{y}_{ro}^t \\
& \lambda_j \ge 0.
\end{aligned} \tag{1.19}$$

Distance function at time $t+1$ using the reference technology for the period t:

$$\begin{aligned}
\left[\hat{D}_i^t(\mathbf{y}_o^{t+1}, \mathbf{x}_o^{t+1})\right]^{-1} \quad & = \min_{\theta,\lambda} \theta_o \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j \mathbf{x}_{ij}^t \leq \theta_o \mathbf{x}_{io}^{t+1} \\
& \sum_{j=1}^{n} \lambda_j \mathbf{y}_{rj}^t \geq \mathbf{y}_{ro}^{t+1} \\
& \lambda_j \geq 0.
\end{aligned} \tag{1.20}$$

1.4 A survey of empirical literature on bank efficiency

The purpose of this part of the publication is to provide a brief overview of the empirical literature on the topic - the banks efficiency. It is focused on the study of European banks, including the V4 countries, especially after the year 2000.

An important criterion for the comparison of the banks efficiency is the cross-region efficiency. The main objective of these studies is to provide information about the competitiveness of banks in selected regions or countries to professional and unprofessional public.

A large group of empirical studies is constituted by publications that show the positive effect of deregulation's impact on the efficiency of European banks. Stavárek (2006) examined the efficiency frontier based on the intermediate approach (not including the output of non-interest income) and classic DEA in the time period 2001-2003 for 17 CEEC banks plus Portuguese and Greek banks. Results of these analysis showed that the most efficient was the V4, followed by other countries. Interestingly, it was found that larger banks are more efficient. Another study, published by Řepková (2014) was focused on the evaluation of the technical efficiency of 11 Czech commercial banks. The author had based her research on input-oriented DEA model (CCR and BCC). She analyzed her results by using the moving window analysis for years from 2003 to 2012. The average technical efficiency for CCR model was 70-78 % and around 84-89 % it was for BCC model. Zimková (2014) had examined the evaluation of technical efficiency of 16 Slovak banks in 2012. She used the input-oriented BCC and SBM models and transmission approach. As the most efficient banks were defined - Komercni banka, ING Bank and Tatra Bank. The least efficient bank was OTP bank. Weill (2007) had devoted the research to the CEE countries and 11 Western European countries in 1996-2000. Based on the stochastic frontier analysis (SFA), he had found that the bank efficiency in these countries had improved. The increase of cost efficiency was higher in the CEE countries than in Western European countries. Košak et al. (2009) also examined the cost efficiency of 8 new EU member states in time period 1996-2006. They used the SFA. The conclusions of the study showed that the cost efficiency had increased over the time. Pančurová and Lyócsa (2013) had examined the cost and the revenue efficiency using DEA method for a sample of 11 Central and Eastern European countries over the 2005-2008 period. They had found out that the size of bank is positively associated with the cost efficiency and the revenue. They had also examined that foreign banks are more cost effective than in the case of the revenue efficiency.

Koutsomanoli-Filippaki et al. (2009) had analyzed the profit efficiency for the V4 before joining the EU (1999-2003). They used SFA. Their major finding was that financial reforms had a significant positive impact on the profit and efficiency of the banking industry in the V4.

Titko et al. (2014) had deal in their study with performance and efficiency of 97 banks. They had connected all these calculations to the competitiveness. They had used the input-oriented DEA model under variable return to scale. All analyses had been done for the time period 2006-2012 and for the banking sector of all new member states of the EU. The results had shown a strong negative relationship between competition and efficiency in the banking sector. Roman and Sargu (2013) had devoted to analysis of the technical efficiency of European banks, especially in the case of the new EU member countries during the period 2003-2010. They were using two stage non-parametric approach. The obtained results suggest that the technical efficiency had slightly increased and the Czech banks are the best. They are followed by banks from Hungary and Romania. Kenjegalieva and Simper (2011) had studied changes in the productivity development of Central and Eastern European banks during 1998-2003. They used the intermediation approach and Luenberger productivity index, that is applied to technology, where the desirable and undesirable outputs are jointly produced and they are possible negative. They had found that the main driver of the productivity change is the technical improvement. However, other external risk factors are risks in the economy, corruption and also corruption perception.

There are also studies that show the **negative effect of the deregulation**. Fries and Taci (2005) studied the cost efficiency of 15 East European countries for the time period 1994-2001. They were using the SFA method and they had concluded that with the advancement of reforms the cost efficiency of transition nations had declined significantly.

1.5 Efficiency evaluation - approaches

The process of globalization brings the financial liberalization. However the control of the capital inflow and outflow must be in line with the market economy and under the control. There are the limitations of financial vulnerability.

The scientific literature that is dedicated to the efficiency evaluation of banks is based on two approaches. These approaches are related to the selection of inputs and outputs of the production units. First, the **production approach** had appeared. It is sometimes called the **service provision** or the **value added approach**. The second approach is known as the **intermediation approach**, sometimes referred as the **asses approach**. Both approaches are based on the classical microeconomic theory of the firm (banks) and they vary with the specification of banking activities.

The production approach was founded by Benston (1965). Benston had based the approach on the assumption that banks provide services to customers. The outputs of the production units are services provided to customers, which are represented by the number and the type of processed documents, transactions or special provided services over the considered period. The inputs are labour material, space and information systems expressed in terms of physical units or associated cost. The disadvantage of this approach is the focus only on the operating cost and ignorance of the interest expenses.

The second approach - the intermediation approach was proposed at work by Sealy and Lindley (1977). Authors had defined banks as a financial intermediaries channelling the funds between depositors and

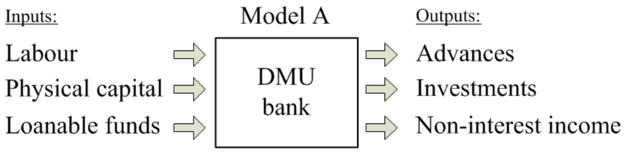


Figure 1–1 The structure of Model A

creditors. Banks are seen as production units which produce the intermediation services through the collection of deposits or other liabilities. They are also interested in the use of interest-earning assets and loans, securities and similar investments. This approach includes the operating cost and the interest cost, as the deposits are add to inputs.

Authors Berger and Humphrey (1997) had criticized in their work both approaches. But at the end, they had rather recommended the intermediation approach for the efficiency evaluation and the efficiency measurements of banking sectors. The reason is, that management of banks is trying to reduce just the cost and they do not reduce the non-interest expenses. In the case of the banking sector, there are processed large numbers of customer service, as well as bank fundings and investment decisions are not generally under the control of the branch. Also, authors Elyasiani and Mehdian (1990) pointed out on three advantages of the intermediation approach:

- it includes the total banking costs in better way, because interest expense on deposits and other liabilities are omitted the;
- it gives more convenient way of the deposits inclusion (input variable);
- it allows to monitor quality data considerations.

1.6 General models and specification of data

Based on the literature overview, we have chosen to use the intermedia approach. The two basic models have been formulated for the subsequent empirical studies of banks' efficiency evaluations. *Model A* and *Model B* have each the same three inputs for the bank (physical capital, labor, loanable funds). The *Model A* includes three outputs (advances,

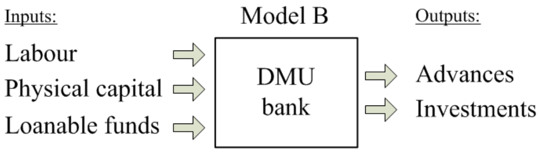


Figure 1–2 The structure of Model B

Table 1–1 Description of inputs and outputs and input prices

Variables	Description in the balance sheet	Unit of measurement
Input Variables		
Labour (x_1 - EMP)	Number of employees	Number
Physical capital (x_2 - FA)	Fixed assets = Tangible + Intangible assets	Thousands of Euro
Loanable funds (x_3 - DEP)	Deposits + Short term funding	Thousands of Euro
Input prices		
Price of Labour (p_1 - p_ EMP)	Personnel expenses	Thousands of Euro
Price of Physical capital (p_2 - p_FA)	Other operating expenses	Thousands of Euro
Price of Loanable funds (p_3 - p_DEP)	Interest expense on customer deposits	Thousands of Euro
Output Variables		
Advances (y_1 - ADV)	Loans and advances to banks	Thousands of Euro
Investments (y_2 - SEC)	Other securities	Thousands of Euro
Non-interest income (y_3 - NEA)	Non-earning assets	Thousands of Euro

Interests, non-interest income). The *Model B* is reduced and has just two outputs. The non-interest income is not included. The structure of both models is illustrated in Figure 1−1 and Figure 1−2.

Model B is the standard specification for the intermediation approach. It provides the assessment of banks' efficiency only in terms of the financial intermediation. This model does not take into account the non-traditional activities. The influence of output - non-interest income, is found out by comparing the results of *Model A* and *Model B* for the evaluation of banks' efficiency or eventually the intensity of the development of the indicator.

The required data set of inputs and outputs have been collected from the Bankscope[2]. Table 1–1 provides the description of inputs and outputs. All variables had been given in thousand Euros.

[2]https://bankscope.bvdinfo.com/

Table 1–2 List of analyzed banks

DMU	Name of banks	Country
1	Bank Ochrony Srodowiska SA - BOS SA	PL
2	Bank Polska Kasa Opieki SA-Bank Pekao SA	PL
3	Bank Zachodni WBK S.A.	PL
4	BNP Paribas Bank Polska SA	PL
5	Ceska Sporitelna a.s.	CZ
6	Ceskomoravska Zarucni a Rozvojova Banka a.s.	CZ
7	Ceskoslovenska Obchodni Banka A.S.- CSOB	CZ
8	ING Bank Slaski S.A. - Capital Group	PL
9	J&T Banka as	CZ
10	K&H Bank Zrt	HU
11	MBank Hipoteczny SA	PL
12	mBank SA	PL
13	Nordea Bank Polska SA	PL
14	OTP Bank Plc	HU
15	OTP Banka Slovensko, as	SK
16	PPF banka a.s.	CZ
17	Prima banka Slovensko a.s.	SK
18	Raiffeisen Bank Zrt	HU
19	Raiffeisen stavebni sporitelna AS	CZ
20	Stavebni Sporitelna Ceske Sporitelny as	CZ
21	Tatra Banka a.s.	SK
22	Unicredit Bank Czech Republic and Slovakia AS	CZ
23	UniCredit Bank Hungary Zrt	HU
24	UniCredit Bank Slovakia a.s.	SK
25	Vseobecna Uverova Banka a.s.	SK

The analysis of the efficiency is done based on the size and the location of the bank (country). Table 1-2 shows the selected sample of 25 financial institutions, mainly banks, of the V4 countries. The sample contains 8 financial institutions from the Czech Republic (CZ), 4 financial institutions from Hungary (HU), 8 financial institutions from Poland (PL) and 5 financial institutions from the Slovak Republic (SK). The sample was selected according to the availability of all data for all institutions in the period. Categorization by size was made on the basis of the quartile value of the fixed assets:

- **micro banks** - the fixed assets are less than first quartile Q1;

Table 1–3 The number of banks by the size

	Bank size				Quartile value of fixed assets		
year	**micro**	**small**	**medium**	**lage**	**FA_Q1**	**FA_Q2**	**FA_Q3**
2004	6	6	7	6	10 851	47 400	146 198
2005	6	6	7	6	9 996	52 800	150 674
2006	6	6	7	6	13 047	50 300	150 164
2007	6	6	7	6	18 754	62 451	169 253
2008	6	6	7	6	21 494	62 770	168 481
2009	6	6	7	6	19 086	61 147	154 649
2010	6	7	6	6	18 532	62 982	172 617
2011	6	7	6	6	17 940	43 684	167 492
2012	6	6	7	6	17 895	43 155	155 663
2013	6	6	7	6	17 134	37 672	149 799

- **small banks** - the fixed assets are equal to or higher than first quartile and less than median Q2;
- **medium banks** - the fixed assets are between the median and third quartile Q3;
- **large banks** - the fixed assets are greater than third quartile.

The following Table 1-3 shows the number of banks according to the size of the fixed assets in the period 2004-2013. The distribution frequency is stable. Just in 2010 and 2011, the bank - UniCredit Bank Czech Republic and Slovakia; has decreased the level of fixed assets, so the bank had been moved from the medium group to the small group.

1.7 Empirical results of Model A and Model B for CCR

This part of the book deals with the influence of the non-traditional activities such as the output in the production model for evaluating the banks efficiency. The development of the share of the non-traditional activities can be characterized by using a percentage indicator of the non-interest income in total income and this is approximated by the following share of the non-traditional activities at the output of the bank production system:

$$\text{Share\_NEA} = 100 \cdot \frac{\text{NEA}}{\text{ADV} + \text{SEC} + \text{NEA}} = 100 \cdot \frac{y_3}{y_1 + y_2 + y_3}. \quad (1.17)$$

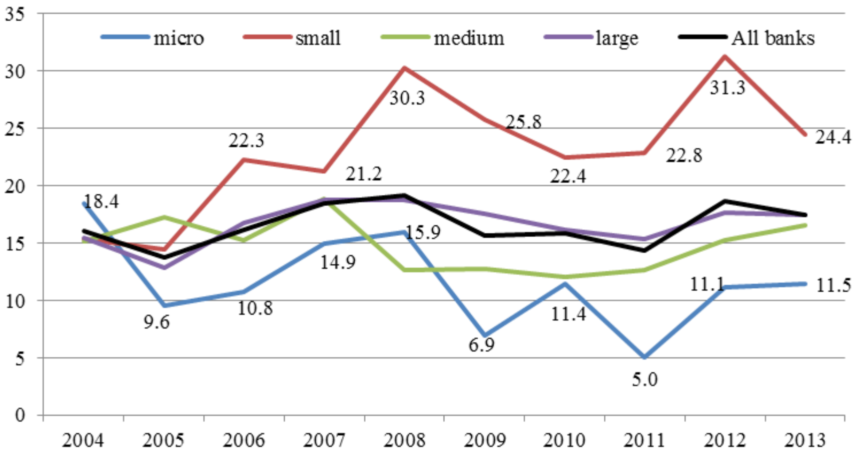


Figure 1–3 The development of the share of the non-traditional activities depending on the size of banks

The development of the average level of the indicator **Share_NEA** is shown in Figure 1–3 for all four groups of banks. They vary in the size as it is defined in Table 1–3. The development of the average share of the non-traditional activities for all banks is ranged from 13.7 % to 19.1 % with an average level of 16.5 %. In terms of the development, there is apparent increasing trend till the crisis period in 2008, after the falling and rising again after the year 2011. This corresponds with the development of large banks. The development of the share of the non-traditional activities in group of micro banks is slightly decreasing from 18.4 % to 11.5 %. The opposite trend is discovered for the development of small banks, where the share of the non-traditional activities had increased from 15.3 % in 2004 to 24.4 % in 2013.

Figure 1–4 compares the level and the average indicator trend of the share of the non-traditional activities for banks according to the regional classification. The Figure 1–4 shows that the highest average level of the Share_NEA has Hungary (22.6 %). However, this percentage significantly decreases from the initial value 27.2 % to 18.7 %. The lowest level of the share of the nontraditional activities is identified for banks in the Czech Republic (10.6 %). Their trend is slightly declining

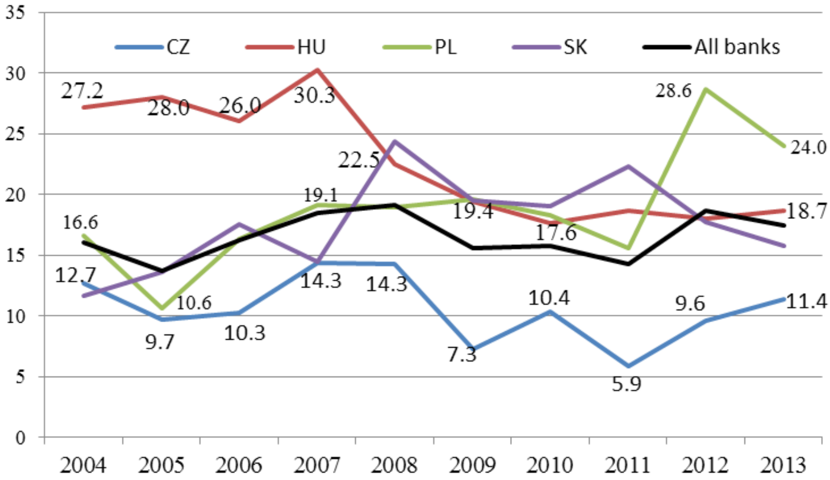


Figure 1–4 The development of the share of the non-traditional activities depending on country

from the value of 12.7 % to the value of 11.4 %. Polish and Slovak banks recorded rather growing trend.

During the examination, we had focused on the scientific literature that deals with the measuring and assessing the impact of the inclusion of the non-traditional activities at the bank efficiency and productivity, which is measured by using a non-parametric approach - DEA models. This all especially for the non-US surveys. Most studies had concluded that the inclusion of the non-traditional activities as output had resulted the strengthening of the average efficiency and also productivity scores. These conclusions were published in studies by Isik and Hassan (2003) for Turkish banking industry for the period 1981-1990, Sufian and Habibullah (2009) for Chinese banks for the period 2000-2005, Tortosa-Ausin (2003) for the cost efficiency of Spanish banks in the period 1986-1997, Huang and Chen (2006) for the cost efficiency of Taiwanese banks in the period 1992-2004. Casu and Girardon (2005) had examined the productivity of the European banks in the period 1994-2000 and also Sufian and Ibrahim (2005) had devoted to

Table 1–4 Mean efficiencies for the banks in the period 2004-2013

model		Model A			Model B	
year	**A_TE_CRS**	**A_CE_CRS**	**A_AE_CRS**	**B_TE_CRS**	**B_CE_CRS**	**B_AE_CRS**
2004	0.744	0.715	0.965	0.723	0.695	0.907
2005	0.716	0.685	0.963	0.617	0.587	0.846
2006	0.743	0.721	0.973	0.658	0.635	0.893
2007	0.716	0.694	0.968	0.671	0.641	0.842
2008	0.650	0.632	0.976	0.548	0.533	0.763
2009	0.844	0.793	0.937	0.627	0.612	0.843
2010	0.610	0.528	0.876	0.563	0.439	0.652
2011	0.799	0.659	0.822	0.583	0.453	0.645
2012	0.784	0.769	0.983	0.684	0.667	0.874
2013	0.738	0.720	0.980	0.683	0.661	0.839
min	0.610	0.528	0.822	0.548	0.439	0.645
max	0.844	0.793	0.983	0.723	0.695	0.907
mean	0.734	0.692	0.944	0.636	0.592	0.811

Note: TE (technical efficiency), AE (allocative efficiency), CE (cost efficiency).

evaluation of the productivity of Malaysian Banks in 2001-2003. Both publications also had come to the conclusion that the inclusion of the non-tradiitonal activities have increased the level of the productivity due to the technological change rather than the efficiency change.

Pasiouras (2008) had focused on the efficiency analysis of Greek banks over the period 2000-2004. He discovered that the inclusion off-balance sheet items in the output vector does not have impact on the efficiency scores, while inclusion of loan loss provisions in the input vector contributes to highest efficiency scores.

The next empirical part deals with the examination of the influence of (non)inclusion of the non-traditional activities for the non-allocation and allocation DEA models. All models are **input-oriented** and they formulated under constant return to scale (**CRS**). These models were formulated in section 1.3.2 as the equation (1.4) and in section 1.3.3 as the equation (1.10). Model A includes the non-traditional activities into the input vector as the non-earning assets NEA (y_3). The Model B does not take into account this output. First, let the efficiencies for **Model A** are calculated - technical efficiency (A_TE_CRS), cost efficiency (A_CE_CRS) and allocative efficiency (A_AE_CRS). Efficiencies are calculated by the equation (1.9) for each bank of the V4 for the period 2004-2013. Similarly, calculations for **Model B** are done (B_TE_CRS, B_CE_CRS, B_AE_CRS). The results are summarized in Table 1-4 and graphical presentation is shown in Figure 1–5 (Model A on the left side and Model B on the right side).

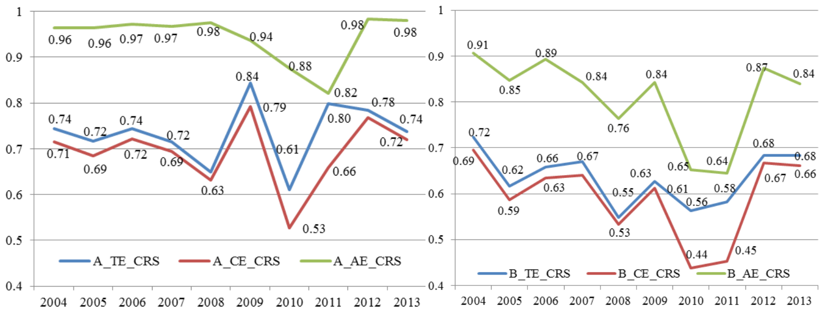


Figure 1–5 The development of TE, CE and AE for Model A and Model B for CRS

The results indicate that the average technical efficiency of surveyed banks is higher every year for Model A. It is in the range from 0.61 to 0.84. Model B has the range from 0.55 to 0.72. The same conclusions are also for the cost efficiency. The range of the cost efficiency for Model A moves from 0.53 to 0.79 and for Model B from 0.44 to 0.70. These findings lead to the conclusion that the exclusion of the non-traditional activities in Model B leads to an underestimation of the true efficiency of banks. Figure 1–5 shows the development of different efficiencies. Technical and cost efficiencies for Model A have slightly downward trend until 2008. After the financial crisis, the significant oscillation and increase of the difference between technical and cost efficiency is seen in 2013. The values of efficiencies in the end of the period are almost same as in the beginning. The decline of technical and cost efficiency is more intensive in Model B, particularly in 2010 and 2011. Also the significant reduction of cost efficiency in comparison with technical efficiency is more deeper. In 2013 levels of both efficiencies are below the original score.

A more detailed analysis for each bank had pointed out that the main reason for the technical inefficiency is the need to reduce the number of employees and fixed assets in comparison with efficient banks up an average of 58 %. Analysis of the cost efficiency had shown that it is necessary to reduce other operating expenses by up to 65 %, personal expenses by 50 % and interest expenses on customer deposits by 25 %.

The inclusion of the non-traditional activities into the output vector allows banks to obtain unbiased estimates of the average level of bank efficiency. This also contributes to reduce the cost inefficiency in employing the best practice production methods and achieving the maximum outputs from the minimum cost of inputs, especially during and immediately after the financial crisis. Our results are completely in line with Isik and Hassan (2003), Sumar and Gulati (2014) and Košak et al. (2009).

Parametric and non-parametric test have been used to test the statistical significance of the differences above mentioned models for technical, cost and allocative efficiency. Table 1-5 summarizes the results of these tests. The results of paired t-test with null hypothesis that the estimated mean of differences A_EFF-B_EFF is zero, were rejected at 1% significant level for all types of efficiencies EFF = TE, CE and AE. This indicates that the mean efficiencies of banks are significantly different if the non-interest income is not included in the output. This may lead to biased conclusions. The average technical efficiency for Model A was 73.43 % and 63.57 % for Model B. Therefore, the difference 9.86 % is statistically significant. The average cost efficiency in Model A was higher by 9.94 %, while the average of cost efficiency for Model A was 69.15 % and 59.21 % for Model B.

Table 1-5 also gives results for non-parametric tests - Wilcoxon Signed Ranks Test and Sign Test. The null hypothesis in these cases compare the differences in median of efficiencies for Model A and Model B. It is assumed that there are statistically equal. These tests had confirmed that the null hypothesis is rejected for technical, cost and allocative efficiency at 1% level of significance.

The Kendall rank correlation coefficient (Kendall's tau coefficient) was used to measure the degree of correspondence between the ranking of individual banks for TE, CE or AE between Model A and Model B. This coefficient is better than the classical Spearman's rank correlation coefficient. The results are summarized in Table 1-6. The results for the hypothesis: H_0 Kendall's tau correlation (A_EFF_CRS, B_EFF_CRS) = 0, shows the following findings:

- there is no perfect positive relationship between rankings of individual banks for Model A and Model B for TE, CE or AE;
- the probability of concordance among the order of banks varies.

Table 1–5 Hypothesis testing - efficiency differences across Model A and Model B

	EFF= TE_CRS	EFF= CE_CRS	EFF= AE_CRS
Paired t-test			
H_0: The mean of differences A_EFF - B_EFF = 0.			
Mean of paired differences	0.0986	0.0994	0.1337
t-statistic	9.659	10.086	13.079
Sig. (2-tailed)	5.93E-19	2.80E-20	4.17E-30
Inference	reject H_0	reject H_0	reject H_0
Wilcoxon Signed Ranks Test			
H_0: The median of differences A_EFF - B_EFF = 0.			
No. of pocitive ranks	0	0	12
z-statistic	-10.624	-11.175	-11.953
Sig. (2-tailed)	2.30E-26	5.42E-29	6.23E-33
Inference	reject H_0	reject H_0	reject H_0
Sign Test			
H_0: The median of differences A_EFF - B_EFF = 0.			
No. of positive differences	0	0	12
z-statistic	-12.166	-12.806	-12.766
Sig. (2-tailed)	4.73E-34	1.51E-37	2.53E-37
Inference	reject H_0	reject H_0	reject H_0

Note: Efficiency EFF = TE (technical), AE (allocative), CE (cost) for CRS

It ranges from 0.347 to 0.951 for the case of TE, the interval $[0.094; 0.952]$ is for CE and the range of AE is from 0.205 to 0.599. The value of the Kendall rank correlation coefficient is on the downward trend until 2011. This year there was the significant decline in the value of CE. The value is statistically zero and then increases again. These results had shown that the order of the banks in terms of CE are significantly changing. This corresponds to a different response of banks to the financial crisis, especially for the output of the non-traditional activities.

In terms of cost efficiency for Model A, the lowest value was 0.311 for *MBank Hipoteczny SA* (PL) in 2009. Another low values had been calculated for *UniCredit Bank Hungary Zrt* (HU, CE=0.332) and *MBank Hipoteczny SA* (PL, CE=0.242) in 2011. The number of cost efficient units have been in range from 3 to 5 banks in the period. Table 1-7 shows the number of cost efficient banks. The least cost efficient banks were in 2008, 2010 and 2013. Table 1-7 shows that the most cost efficient banks are in the Czech Republic, especially *PPF banka a.s.*. Also *Ceskoslovenska Obchodni Banka A.S. - CSOB* was cost efficient until 2006, *Raiffeisen stavebni sporitelna AS* in the middle of the period and

Table 1–6 Kendall rank correlation coefficient

year	**A_TE_CRS B_TE_CRS**	**A_CE_CRS B_CE_CRS**	**A_AE_CRS B_AE_CRS**
2004	0.951**	0.952**	0.599**
2005	0.657**	0.596**	0.352*
2006	0.741**	0.716**	0.436**
2007	0.918**	0.900**	0.484**
2008	0.699**	0.695**	0.205
2009	0.441**	0.507**	0.308*
2010	0.731**	0.628**	0.339*
2011	0.347**	0.094	0.331*
2012	0.622**	0.607**	0.243
2013	0.848**	0.842**	0.205

Note: **Correlation is significant at the 0.01 level (2-tailed).
*Correlation is significant at the 0.05 level (2-tailed).

Table 1–7 Number of cost efficient banks according to A_CRS model

	year	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**
No. of A_CE_CRS efficient banks		4	4	5	5	3	5	3	4	4	3
Ceska Sporitelna a.s.	CZ										
Ceskomoravska Zarucni a Rozvojova Banka a.s.	CZ						X	X	X	X	X
Ceskoslovenska Obchodni Banka A.S.- CSOB	CZ	X	X	X							
J&T Banka as	CZ										
PPF banka a.s.	CZ	X	X	X	X	X	X	X		X	X
Raiffeisen stavebni sporitelna AS	CZ			X	X	X	X				
Stavebni Sporitelna Ceske Sporitelny as	CZ	X			X						
Unicredit Bank Czech Republic and Slovakia AS	CZ										
K&H Bank Zrt	HU							X	X	X	X
OTP Bank Plc	HU						X		X		
Raiffeisen Bank Zrt	HU										
UniCredit Bank Hungary Zrt	HU										
Bank Ochrony Srodowiska SA - BOS SA	PL										
Bank Polska Kasa Opieki SA-Bank Pekao SA	PL										
Bank Zachodni WBK S.A.	PL										
BNP Paribas Bank Polska SA	PL	X	X							X	
ING Bank Slaski S.A. - Capital Group	PL		X	X	X		X				
MBank Hipoteczny SA	PL										
mBank SA	PL										
Nordea Bank Polska SA	PL										
OTP Banka Slovensko, as	SK										
Prima banka Slovensko a.s.	SK			X					X		
Tatra Banka a.s.	SK										
UniCredit Bank Slovakia a.s.	SK				X	X					
Vseobecna Uverova Banka a.s.	SK										

Ceskomoravska Zarucni a Rozvojova Banka a.s. in the end of the period. Polish banks *BNP Paribas Bank Polska SA* and *ING Bank Slaski S.A. - Capital Group* were cost efficient mainly in the beginning of the period. Hungarian bank *K&H Bank Zrt* has been cost efficient since 2010.

For better understanding of the inclusion of the non-traditional activities into the output vector of banks, the efficiency will be examined according to the size of banks and the regional jurisdiction.

Figure 1−6 presents the results of average technical (TE), cost (CE) and allocation (EA) efficiency for Model A with the non-traditional activities in output vector and Model B without non-traditional activities for the period 2004-2013. Bar graphs confirm that the omission of the non-traditional activities led to the underestimation of banks efficiency. In the terms of the average level of banks efficiency by both models under CRS conditions, banks may be arrange:

TE_micro banks > TE_large banks > TE_medium banks > TE_small banks.

Due to the previous analysis of the share of the non-traditional activities in the overall output according to the size of the banks, Figure 1−6 shows controversial conclusions. Micro banks with the lowest share of non-traditional activities, with average of 11.6 %, have the highest average technical efficiency for Model A (0.85) and Model B (0.82) as well. The difference is only 3 %. However, small banks with the largest average share of Share_NEA, around 23 %, have the lowest average technical efficiency for Model A (0.63) and Model B (0.46). The difference between models is larger 17 %. A similar situation is also observed for the cost efficiency. These results confirm that the inclusion of the non-traditional activities, such as the output of services provided by banks, are correct. Otherwise, there is a bias estimation for the efficiency. It is also necessary to take into account the size of banks and the share of the non-traditional activities in the total output. Likewise the cost efficiency, if there is a low share of the non-traditional activities at the output then the high degree of technical and cost efficiency is detected, but the difference between technical and the cost efficiency is higher. For example, micro banks have the low average share of Share_NEA (11.6 %) and small banks have the average of Share_NEA around 23 %. Micro banks have the highest average of the technical efficiency respectively cost efficiency for Model A (0.85 resp. 0.76) with difference of 9 % and for Model B (0.82 resp. 0.72) with difference of 10 %. Small banks have the lowest average technical efficiency respectively cost efficiency 0.63 resp. 0.60 with difference of 3 % for Model A and for Model B it is similar - 0.46 and 0.43 with difference 3 %. According to empirical observation, it can be assumed that the inclusion of the non-traditional activities would had to increase technical and cost efficiency. However,

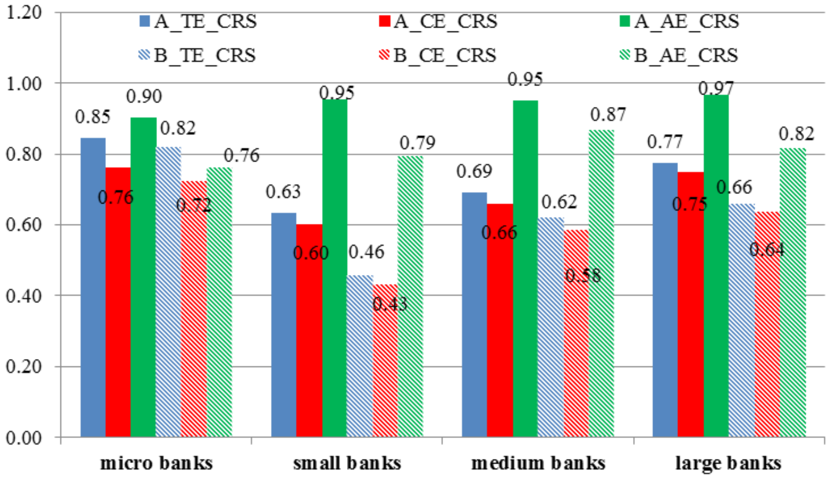
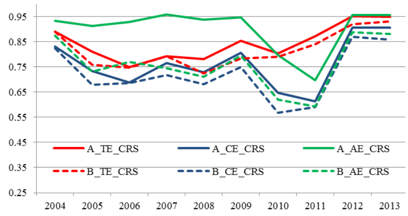


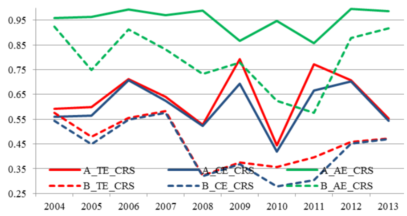
Figure 1–6 The development of technical, cost and allocation efficiency under condition of CRS

if there is higher proportion of the non-traditional activities in the total output, there is also greater decrease in the difference between technical and cost efficiency in the investigation for banks of the V4.

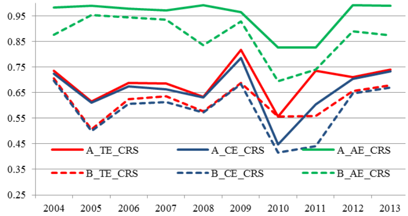
Figure 1−7 shows the development of the average technical, cost and allocative banks efficiency according to their size in the period 2004-2013. Table A_1 in Annex A gives more details about the figure. Presented charts support the hypothesis that micro banks have the highest average level of technical and cost efficiency. Since 2006 the average technical efficiency had increased from 0.75 to 0.95 in 2013. The development of cost efficiency had decreased after the stagnation in 2009 from 0.81 to 0.61. Then there was sharp rise to 0.91. Differences between both models are not very significant, even in the time of financial crisis. The second strongest group is the group of large banks. These banks had responded to the financial crisis by reduction of efficiency and by increase of efficiency oscillation. There is also a significant difference between the efficiency of Model A and Model B in comparison with micro banks. Until 2008, the group of medium banks have relatively stable technical and cost efficiency, as well as micro banks.



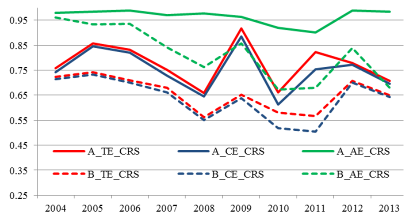
a) micro-banks



b) small-banks



c) medium-banks



d) large-banks

Figure 1–7 The development of technical, cost and allocation efficiency for Model A and Model B under CRS

Then the high variability had followed until 2012 with the significant decline of cost efficiency compared to technical efficiency. Small banks had the lowest technical and cost efficiency, especially in Model B - this model had excluded the non-traditional activities and the level of efficiency was 0.28 for cost efficiency and 0.36 technical efficiency. For those banks (they have the highest average of Shera_NEA for the outputs of banks) it is shown that the elimination of the non-traditional activities may significantly underestimate the correctly estimated efficiency.

Overall, the obtained results demonstrate that micro banks are the most technical and cost efficient. They are followed by large banks. Small banks had recorded decline during the financial crisis, mainly in the cost efficiency. The greatest differences between efficiencies of Model A and Model B had appeared for small banks, where the share of the non-traditional activities in output vector is the highest.

The evaluation of the average level of bank efficiency in term of the place of action is expressed in Figure 1–8. More precise values are shown in Table A_2 in Annex A. The results had shown that the inclusion of the non-traditional activities (Model A) gives the highest

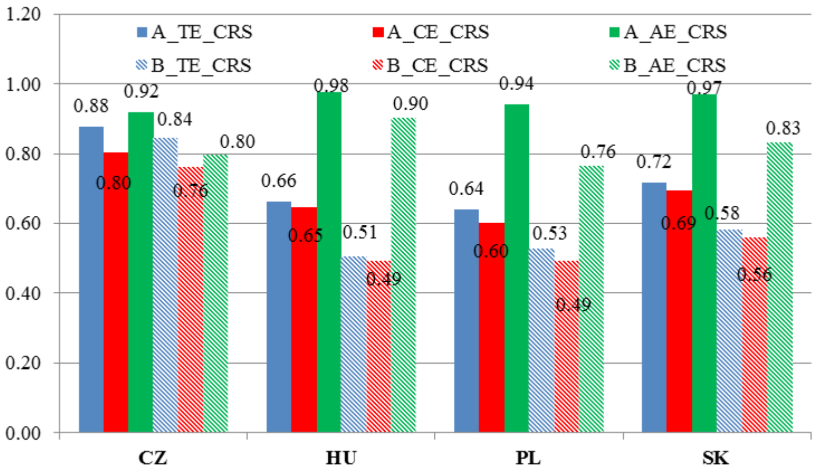


Figure 1–8 The development of technical, cost and allocation efficiency under CRS by regions

technical and cost efficiency for banks in the Czech Republic (0.88 resp. 0.80). The banks from the Slovak Republic (0.72 resp. 0.69), Hungary and Poland follow. These results correspond with the analysis of cost efficient banks in Table 1-7. However, if we take into account Model B (model without the non-traditional activities), Polish banks are more efficient than the Hungarian banks. This is true only for technical efficiency. In the case of the average cost efficiency, they are both equal to 0.49.

A more detailed graphical analysis of the development of the average bank efficiency in the period 2004-2013 is shown in Figure 1–9. It is seen that the Czech banks had a stable and high technical and cost efficiency. There is just one exception - the time after the financial crisis (2009-2011). This is seen in both models, especially for the cost efficiency. The development of technical and cost efficiency of Slovak banks involves downward trend with the high variability and with growing difference between results of Model A and Model B. Hungarian banks had the opposite trend of development. The banks had growing trend in the average technical and cost efficiency for both models. Al-

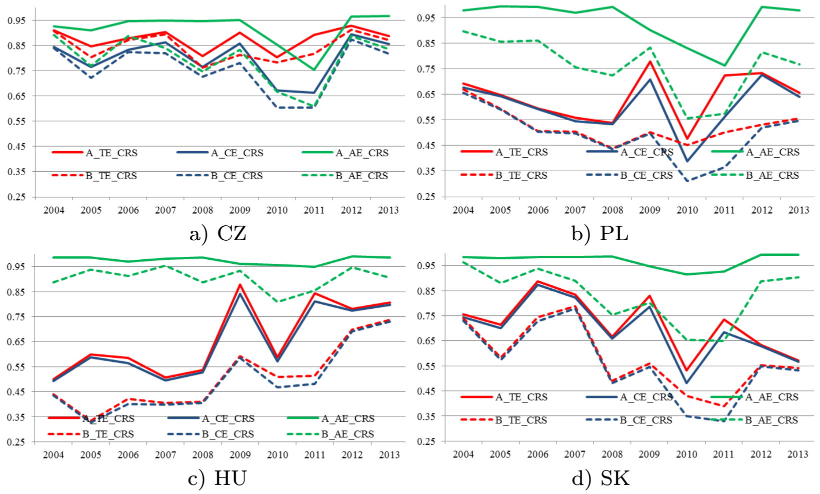


Figure 1–9 The development of technical, cost and allocation efficiency for Model A and Model B under CRS by region

though there was a noticeable difference if the non-traditional activities were or were not included. In case of Polish banks there had been observed decreasing trend for the technical as well as the cost efficiency until 2008. After considerable variability the stable efficiency occurs in 2013 and it is the same level as it is in 2004. The gap between technical and cost efficiency for both models is really growing in size in the unstable period in Poland.

These findings demonstrate that the highest and the most stable technical and cost efficiency is for banks in the Czech Republic. The inclusion of the non-traditional activities do not affect the results so much. Technical and cost efficiency of Slovak banks in the period decreases and for Hungarian banks is increasing. The omissions of the non-traditional activities provide underestimation of efficiency estimation. A notable variability of banking efficiency is seen during the financial crisis in 2008 for Polish, Hungarian and Slovak banks.

1.8 Empirical results of Model A and B for VRS

The previous empirical analysis of banking efficiency for the V4 was based on the assumption of the constant return to scale for the production function (CRS). There exists question, whether Model A or Model B would be more appropriate if the condition of variable return to scale (VRS) would be used. The **LCS'scale deficiency index (LCSSDI)** is used in the scientific literature for the examination of the form of scale economies in the V4 banking. LCSSDI was published by López-Cortés and Snowden (1998):

$$\text{LCS'scale deficiency index} = \frac{\text{Banks with DRS}}{\text{Banks with IRS + banks with DRS}}.$$

This index reflects the proportion of banks that are characterized by deceasing return to scale (DRS) to the total number of scale-deficient banks. If the value of deficiency index is greater than 0.5, them a large number of scale deficient banks experience DRS, while a value less than 0.5 indicates that a larger number of banks experience IRS in each year. The numbers of CRS, DRS, IRS units and calculated LCS'scale deficiency index for Model A and Model B for the time period are shown in Table 1-8.

The calculated values of LCS'scale deficiency indices of Model A (with the non-traditional activity) are in the range from 55.0 % to 66.7 %, with the mean value of 63 %. These results support the use of DEA model with variable return to scale. Table 1-8 shows a very low share of the index in 2005 (18.8 %) and 2006 (15.4 %) for Model B. The calculated values of LCS'scale deficiency indices of Model B have the average value equal to 43.7 %. This leads to the conclusion that Model B is suitable to leave under the conditions of CRS. Overall, the results lead to the conclusion that it is more appropriate to use Model A. This model does not underestimate the efficiency. Also it is seen that it is more appropriate to use the model with variable return to scale. Note, this model is denoted as A_VRS.

In this part of the publication, it was decided that the evaluation of the efficiency is based on previous results:

- structure of inputs and outputs depends on Model A - the inclusion of the non-traditional activities into the output vector of banks (Figure 1-1);

Table 1–8 LCS'scale deficiency index for Model A and Model B

	A_RTS			B_RTS				
year	**CRS**	**DRS**	**IRS**	**CRS**	**DRS**	**IRS**	**A_LCSSDI**	**B_LCSSDI**
2004	7	12	6	9	7	9	66.7	43.8
2005	8	10	7	9	3	13	58.8	18.8
2006	11	9	5	12	2	11	64.3	15.4
2007	8	11	6	10	8	7	64.7	53.3
2008	7	12	6	5	11	9	66.7	55.0
2009	9	10	6	8	9	8	62.5	52.9
2010	5	12	8	5	11	9	60.0	55.0
2011	8	11	6	4	10	11	64.7	47.6
2012	7	12	6	5	10	10	66.7	50.0
2013	5	11	9	5	9	11	55.0	45.0
						min	55.0	15.4
						max	66.7	55.0
						mean	63.0	43.7

- variable return to scale - A_VRS model (equation 1.6);
- input-oriented DEA model.

Table 1−9 summarizes the mean cost (A_CE_VRS), technical (A_TE_-CRS), pure technical (A_TE_VRS), scale (A_SE) and allocative efficiency scores (A_AE_VRS) of Model A for the time period. Figure 1−10 presents the trend of these efficiencies.

The **average cost efficiency** varies between 70.6 % and 86.5 % with the average value of 80.2 %. In comparison with A_CE_CRS from Table 1−4, this value is about 11 % higher. The average cost efficiency is equal to 80.2 %. This indicates that the typical bank in the sample would have to produce the same level of outputs using only 80.2 % of the cost actually incurred if it was producing on the cost frontier rather than at its current location. On the other hand, the cost inefficiency is equal to 19.8 % and it implies that in each year of the study period, the typical bank needs 19.8 % more resources and it entails more costs for the production of the same output relative to the best bank. The comparison of the average cost efficiency with the work by Kumar and Gulati (2014) shows that the estimated level of cost banks inefficiency in the V4 (19.8 %) is higher than it is for Indian banks (17.8 % in 1995-1998). On the other hand, it is less than for Turkish banks - 28 % (Iasik and Hassan (2003)) or the world mean inefficiency - 27 % (Berger and Humphrey, 1997). The average cost efficiency in 2004 was equal to

Table 1–9 Average efficiencies for banks under VRS

year	A_CE_VRS	A_TE_CRS	A_TE_VRS	A_SE	A_AE_VRS
2004	0.802	0.744	0.859	0.875	0.939
2005	0.771	0.716	0.840	0.867	0.922
2006	0.793	0.743	0.844	0.864	0.937
2007	0.841	0.716	0.884	0.737	0.950
2008	0.798	0.650	0.827	0.792	0.967
2009	0.865	0.848	0.918	0.921	0.940
2010	0.706	0.610	0.780	0.798	0.909
2011	0.769	0.799	0.898	0.893	0.855
2012	0.864	0.784	0.908	0.867	0.952
2013	0.815	0.738	0.844	0.883	0.969
min	0.706	0.610	0.780	0.737	0.855
max	0.865	0.844	0.918	0.921	0.969
mean	0.802	0.734	0.860	0.849	0.934

80 %. In terms of the development, it had rather a growing trend, but thanks to the financial crisis the significant decline to 71 % had been seen after 2009.

The **average technical efficiency** for each individual year was in range from 61.0 to 84.4 % with the average level of 73.4 %. The results are comparable to similar studies of the Czech banks (Řepková, 2014), where the efficiency had achieved the level of 70-78 % in the period 2003-2012. Also the previous results had shown that the Czech banks are technically more efficient than another banks from the V4. In terms of the development, it is interesting that the average pure technical efficiency (86.0 %) is lower in the V4 than the average allocative efficiency (93.4 %). This is true for the entire period 2004-2013, as it is shown in Figure 1–10. These relationships had supported the findings that the average allocative inefficiency (6.6 %) (i.e. select the optimal input combination for given input prices) should be less important than technical inefficiency (14 %) (i.e. under-utilization or wasting of inputs) as a source of the cost inefficiency within all inefficient banks. It is also evident that the average cost inefficiency (19.8 %) had contributed by 6.6 % on inappropriate selection of the optimal combinations of inputs given their prices and technology. Remaining inefficiency is due to the wastage of inputs in the production process. This findings suggest that managers of banks in the V4 should pay more attention to the use of all factor inputs rather than choosing the proper input mix given prices.

Table 1–10 Average efficiencies under VRS according to the size of banks

bank size	A_CE_VRS	A_TE_CRS	A_TE_VRS	A_SE	A_AE_VRS
micro	0.899	0.845	0.965	0.879	0.933
small	0.667	0.633	0.716	0.865	0.941
medium	0.730	0.694	0.810	0.860	0.903
large	0.928	0.774	0.962	0.789	0.963

Furthermore, the **average scale efficiency** was 84.9 % and the average pure technical efficiency was 86 %. The development of these efficiencies was similar. Just in 2007 the significant increase of the scale inefficiency had appeared. The main sources of the average overall technical inefficiency (26.6 %) were the pure technical inefficiency (14 %) (related to input) and the scale inefficiency (15.1 %) (related to output). These results had implied that technical inefficiency emanates due to managerial underperformance in controlling the way of inputs in production process and due to the failure to operate at optimum scale size. The results suggest that there are more opportunities to gain the technical efficiency - better utilization of existing resources by management or taking advantage of scale economies.

A similar analysis had been done for the efficiency levels and the efficiency development for A_VRS model according to the size of banks (Table 1–10) and according to country exposure (Table 1–11).

The calculated values of the average cost efficiency had shown that the most cost efficient are large banks (92.8 %). They are followed by micro banks (89.9 %). This is different compared to A_CRS model. Least cost efficient are again small banks (66.7 %). The low cost inefficiency of large banks is due to the technical inefficiency (3.8 %) and the allocative inefficiency. While in the group of small banks, the main reason of the cost inefficiency (33.3 %) is mainly technical inefficiency (28.4 %). This finding suggests the main problem is badly provided input combination of given input prices. Analysis of the average technical efficiency provides discovery that technically the most efficient group is the group of micro banks (84.5 %), followed by large banks (77.4 %). The main source of technical inefficiency for micro banks is scale inefficiency (12.1 %), while the pure technical inefficiency is just about 3.5 %. In the group of large banks, however, the main sources for technical inefficiency are the scale inefficiency (21.1 %) and the pure technical inefficiency (3.8 %). Management of micro banks should focus on con-

Table 1–11 Average efficiencies for VRS according to countries

country	A_CE_VRS	A_TE_CRS	A_TE_VRS	A_SE	A_AE_VRS
CZ	0.891	0.876	0.966	0.899	0.922
HU	0.752	0.662	0.818	0.815	0.919
PL	0.768	0.640	0.828	0.784	0.930
SK	0.757	0.716	0.777	0.898	0.972

trolling the waste of inputs in the production process. Management of large banks should avoided the failure of operating with non-optimum scale size.

Table 1–11 contains the average efficiency scores for Model A for variable return to scale and according to the classification of country exposure. The highest level of the average cost efficiency is seen for banks in the Czech Republic (89.1 %). The lowest level is in Hungary (75.2 %). The main cause of the cost inefficiency of Hungarian banks is the pure technical inefficiency (33.8 %), scale inefficiency affects only 7.1 %. Slovak banks have the average cost inefficiency 24.3 %. The key determinant is the pure technical inefficiency 28.4 %. The scale inefficiency is only 2.8 %. This is the lowest rate of the average scale inefficiency in the V4. For all V4 countries, the average pure technical efficiency is lower than the scale efficiency. Therefore, the main source of cost inefficiency is the incorrect use of scale economies. The overall technical inefficiency is highest for Polish banks (21.6 %). The main reason is the allocative inefficiency and especially the pure technical inefficiency (36 %). Due to the fact that almost all values of the average scale efficiency (A_SE) are usually smaller than the values of the average pure technical efficiency (A_TE_VRS), it can be provide the conclusion that the major source of the overall technical inefficiency in banks of the V4 is caused by the scale inefficiency (related to output).

Table 1–12 shows a more detailed analysis of the cost efficiency for each individual bank for VRS assumption. The number of cost efficient banks is significantly higher then for CRS assumption. Three Czech banks (Ceska sporitelna, Czechoslovak Commercial Bank AS-CSOB and PPF banka as), one Hungarian bank (OTP Bank Plc) and one Polish bank (mBank Hipoteczny SA) had been detected as efficient for almost entire period 2004-2013. The most cost efficient banks were in the Czech Republic with the average of 54 %, followed by Hungary (33 %), Poland (31 %) and finally by the Slovak Republic (22 %).

Table 1–12 Number of cost efficient banks according to A_VRS model

	year	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
	No. of A_CE_CRS efficient banks	9	7	9	11	11	11	9	7	10	8
Ceska Sporitelna a.s.	CZ	X	X	X		X	X	X	X	X	X
Ceskomoravska Zarucni a Rozvojova Banka a.s.	CZ				X	X	XX	XX	XX	XX	XX
Ceskoslovenska Obchodni Banka A.S.- CSOB	CZ	XX	XX	XX	X	X	X	X	X	X	X
J&T Banka as	CZ										
PPF banka a.s.	CZ	XX	XX	XX	XX	XX	XX	XX		XX	XX
Raiffeisen stavebni sporitelna AS	CZ			XX	XX	XX	XX				X
Stavebni Sporitelna Ceske Sporitelny as	CZ	XX		X	XX						
Unicredit Bank Czech Republic and Slovakia AS	CZ										
K&H Bank Zrt	HU							XX	XX	XX	XX
OTP Bank Plc	HU	X	X	X	X	X	XX	X	XX	X	
Raiffeisen Bank Zrt	HU										
UniCredit Bank Hungary Zrt	HU										
Bank Ochrony Srodowiska SA - BOS SA	PL										
Bank Polska Kasa Opieki SA-Bank Pekao SA	PL					X	X	X		X	
Bank Zachodni WBK S.A.	PL										X
BNP Paribas Bank Polska SA	PL	XX	XX							XX	
ING Bank Slaski S.A. - Capital Group	PL		XX	XX	XX	X	XX			X	
MBank Hipoteczny SA	PL	X	X	X	X	X	X	X	X	X	X
mBank SA	PL	X									
Nordea Bank Polska SA	PL										
OTP Banka Slovensko, as	SK				X		X				
Prima banka Slovensko a.s.	SK			XX			X		XX		
Tatra Banka a.s.	SK					X					
UniCredit Bank Slovakia a.s.	SK	X			XX	XX		X			
Vseobecna Uverova Banka a.s.	SK				X						

XX = cost efficient bank according to A_CRS and A_VRS model.
X = cost efficient bank according to A_CRS.

Table 1–13 Average through window with length of window=3

	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	Aver.	C-Aver
B1	0.333	0.278	0.327								0.313	
		0.328	0.387	0.514							0.410	
			0.421	0.563	0.511						0.498	
				0.649	0.593	0.471					0.571	
					0.518	0.424	0.487				0.476	
						0.424	0.497	0.567			0.496	
							0.483	0.545	0.617		0.548	
								0.631	0.779	0.785	0.732	0.506

1.9 Moving window analysis

In section 1.3.4 the attention was paid to the analysis of DEA models for the panel data. Based on Moving window analysis, the trends of dynamic efficiency can also be analyzed in the time period 2004-2013. The software *DEA Solver v. 8.0* was used to conducted the analysis of the average technical efficiency (A_TE_VRS) by input-oriented Model A using 3-year window.

The calculation process of the three-year average is presented in Table 1−13 for the Polish bank B1 (Environmental Protection Bank SA - BOS SA). The average value of the technical efficiency was equal to

Table 1−14 Average through window with length of window=3

	2004-2006	2005-2007	2006-2008	2007-2009	2008-2010	2009-2011	2010-2012	2011-2013
B1	0.313	0.410	0.498	0.571	0.476	0.496	0.548	0.732
B2	0.794	0.805	0.900	0.959	0.985	0.981	0.937	0.654
B3	0.666	0.656	0.644	0.685	0.633	0.702	0.791	0.831
B4	0.875	0.803	0.482	0.504	0.412	0.394	0.590	0.604
B5	0.924	0.844	0.787	0.868	0.980	1.000	0.991	0.974
B6	0.682	0.765	0.821	0.923	1.000	0.972	1.000	0.972
B7	1.000	1.000	0.951	0.989	0.978	1.000	0.985	1.000
B8	0.905	1.000	1.000	1.000	0.997	0.957	0.957	0.990
B9	0.740	0.620	0.470	0.573	0.623	0.682	0.819	0.929
B10	0.586	0.592	0.606	0.814	0.910	0.957	0.843	0.886
B11	0.996	0.962	0.938	0.997	1.000	1.000	1.000	1.000
B12	0.966	0.790	0.623	0.701	0.828	0.791	0.791	0.807
B13	0.340	0.348	0.243	0.261	0.276	0.475	0.475	0.378
B14	0.926	0.962	0.934	0.935	1.000	0.995	0.995	1.000
B15	0.470	0.645	0.604	0.684	0.681	0.580	0.580	0.532
B16	0.935	0.974	0.968	1.000	0.952	1.000	1.000	1.000
B17	0.713	0.621	0.571	0.501	0.544	0.545	0.545	0.643
B18	0.443	0.468	0.367	0.479	0.586	0.606	0.606	0.609
B19	0.916	0.967	0.981	1.000	0.993	1.000	1.000	0.986
B20	1.000	1.000	0.962	0.964	0.976	0.961	0.961	0.966
B21	0.705	0.702	0.785	0.809	0.721	0.519	0.517	0.581
B22	0.934	1.000	1.000	0.895	0.780	0.686	0.774	0.853
B23	0.659	0.532	0.476	0.493	0.380	0.378	0.379	0.494
B24	0.707	0.772	0.779	0.709	0.674	0.602	0.644	0.501
B25	0.803	0.835	0.828	0.890	0.650	0.575	0.542	0.557

0.313 for the period 2004-2005. This is indicated in column Average and continues with another window - the average for the displaced between 2005 and 2006 with a value of 0.410. These values are then successively listed in the summary Table 1−14 for all 25 banks.

The results can be graphically depicted. For example, the graphical results are seen for Hungarian banks in Figure 1−10. The best technical efficiency was reached by the bank B14 (OTP Bank Plc). B14 was also cost efficient according to previous cost efficiency analysis. The bank B10 (K&H Bank Zrt) had growing trend in technical efficiency. Since 2010 the bank B10 was also cost efficient. Conversely, the bank B23 (UniCredit Bank Hungary Zrt) had recorded a downward trend throughout the period. B23 was not cost efficient. The bank B18 (Raiffeisen Bank Zrt), after the initial oscilation during the financial crisis, began to be directed toward the growth of the technical efficiency. Such detailed analysis can be done for different efficiency scores and the detailed analysis of individual banks may be done as well.

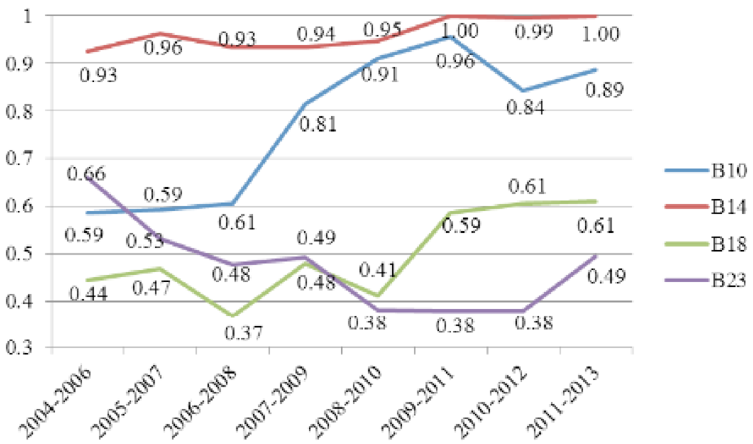


Figure 1–10 Moving window analysis for Hungarian banks

1.10 MPI and its components

The Malmquist productivity index ($\mathrm{MPI}^{t,t+1}$) represents the change in productivity in two examined periods t and $(t+1)$. This index was defined in section 1.3.4 in equation (1.13). The development of this index is analyzed and the affect of two components of the index - $\mathrm{EFFC}^{t,t+1}$ (change in technical efficiency) and $\mathrm{TECHC}^{t,t+1}$ (change in production technology) is closely investigated. Both indices have been defined in equation (1.14). The software *DEA Solver v. 8.0* was used to calculate values of these indices. The results are presented in Table 1−15 for the time period 2004-2013.

The results of the MPI index and its components are sorted according to the inter-quartile values of the MPI in descending order. In the column Q the number of the quartile is presented:

- $Q = 1$ represents 1. group - $\mathrm{MPI}^{2004,2013} < 0.87341$;
- $Q = 2$ represents 2. group - $0.87341 \leq \mathrm{MPI}^{2004,2013} < 1.09493$;
- $Q = 3$ represents 3. group - $1.09493 \leq \mathrm{MPI}^{2004,2013} < 2.31488$;
- $Q = 4$ represents 4. group - the rest of banks.

Table 1–15 The evaluation of change for banks by MPI, EFFC and TECHC

DMU	Bank name	Country	MPI	Q	EFFC	TECHC
B6	Ceskomoravska Zarucni a Rozvojova Banka a.s.	CZ	**8.869**	4	1.452	**6.107**
B16	PPF banka a.s.	CZ	**5.574**	4	1.000	**5.574**
B20	Stavebni Sporitelna Ceske Sporitelny as	CZ	2.985	4	1.000	**2.985**
B19	Raiffeisen stavebni sporitelna AS	CZ	2.770	4	1.000	**2.770**
B1	Bank Ochrony Srodowiska SA - BOS SA	PL	2.517	4	**2.134**	1.179
B13	Nordea Bank Polska SA	PL	2.474	4	1.002	2.468
B10	K&H Bank Zrt	HU	2.156	3	1.701	1.267
B9	J&T Banka as	CZ	2.126	3	0.943	**2.254**
B18	Raiffeisen Bank Zrt	HU	2.026	3	0.740	**2.736**
B22	Unicredit Bank Czech Republic and Slovakia AS	CZ	1.985	3	0.984	**2.017**
B12	mBank SA	PL	1.412	3	0.966	1.461
B3	Bank Zachodni WBK S.A.	PL	1.296	3	1.298	0.999
B21	Tatra Banka a.s.	SK	1.095	3	0.687	1.594
B8	ING Bank Slaski S.A. - Capital Group	PL	1.087	2	1.049	1.036
B23	UniCredit Bank Hungary Zrt	HU	1.012	2	0.830	1.220
B15	OTP Banka Slovensko, as	SK	0.914	2	1.478	0.618
B14	OTP Bank Plc	HU	0.893	2	1.000	0.893
B2	Bank Polska Kasa Opieki SA-Bank Pekao SA	PL	0.888	2	0.839	1.058
B5	Ceska Sporitelna a.s.	CZ	0.877	2	1.000	0.877
B17	Prima banka Slovensko a.s.	SK	0.870	1	1.118	0.778
B7	Ceskoslovenska Obchodni Banka A.S.- CSOB	CZ	0.794	1	1.000	0.794
B25	Vseobecna Uverova Banka a.s.	SK	0.697	1	0.755	0.923
B4	BNP Paribas Bank Polska SA	PL	0.540	1	0.574	0.942
B24	UniCredit Bank Slovakia a.s.	SK	0.512	1	0.530	0.965
B11	MBank Hipoteczny SA	PL	0.237	1	1.000	0.237

The group with the biggest change in the productivity MPI ($Q = 4$) is represented by the group of four Czech and two Polish banks. The highest increase of the productivity for the period 2004-2013 had been in the Czech bank B6 (Moravian Guarantee and Development Bank Inc.) at 8.87 and in Polish bank B1 (Environmental Protection Bank SA - BOS SA) at 2.52. For both banks, it is typical that the reason of the productivity increased is the improvement of efficiency change and technical change. In case of B6, there is significantly big increase for TECHC (6.11). The higher catching-up effect (2.13) prevails for B1. Other banks in the group $Q = 4$ are characterized by stability in terms of approaching the benchmark production frontier. However, there had been an increase of this limit over the period.

The 3. group contains 7 banks. The range of the MPI is from 1.10 to 2.16. Improvement of the efficiency change and the technical progress is evident only for Hungarian bank B10 (K&H Bank Zrt). Polish bank B3 (Bank Zachodni WBK SA) is distinguished by approaching the efficient frontier (1.30), while the technological change is almost stable (0.999). Other banks in the group are rather falling behind the benchmark production frontier if there is the increase of the production boundary.

Another group, 2. group contains 6 banks, where the change of the productivity is in interval from 0.88 to 1.09. This means that it has very slight increasing tendency. More precisely, the tendency is actually decreasing. Slight increase of the productivity is seen for bank B8 (ING Bank Slaski SA - Capital Group; 1.09). This is due to the increasing effect of EFFC and TCHC. Banks B2 and B23 are moving away from the production boundary by improving of technological progress.

In the last group ($Q = 1$), the productivity decline is from 0.87 to 0.24. There are 6 banks. These banks are moving away from the efficient frontier mainly by wrong use of the technology in the production process. The declining efficiency change (B25, B4, B24) is helping too. Polish bank B11 (mBank Hipoteczny SA) is relatively stable and the distance from efficiency frontier is almost all the time same, but the bad technological progress reduce TECHC and also MPI up to 0.24.

Figure 1−11, 1−12 and 1−13 demonstrate the above results graphically. It shows that the highest productivity growth is for Czech banks, mainly due to the technological changes. Conversely, Slovak banks tend to have slightly declining productivity, due to the changes in the technological advances (usually less than 1) and the greater variability in terms of approaching the benchmark of the production frontier.

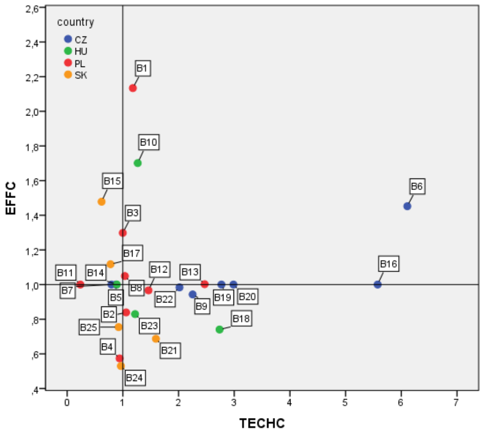


Figure 1–11 Classification of banks according to the level of the average value of MPI

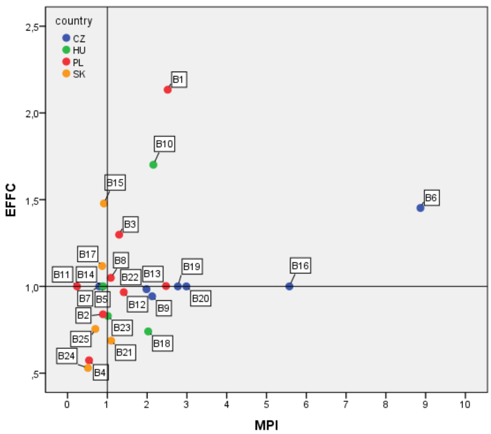


Figure 1–12 Classification of banks according to the level of the average value of EFFC

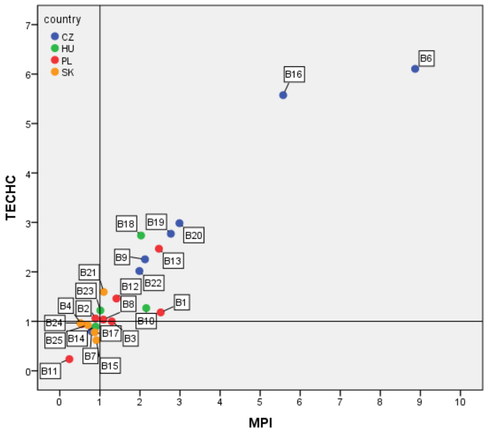


Figure 1–13 Classification of banks according to the level of the average value of TECHC

1.11 Conclusions

The first chapter of the book deals with the banks efficiency through the intermediation approach with the inclusion of the non-traditional activities into the output vector. The empirical analysis had focused on 25 banks from the V4 countries at the time period 2004-2013.

On the basis of characteristics of the development and transformation of the banking system in the V4 countries, the basic analytical tools have been specified for calculating and assessing the efficiency of banks using non-parametric approach data modeling. The technical, cost and allocative efficiency have been examined. These efficiencies have been obtained for input-oriented models with constant and variable returns to scale. Moving windows analysis was used to monitor the trends of the efficiencies. The Malmquist index was used for the productivity analysis. Also its components (change in technical efficiency and technological progress) were analyzed as well.

The research results give answers to presented research questions. Two DEA models - Model A and Model B; have been use to investigate the role of the non-traditional activities in evaluating the efficiency of banks. Model A contained the non-traditional activities as indicator of the non-interest income in its output vector. The results confirmed that the inclusion of the non-traditional activities into the output vector of banks provides an unbiased estimation of the average efficiency of the banks. It also helped to reduce the cost efficiency of banks with employing the best practice production method and to achieve the maximum outputs from minimum input costs. This was important to observe, especially during and immediately after the financial crisis. The test of statistical significance of differences in efficiency for both models was confirmed using parametric and non-parametric tests. A positive relationship between individual rankings of banks for the technical, cost and allocative efficiency have been detected. Analysis of cost efficiency for Model A (with the non-traditional activities) have showed that 3-5 banks were cost efficient throughout the analyzed period 2004-2013.

Analysis for the return to scale showed that it is appropriate to use Mode A with the variable returns to scale (LCS'scale deficiency index was from 55 % to 66.7 %). The average cost efficiency for the BCC-I model was in the range from 70.6 to 86.5 %. It was shown that the average cost inefficiency (19.8 %) was caused by 6.6 % of inappropriate selection of the optimal combinations of inputs given their prices and technology, and remaining is due to the wastage of inputs in the production process.

Another research question was how the bank size affects their efficiency. The first analysis of the development of the average share of the non-traditional activities to the total output in the period 2004-2013 showed an increasing trend among small banks and slightly decreasing trend for micro banks. In terms of the average technical efficiency both models under CRS have confirmed that the average TE_micro banks > TE_large banks > TE_medium banks > TE_small banks. This relationship also holds true for the average technical efficiency for Model A under VRS. Other empirical results have shown that the inclusion of non-traditional activities is increasing the technical and the cost efficiency. On other hand, there is relationship between the proportion - the higher proportion of the non-traditional activities in the total output gives the smaller difference between the technical and the cost efficiency for banks. Further analysis of the cost efficiency of Model A under VRS led to find that the most efficient banks are large banks, followed by micro banks, medium and small banks. The low cost inefficiency of large banks (7.2 %) is due by technical and allocative inefficiency, equally. The main source of technical inefficiency is the scale inefficiency compared to pure technical inefficiency. The main reason for the high average cost inefficiency for small banks (33.3 %) is mostly in technical inefficiency.

The comparison of banks efficiency by the country where the bank operates led to find that there are regional differences. The share of the non-traditional activities to total output was highest for Hungarian banks. However, this share have declined mostly during the financial crisis. In terms of the number of banks lying on the cost effective threshold (depending on model with CRS) it can be stated that most of them were from the Czech Republic and least of them were from the Slovak Republic in the time period 2004-2013. The number of cost efficient banks have increased for model under variable returns to scale and have stood in the Czech Republic - 54 %, Hungary - 33 %, Poland - 31 % and 22 % of the Slovak Republic.

The analysis of the dynamics indicators for the technical efficiency of Model A under VRS was performed using a 3-year moving windows analysis in the time period from 2004 to 2013 for Hungarian banks. Outcomes have shown that the most technically efficient bank was OTP Bank Plc. A progressive increase of technical efficiency was seen for K & H Bank Zrt and decline was identified for UniCredit Bank Hungary Zrt, which was not cost efficient.

The change of banks productivity was analyzed using Malmquist index and its components throughout the period. These results made it possible to classify the banks into four groups based on the values of quartiles productivity changes. In the group with the highest change in productivity (higher than 2.47) were four Czech and two Polish banks due to the particularly marked improvement in the technological process. Also one Polish bank (B1) and one Czech bank (B6) were improving in efficiency changes. Banks with declining productivity were mostly from Slovakia and the range was from 0.24 to 0.87. The main source of this adverse trend has been the backward technological progress and the growing distance from the efficient frontier.

Finally it may be said that the inclusion of the non-traditional activities into the output vector of banks provides the accurate non-deflected efficiency evaluation. Also it is better and preferable to use DEA model with variable returns to scale for banks from the V4 in time period 2004-2013. All findings and results suggest that banking managers in the V4 countries should pay more attention to all inputs rather than just to choose the proper input mix given by prices. The average scale efficiency was 84.9 %. The results have also demonstrated that the technical inefficiency emanates due to managerial underperformance in controlling where is the waste of inputs for the production process and also due to the failure to operate with the optimum scale size. Moreover, the results have suggested that there are more opportunities for the technical efficiency gains - better utilization of existing resources by management or taking advantage of the scale economies. The most efficient banks were identified in the Czech Republic and least efficient banks were found in the Slovak Republic. The main source of decreasing productivity was primarily the deteriorating technological change.